ATLANTA PUBLIC SCHOOLS
Mathematics & Science Initiative
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Atlanta Public Schools

Teacher's Curriculum Supplement

Mathematics II: Unit 4
Statistics

GE Foundation

This document has been made possible by funding from the GE Foundation Developing Futures grant, in partnership with Atlanta Public Schools. It is derived from the Georgia Department of Education Math II Framework and includes contributions from Georgia teachers. It is intended to serve as a companion to the GA DOE Math II Framework Teacher Edition.

Preface

We are pleased to provide this supplement to the Georgia Department of Education's Mathematics II Framework. It has been written in the hope that it will assist teachers in the planning and delivery of the new curriculum, particularly in these first years of implementation. This document should be used with the following considerations.

- The importance of working the tasks used in these lessons cannot be overstated. In planning for the teaching of the Georgia Performance Standards in Mathematics teachers should work the tasks, read the teacher notes provided in the Georgia Department of Education's Mathematics II Framework Teacher Edition, and *then* examine the lessons provided here.

- This guide provides day-by-day lesson plans. While a detailed scope and sequence and established lessons may help in the implementation of a new and more rigorous curriculum, it is hoped that teachers will assess their students informally on an on-going basis and use the results of these assessments to determine (or modify) what happens in the classroom from one day to the next. Planning based on student need is much more effective than following a pre-determined timeline.

- It is important to remember that the Georgia Performance Standards provide a balance of concepts, skills, and problem solving. Although this document is primarily based on the tasks of the Framework, we have attempted to help teachers achieve this all important balance by embedding necessary skills in the lessons and including skills in specific or suggested homework assignments. The teachers and writers who developed these lessons, however, are not in your classrooms. It is incumbent upon the classroom teacher to assess the skill level of students on every topic addressed in the standards and provide the opportunities needed to master those skills.

- In most of the lesson templates, the sections labeled *Differentiated support/enrichment* have been left blank. This is a result of several factors, the most significant of which was time. It is hoped that as teachers use these lessons, they will contribute their own ideas, not only in the areas of differentiation and enrichment, but in other areas as well. Materials and resources abound that can be used to contribute to the teaching of the standards.

On the topic of differentiation, it is critical to reiterate that many of the strategies used in a standards-based mathematics classroom promote differentiation. These strategies include

- the use of rich tasks with multiple points of entry and more than one path to a solution,
- flexible grouping of students,
- multiple representations of mathematical concepts,
- writing in mathematics,
- monitoring of progress through on-going informal and formative assessments, and
- analysis of student work.

We hope that teachers will incorporate these strategies in each and every lesson.

It is hoped that you find this document useful as we strive to raise the mathematics achievement of all students in our district and state. Comments, questions, and suggestions for inclusions related to this document may be emailed to Dr. Dottie Whitlow, Executive Director, Mathematics and Science Department, Atlanta Public Schools, dwhitlow@atlantapublicschools.us.

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Explanation of the Terms and Categories Used in the Lesson Template

Task: This section gives the suggested number of days needed to teach the concepts addressed in a task, the task name, and the problem numbers of the task as listed in the Georgia Department of Education's Mathematics II Framework Teacher Edition (GaDOE TE).

In some cases new tasks or activities have been developed. These activities have been named by the writers.

Standard(s): Although each task addresses many Math II standards and uses mathematics learned in earlier grades, in this section, only the key standards addressed in the lesson are listed.

New Vocabulary: Vocabulary is listed here the *first* time it is used. It is strongly recommended that teachers, particularly those teaching Math Support, use interactive word walls. Vocabulary listed in this section should be included on the word walls and previewed in Math Support.

Mathematical concepts/skills: Major concepts addressed in the lesson are listed in this section whether they are Math II concepts or were addressed in earlier grades or courses.

Prior knowledge: Prior knowledge includes only those topics studied in previous grades or courses. It does not include Math II content taught in previous lessons.

Essential Question(s): Essential questions may be daily and/or unit questions.

Suggested materials: This is an attempt to list all materials that will be needed for the lesson, including consumable items, such as graph paper; and tools, such as graphing calculators and compasses. This list does not include those items that should always be present in a standards-based mathematics classroom such as markers, chart paper, and rulers.

Warm-up: A suggested warm-up is included with every lesson. Warm-ups should be brief and should focus student thinking on the concepts that are to be addressed in the lesson.

Opening: Openings should set the stage for the mathematics to be done during the work time. The amount of class time used for an opening will vary from day-to-day but this should not be the longest part of the lesson.

Worktime: The problem numbers have been listed and the work that students are to do during the worktime has been described. A student version of the day's activity follows the lesson template in every case. In order to address all of the standards in Math II, some of the problems in some of the original GaDOE tasks have been omitted and less time consuming activities have been substituted for those problems. In many instances, in the student versions of the tasks, the writing of the original tasks has been simplified. In order to preserve all vocabulary, content, and meaning it is important that teachers work the original tasks as well as the student versions included here.

Teachers are expected to both facilitate and provide some direct instruction, when necessary, during the work time. Suggestions related to student misconceptions, difficult concepts, and deeper meaning have been included in this section. However, the teacher notes in the GaDOE Math II Framework are exceptional. In most cases, there is no need to repeat the information provided there. Again, it is imperative that teachers work the tasks and read the teacher notes that are provided in GaDOE support materials.

Questioning is extremely important in every part of a standards-based lesson. We included suggestions for questions in some cases but did not focus on providing good questions as extensively as we would have liked. Developing good questions related to a specific lesson should be a focus of collaborative planning time.

Closing: The closing may be the most important part of the lesson. This is where the mathematics is formalized. Even when a lesson must be continued to the next day, teachers should stop, leaving enough time to "close", summarizing and formalizing what students have done to that point. As much as possible students should assist in presenting the mathematics learned in the lesson. The teacher notes are all important in determining what mathematics should be included in the closing.

Homework: In some cases, homework suggestions are provided. Teachers should use their resources, including the textbook, to assign homework that addresses the needs of their students.

Homework should be focused on the skills and concepts presented in class, relatively short (30 to 45 minutes), and include a balance of skills and thought-provoking problems.

Differentiated support/enrichment: On the topic of differentiation, it is critical to reiterate that many of the strategies used in a standards-based mathematics classroom promote differentiation. These strategies include

- the use of rich tasks with multiple points of entry and more than one path to a solution
- flexible grouping of students
- multiple representations of mathematical concepts
- writing in mathematics
- monitoring of progress through on-going informal and formative assessments
- and analysis of student work.

Check for understanding: A check for understanding is a short, focused assessment-a ticket out the door, for example. There are many good resources for these items, including the GaDOE culminating task at the end of each unit and the *Mathematics II End-of-Course Study Guide*. Both resources can be found on-line at www.georgiastandards.org, along with other GaDOE materials related to the standards. Problem numbers from the GaDOE culminating task have been listed with the appropriate lessons in this document.

Resources/materials for Math Support: Again, in some cases, we have provided materials and/or suggestions for Math Support. This section should be personalized to your students, class, and/or school, based on your resources.

Unit 4 Content Notes

As students work through this unit, it is important that, given a distribution, they are able to plot the distribution; describe its shape; and choose, compute, and interpret appropriate summary measures. Many of the concepts related to these skills have been addressed in earlier grades and should be used in conjunction with the new materials presented in this unit. These content notes revisit a few of those ideas.

- A distribution is **uniform** (or rectangular) when all values occur with the same frequency. In a uniform distribution the mean and the median are equal.

- A distribution is **normal** when it is symmetric, unimodal, and bell-shaped. The mean and standard deviation are good summary measures for a normal distribution.

- A distribution is **skewed** when much of the data bunches or clusters at one end and a long tail stretches out in either direction. The distribution is said to be skewed in the direction of the tail. When a distribution is **left skewed,** the mean is smaller than the median. If the distribution is **right skewed**, the mean is greater than the median. The median and interquartile range are often considered better measures of a skewed distribution. However, which summary measures are better, ultimately depends on context. Suppose, for example, that we consider salaries for players on a major league baseball team. Despite the fact that the distribution is skewed right (most players' salaries cluster to the left but there are a few salaries that are very high, creating a tail to the right) the better measure of center depends on who is asking the question and for what reason. The players may benefit more by considering the median as the "typical" players' salary whereas the owners of the team may benefit more from considering the mean.

- When a distribution has one peak or "mound", as in the case of a normal or skewed distribution, the distribution is said to be **unimodal.** Distributions with two peaks are described as **bimodal.** Bimodal distributions often represent more than one population. It may not be useful to find measures of center and spread for bimodal distributions.

- When analyzing data sets, it may be wise to consider more than one set of summary measures. Different information can be gained by examining the mean and the standard deviation as opposed to the median and the interquartile range. Information gained from both sets of summary measures may be useful.

Copies of *Statistics: The Art and Science of Learning from Data,* by Christine Franklin and Alan Agresti, have been provided to Math I and Math II teachers in Atlanta Public Schools. As preparation for this unit, we suggest that teachers read at least Chapters 1 and 2 of this text.

The use of graphing calculators or a statistical software program is a *must* for teaching this unit. A list of data related calculator operations is provided beginning on page 61 of this document.

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Unit 1 Timeline

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Task 6: Your Class, Celebrities, and the Golden Ratio	2 days

Task Notes

The importance of working the tasks used in these lessons cannot be overstated. In planning for the teaching of the Georgia Performance Standards in Mathematics, teachers should work the Student Tasks, read any corresponding teacher notes provided in the Georgia Department of Education's Mathematics II Framework Teacher Edition, and *then* examine the lessons provided here.

The tasks provided in this Supplement are based on the content of Unit 4 of the Georgia Department of Education's Mathematics II Framework. We suggest, as always, that teachers use this Supplement along with the GaDOE Teacher Edition and the *Mathematics II End-of-Course Study Guide* which can be found on-line at www.georgiastandards.org.

Task 1: Measures of Spread
The big ideas presented in this task include:

- calculation of variance and standard deviation;
- development of initial understandings of standard deviation as a measure of spread; and,
- using measures of spread to help answer questions related to data.

The initial GaDOE task has been revised to include graphs of data and additional questions that might be answered using these graphs. Logistical changes such as the numbering of problems and summary rows for tables have also been made.

The homework assigned at the end of this task is extremely important. It provides opportunities to establish a model for analyzing data, compare groups, and familiarize students with the graphing calculator. Depending on the expertise of your students in these areas, it may be necessary to treat this homework assignment as a task. See the notes in the *Homework* section of the lesson plans on page 12.

Task 2: "Picturing" Standard Deviation Using Dotplots
The big ideas presented in this task include:

- the mean as a balancing point
- calculation of variance and standard deviation
- understanding of the standard deviation as a measure of spread

Problems 2 – 5 of the original GaDOE task have been revised to ensure that students investigate sets of data other than symmetric sets, with the given variance and standard deviation.

Task 3: Frequency, Mean, and Standard Deviation
The big ideas presented in this task include:

- calculations of the mean and standard deviation of a frequency distribution, and
- development of formulas for calculating the mean and standard deviation of frequency distributions.

Problems of the original GaDOE task have been numbered and in some cases re-worded for clarification. Problem 6 was revised to include an additional mistake. In her work, Maria has squared the frequency of each data value as well as the deviation from the mean. This is a mistake that students commonly make and thus was added to the task.

Task 4: Normal Distributions
The big ideas presented in this task include:

- estimating the mean and standard deviation of a normal curve
- determining whether a curve is approximately normal using the Empirical Rule
- using appropriate technology for data analysis

This is an original task. It does not appear in the GaDOE Mathematics II Framework. It addresses the content of Task 4 of the Framework.

Task 5: Does a Sample Really Represent a Population?
The big ideas presented in this task include:

- samples are often used to help determine characteristics of a population
- sample means vary from one sample to the next
- a distribution of sample means has less variability than the corresponding population distribution
- there is less variability among sample means as sample size increases
- the shape of a sampling distribution becomes more normal as the size of the samples increases despite the shape of the population distribution
- the mean of the sample means is a good approximation of the population mean
- if the standard deviation of the sample means is $S_{\bar{x}}$, then the standard deviation of the population can be estimated by $S_{\bar{x}}\sqrt{n}$, where n is the sample size

This is an original task. It does not appear in the GaDOE Mathematics II Framework. It addresses the content of tasks 7 and 8 of the Framework.

Task 6: Your Class, Celebrities, and the Golden Ratio
The big ideas presented in this task include:

- describing a distribution using shape, center, and spread (variability)
- posing questions that can be answered by data
- using data to comparing groups

This task was adapted from Tasks 6 and 7 of the GaDOE Mathematics II Framework.

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Task 1: Measures of Spread

Mathematics II
Task 1: *Measures of Spread* **Day 1/1**
(GaDOE TE Task 1)

Standard(s): MM2D1: Using sample data, students will make informal inferences about population means and standard deviations

b. Understand and calculate the means and standard deviations of sets of data
c. Use means and standard deviations to compare data sets

New vocabulary: variance, standard deviation

Mathematical concepts/skills:

- calculation of mean, mean absolute deviation, variance, and standard deviation
- developing conceptual understanding of measures of spread

Prior knowledge:

- absolute value
- mean
- measures of spread including range, interquartile range, and mean absolute deviation

Essential question(s): How do measures of spread help me answer questions related to data?

Suggested materials: calculators

Warm-up: Post the following.

> *In this task, you will compare two sets of data, one of which is shown below. Use this data set to find and discuss measures of center and measures of spread you have learned to this point.*
>
> *90, 90, 80, 100, 99, 81, 98, 82*

Opening: The warm-up is designed to help students review two measures of center - mean and median - and the three measures of spread - range, interquartile range, and mean absolute deviation - learned in previous grades. (*Note: The mode of a set of data is a measure of frequency rather than a measure of center.)*

Discuss the warm-up, allowing students to give explanations of mean, median, range, and interquartile range. Calculation of the mean absolute deviation is reviewed in the task itself so it is not necessary to extend the opening to this discussion if students do not remember the formula.

In this task students will calculate standard deviation and begin to use this measure of spread to compare sets of data. Explain to students that, in this task, they will review mean absolute deviation and investigate a new measure of spread commonly used by statisticians.

Worktime: Students should complete problems 1- 10 of the task.
After students have had time to complete problem 4, have a brief, whole-class discussion to ensure that students are performing calculations correctly. Make an **Anchor Chart** including notation and formulas for mean absolute deviation, variance, and standard deviation. This Anchor Chart will be particularly useful for subsequent lessons.

Closing: Note that there are two different questions that students must answer in problems 9 and 10. The first is "Who should get the math award?" The second is, "Who should the teacher take to the math competition?" The questions should spur some significant debate. Most students will realize that Brianna is more consistent and may choose her in both situations. Others may choose Brianna for the award because she is more consistent but Bryce for the competition because he seems to have the potential for making a higher score-"hitting it out of the park"- so to speak.

Remember that this is students' initial introduction to standard deviation. In no way should we expect them to completely understand the concept at this point. In this task, students should do the following:

- learn to calculate variance and standard deviation;
- recognize that Brianna's scores have less variability than Bryce's scores, and understand how this relates to the fact that the standard deviation of Brianna's scores is much smaller than that of Bryce's scores; and
- use measures of spread to help answer questions related to data.

See GaDOE teacher notes.

Homework: Students may need extra practice in calculating mean absolute deviation, variance, and standard deviation.

The homework that follows this student task is adapted from the GaDOE TE and is extremely important. It provides opportunities to establish a model for analyzing data, compare groups, and familiarize students with the graphing calculator. Depending on the expertise of your students in these areas, it may be necessary to treat this homework assignment as a task. If you choose to spend a class period on the assignment, we suggest students calculate the means, variances, and standard deviations of the two data sets as the homework assignment following *Task 1*. They can check their work in class by using the graphing calculator on *Problem 2*. We strongly suggest that students enter lists, calculate summary measures and draw boxplots using the graphing calculator.

Differentiated support/enrichment:

Check for Understanding:

Resources/materials for Math Support: Students should preview:

- measures of center, including mean and median;
- computing a 5-number summary, drawing boxplots, and determining the interquartile range; and
- calculating the mean absolute deviation.

Mathematics II
Measures of Spread
Day 1 Student Task

Your teacher has a problem and needs your input. She has to give one math award this year to a deserving student, but she can't make a decision. Here are the test grades for her two best students:

Bryce: 90, 90, 80, 100, 99, 81 98, 82

Brianna: 90, 90, 91, 89, 91, 89, 90, 90

1. Write down which of the two students should get the math award and discuss why they should be the one to receive it.
2. Represent Bryce's grades using a dotplot.
3. Calculate the mean of Bryce's distribution.
4. Calculate the mean deviation, variance, and standard deviation of Bryce's distribution.

The formulas for mean absolute deviation, variance, and standard deviation are below. Fill out the table to help you calculate them by hand.

Mean absolute deviation: $MAD = \frac{\sum_{i=1}^{n} |X_i - \mu|}{n}$, where x_i represents the data values in the distribution and μ represents the mean of a population.

Variance: $\sigma^2 = \frac{\sum_{i=1}^{n} (X_i - \mu)^2}{n}$, where x_i represents the data values in the distribution, μ represents the mean of a population and n represents the number of values in the population.

Standard deviation: $\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}}$, which is the square root of the variance.

X_i for Bryce	$x_i - \mu$	$\lvert x_i - \mu \rvert$	$(x_i - \mu)^2$
90			
90			
80			
100			
99			
81			
98			
82			
Totals	$\sum_{i=1}^{n} (x_i - \mu) =$	$\sum_{i=1}^{n} \lvert x_i - \mu \rvert =$	$\sum_{i=1}^{n} (x_i - \mu)^2 =$

Mean absolute deviation for Bryce:

Variance for Bryce:

Standard deviation for Bryce:

5. What do these *measures of spread* tell you?

6. Represent Brianna's grades using a dotplot.

7. Calculate the mean of Brianna's distribution.

Calculate the mean absolute deviation, variance, and standard deviation of Brianna's distribution.

x_i for Brianna	$x_i - \mu$	$\lvert x_i - \mu \rvert$	$(x_i - \mu)^2$
90			
90			
91			
89			
91			
89			
90			
90			
Totals	$\sum_{i=1}^{n}(x_i - \mu) =$	$\sum_{i=1}^{n} \lvert x_i - \mu \rvert =$	$\sum_{i=1}^{n}(x_i - \mu)^2 =$

Mean deviation for Brianna:

Variance for Brianna:

Standard deviation for Brianna:

8. What do these measures of spread tell you?

9. Based on your calculations, write down which of the two students should get the math award and discuss why they should be the one to receive it.

10. Suppose your teacher needs to take either Bryce or Brianna to the next math competition. Who should she choose and why?

Mathematics II
Measures of Spread
Day 1 Homework

Mr. Turner has two Math 2 classes. With one class, he lectured and the students took notes. In the other class, the students worked in small groups to solve math problems. After the first test, Mr. Turner recorded the student grades to determine if his different styles of teaching might have impacted student learning.

Class 1: 80, 81, 81, 75, 70, 72, 74, 76, 77, 77, 77, 79, 84, 88, 90, 86, 80, 80, 78, 82

Class 2: 70, 90, 88, 89, 86, 86, 86, 86, 84, 82, 77, 79, 84, 84, 84, 86, 87, 88, 88, 88

1. Draw a dotplot for each set of data and describe the shapes of the two distributions.

2. Analyze Mr. Turner's student grades by calculating the mean, variance, and standard deviation. What might these measures of center and spread tell Mr. Turner? Explain your thinking.

3. Calculate a 5-number summary and draw a boxplot for each distribution. Use this information to write 5 true statements comparing the grades of the two classes.

4. Compare the information Mr. Turner gains using the mean and standard deviation as his measures of center and spread to the information gained from the 5-number summary and the boxplots. Discuss why it might be helpful to analyze distributions using more than one set of summary measures.

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Task 2: "Picturing" Standard Deviation Using Dotplots

Mathematics II
Task 2: *Picturing Standard Deviation Using Dotplots* **Day 1/1**
(GaDOE TE Task 2)

Standard(s): MM2D1: Using sample data, students will make informal inferences about population means and standard deviations

b. Understand and calculate the means and standard deviations of sets of data
c. Use means and standard deviations to compare data sets

New vocabulary:

Mathematical concepts/skills:

- the mean as a balancing point
- representing data using dotplots
- creating sets of data with a given variance and standard deviation
- calculation of mean, variance, and standard deviation
- developing conceptual understanding of variance and standard deviation

Prior knowledge:

- the mean as a balancing point
- representing data using dotplots

Essential question(s): What does the standard deviation tell me about a set of data?

Suggested materials:

Warm-up: Have students compare homework from the previous lesson with a partner. Tell them to be prepared to ask questions related to any problems they still do not understand.

Opening: Discuss the homework for Task 1 as a means of reviewing and setting the stage for the current lesson.

Read and discuss *Problem 1* of the task with students. It may be necessary to remind students that the mean is a balancing point for a give set of data. For example, a deviation of x units from the mean in the positive direction requires a deviation of x units from the mean in the negative direction if a set of data is to retain the same mean.

Worktime: Students should complete problems 1- 8 of the task.

In the GaDOE teacher notes, only one solution is presented for problems 2 – 5. In each case, that solution is a symmetric set of data in which half of the values are moved x units from the mean to the right and half of the values are moved x units from the mean to the left. The purpose of this task is to help students begin to understand what is meant by a given standard deviation. Therefore, it is important that students see more than one pattern in each of problems 2 -5. For example, in problem 2, students may have a set of data with a mean of 5 and a variance and standard deviation of 1 by listing the values 4, 4, 4, 6, 6, 6. However, they may also have a mean

of 5 and a variance and standard deviation of 1 with the data set 3, 5, 5, 5, 6, 6. Problems 2 – 5 of the GaDOE ask have been revised to ask for at least two different sets of data with the same mean that meet the required conditions.

Closing: In the case of problems 2 – 5, students should discuss their methods for finding more than one set of data that will meet the required conditions. Students should understand that sets of data other than symmetric sets can yield the required variances and standard deviations.

Homework: The homework that follows this student task is from the GaDOE TE.

Differentiated support/enrichment:

Check for Understanding:

Resources/materials for Math Support: Students should review calculating variance and standard deviation.

Mathematics II
"Picturing" Standard Deviation Using Dotplots
Day 1 Student Task

1. Create a set of 6 data points such that the variance and standard deviation are zero. Make a dotplot of the distribution.

2. Create a set of 6 data points such that the variance and standard deviation are one. Make a dotplot of the distribution. Can this be done in more than one way other than varying the mean? Explain your thinking.

3. Create at least two different sets of 6 data points, both with the same mean, such that the variance is four and the standard deviation is two. Make dotplots of the distributions.

4. Create at least two different sets of 6 data points such that the variance is four, the standard deviation is two, and the mean is seven. Make dotplots of the distributions.

5. Create at least two sets of 6 data points such that the variance is sixteen, the standard deviation is four, and the mean is ten. Make dotplots of the distributions.

6. Describe the process you used to come up with your answers.

7. What is the relationship between the standard deviation and variance?

8. What does the standard deviation measure?

Mathematics II
"Picturing" Standard Deviation Using Dotplots"
Day 1 Homework

1. Create a set of at least 5 data points such that the standard deviation is exactly 5.

2. Create a set of at least 5 data points such that the IQR is less than the standard deviation.

3. Create a set of at least 5 data points such that the IQR is the same as the standard deviation.

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Task 3: Frequency, Mean, and Standard Deviation

Mathematics II
Task 3: *Frequency, Mean, and Standard Deviation* **Day 1/1**
(GaDOE TE Task 3)

Standard(s): MM2D1: Using sample data, students will make informal inferences about population means and standard deviations

b. Understand and calculate the means and standard deviations of sets of data

New vocabulary:

Mathematical concepts/skills:

- calculating the mean and standard deviation using a frequency distribution

Prior knowledge:

- frequency tables
- frequency distributions
- calculation of the mean

Essential question(s): How do I most efficiently calculate the mean and the standard deviation when data values have frequencies greater than one?

Suggested materials:

Warm-up: Post the following:

Calculate the mean of the given data set.

2,2,2,2,2,3,3,3,3,3,3,3,3,3,4,4,4,4,4,5,5,5,5,5,5,5,5,5,5

Opening: Have students discuss how they found the mean of the given set of data. Students have worked with frequency distributions and represented them in frequency tables for a number of years. Hopefully some students will have represented the data for this problem in a frequency table and multiplied data values by their frequencies rather than adding each data value individually. Look for students who are taking this approach and have them discuss their work.

Worktime: After completing the warm-up (*Problem 1* of the task), students should complete problems 2 – 9.

This task offers students another opportunity to practice calculation of mean and standard deviation but this time, using a frequency table. *Problems 1 – 5* ask the same questions asked in the original GaDOE task. Teacher notes are applicable for these problems. In problem 6, not only does Maria not take the square root of the variance to obtain the standard deviation, but she squares the frequency as well as the deviation from the mean. This is a mistake that students commonly make and thus was added to the task.

Closing: In Problems 8 and 9 students are asked to write formulas for finding the mean and standard deviation of a frequency distribution. It is important that students are allowed to develop their own formulas and that these formulas be discussed and revised. There are various ways to write the formulas needed for these calculations. Two formulas for the mean and for the standard deviation are given below.

Mean: $\mu = \frac{\Sigma(X_i F_i)}{n}$, where X_i represents the i^{th} distinct data value, F_i is the frequency of the i^{th} data value, and n is the total number of data values.

Mean: $\mu = \frac{\sum_{i=1}^{n}(X_i F_i)}{\sum_{i=1}^{n} F_i}$, where X_i represents the i^{th} distinct data value, F_i is the frequency of the i^{th} data value, and n is the number of distinct data values.

Standard deviation: $\sigma = \sqrt{\frac{\Sigma(F_i(x_i-\mu)^2)}{n}}$, where X_i represents the i^{th} distinct data value, F_i is the frequency of the i^{th} data value, and n is the number of distinct data values.

Standard deviation: $\sigma = \sqrt{\frac{\sum_{i=1}^{n} F_i(x_i-\mu)^2}{\sum_{i=1}^{n} F_i}}$, where X_i represents the i^{th} distinct data value, F_i is the frequency of the i^{th} data value, and n is the number of distinct data values.

Homework: The homework that follows this student task is adapted from the GaDOE TE. Teacher notes beginning on page 104.

Differentiated support/enrichment:

Check for Understanding:

Resources/materials for Math Support: Students should review calculating variance and standard deviation.

Mathematics II
Frequency, Mean, and Standard Deviation
Day 1 Student Task

1. Calculate the mean of the given set of data by hand.

   2,2,2,2,2,3,3,3,3,3,3,3,3,3,4,4,4,4,4,5,5,5,5,5,5,5,5,5,5,5

2. Cody and Bernice both correctly calculated the mean of the data set in *Problem 1*, but it took Bernice a lot longer. Bernice added all of the values and divided the sum by 30. Cody told her that there was a quicker way to do it. Can you figure out how Cody did it? Show how Cody calculated the mean.

3. Cody's and Bernice's teacher asked the students to calculate the standard deviation of the data set in *Problem 1* by hand.

   2,2,2,2,2,3,3,3,3,3,3,3,3,3,4,4,4,4,4,5,5,5,5,5,5,5,5,5,5,5

Carl made a table, listed every data point, and used the formula that he learned two days ago, $\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}}$, to calculate the standard deviation.

Jessica finished the problem in half of the time that it took Carl because she listed her data as a frequency distribution and calculated the standard deviation in a slightly different manner. Show how Jessica calculated the standard deviation.

X_i	F_i = frequency
2	5
3	9
4	5
5	11

4. Jessica then taught Gabe her method for calculating the standard deviation. Gabe did not get the correct answer. He did the following:

$$\frac{5(2-3.5)^2+9(3-3.5)^2+5(4-3.5)^2+11(5-3.5)^2}{30}$$

What mistake(s) did Gabe make?

5. Jessica also taught Melody her method for calculating the standard deviation. Melody also did not get the correct answer. She did the following:

$$\frac{5(2-3.733)^2+9(3-3.733)^2+5(4-3.733)^2+11(5-3.733)^2}{4}$$

What mistake(s) did Melody make?

6. Jessica also taught Maria her method for calculating the standard deviation. Maria also did not get the correct answer. She did the following:

$$\frac{[5(2-3.733)]^2+[9(3-3.733)]^2+[5(4-3.733)]^2+[11(5-3.733)]^2}{30}$$

What mistake(s) did Maria make?

7. Use Cody's and Jessica's method to calculate the mean and the standard deviation of the frequency distribution below.

X_i	F_i
3	12
5	15
6	8
7	4

8. Make a formula for finding the mean of any frequency distribution. Let F_i stand for the frequency.

9. Make a formula for finding the standard deviation of a frequency distribution.

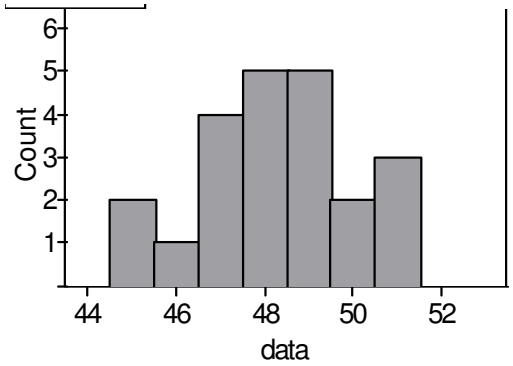
Mathematics II
Frequency, Mean, and Standard Deviation
Day 1 Homework

For each distribution below, calculate the mean and the standard deviation and then calculate the median and the interquartile range. Determine which set of measures (center and spread) would be more representative of the given distribution.

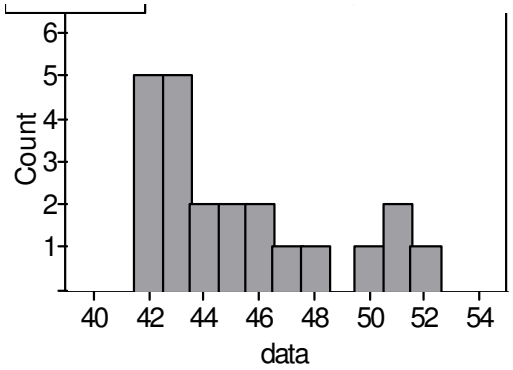
1.

x	F(x)
45	2
46	1
47	4
48	5
49	5
50	2
51	3

2.



3.



ATLANTA PUBLIC SCHOOLS
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Teacher's Curriculum Supplement

Mathematics II: Unit 4
Statistics

Task 4: Normal Distributions

Mathematics II
Task 4: ***Normal distributions*** **Day 1/2**

Standard(s): MM2D1: Using sample data, students will make informal inferences about population means and standard deviations

b. Understand and calculate the means and standard deviations of sets of data.
c. Use means and standard deviations to compare data sets.
d. Compare the means and standard deviations of random samples with the corresponding population parameters, including those population parameters for normal distributions. Observe that the different sample means vary from one sample to the next. Observe that the distribution of the sample means has less variability than the population distribution.

New vocabulary:

Mathematical concepts/skills:

- measuring to the nearest millimeter
- representing data using a dotplot
- determining the shape of a distribution

Prior knowledge:

- measuring to the nearest millimeter
- representing data using a dotplot
- determining the shape of a distribution

Essential question(s): What kind of distribution will result from repeated measurements of approximately congruent objects?

Suggested materials:

- centimeter rulers (1 per student or pair of students)
- tennis balls (1 per student or pair of students)
- small Post-It-Notes
- graph paper

Warm-up: On each desk, provide a tennis ball, a centimeter ruler, 2 small post-it notes and a piece of graph paper. Post the following:

> *Plan a method for measuring the diameter of a tennis ball using a centimeter ruler.*

Opening: Tell students that each person in the classroom is to take two measurements of the diameter of their tennis ball, measuring to the nearest millimeter. Students will have decided on different methods for measuring diameters. Have extra materials and tools (string, extra rulers, scissors, etc.) available for students to use, if they so choose, but do NOT discuss the methods decided upon for taking these measurements.

Worktime: Students should complete *Problems 1 – 5* of the task.

Closing: In Problem 2, students are asked what shape they think the class plot of measures of the diameter will take. Most students will probably think that all of the measures will be the same or very close. Once the plot has been created, it is important to discuss reasons for the variability in the distribution and particularly important to discuss any outliers that may exist.

Hopefully the class plot will be approximately normal. At this point students should describe the shape as unimodal, and approximately symmetric. Some may know that it is bell-shaped.

Homework:

Differentiated support/enrichment:

Check for Understanding:

Resources/materials for Math Support: Students should preview measuring to the nearest millimeter and describing distributions using center, shape, and spread.

Mathematics II
Normal Distributions
Day 1 Student Task

In today's lesson, we will examine a type of distribution that is very important in statistics – the **normal** distribution. We will begin by collecting data from an activity that usually generates this type of distribution.

1. Plan a method for measuring the diameter of a tennis ball using a centimeter ruler.

2. Using your plan, make two measurements for the diameter of the ball. Measure to the nearest millimeter.

3. Your class will compile the measurements taken by each student in a class dotplot. What shape do you think the plot will have? Explain your thinking.

4. Record your data on the class plot. When all students have recorded their data, copy the plot onto a sheet of graph paper.

5. How would you describe the shape of the plot?

6. How do you account for any variability in measurement?

Mathematics II
Task 4: ***Normal distributions*** **Day 2/2**

Standard(s): MM2D1: Using sample data, students will make informal inferences about population means and standard deviations

b. Understand and calculate the means and standard deviations of sets of data.
c. Use means and standard deviations to compare data sets.
d. Compare the means and standard deviations of random samples with the corresponding population parameters, including those population parameters for normal distributions. Observe that the different sample means vary from one sample to the next. Observe that the distribution of the sample means has less variability than the population distribution.

New vocabulary: normal distribution, point of inflection, Empirical Rule

Mathematical concepts/skills:

- estimating the mean and standard deviation of a normal curve
- determining whether a curve is approximately normal using the Empirical Rule
- using appropriate technology for data analysis

Prior knowledge:

- calculating the mean of a distribution

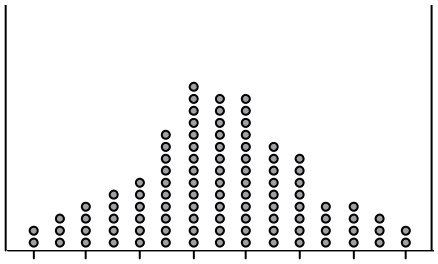
Essential question(s): How can I determine whether a distribution is a normal distribution?

Suggested materials:

- graphing calculator

Warm-up: Post the following:

Describe the shape of the distribution shown here?



Opening: Discuss the warm-up. Draw a smooth curve above the data to help students see the shape of the distribution. Students should use words and phrases such as *one mound*, *unimodal*, and *symmetric*. Discuss the shape of a **normal** distribution described in the task.

Worktime: Students should complete *Problems 7 -15* of the task.

As you monitor student work, you may need to help students estimate points of inflection. It is absolutely unnecessary to discuss derivatives or other calculus concepts at this point. Students simply need to understand that a point of inflection is where the curve changes from being concave down to concave up.

After students have had time to complete problem 13, stop and have a whole class discussion about problems 7 – 13. Discuss the Empirical Rule and examine how it relates to the estimates students have made to this point.

Closing: In problem 14, students are asked to decide whether their class distribution appears to be approximately normal based on the Empirical Rule and their *estimates* of the mean and standard deviation. In problem 15, students are asked to calculate the actual mean and standard deviation of the distribution using a graphing calculator and then decide whether the distribution is normal based on actual values and the Empirical Rule. Both of these problems should be discussed thoroughly.

It is important for students to understand that a distribution is approximately normal if percents fall close to those given in the Empirical Rule.

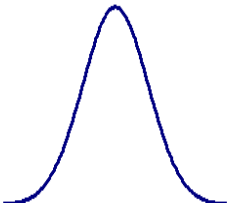
Homework: The homework that follows this student task was taken from the GaDOE TE page 95. Teacher notes are applicable.

Differentiated support/enrichment:

Check for Understanding:

Resources/materials for Math Support: Students should preview/review calculator operations related to statistics, determining points of inflection by "eyeballing", and ratios.

Mathematics II
Normal Distributions
Day 2 Student Task
Normal distributions are unimodal, symmetric, and bell-shaped as in the diagrams below.



In practice, distributions are not perfectly normal but many situations generate data that are *approximately* normal. Taking repeated measurements of the same object (as you did with the tennis ball) is one of those situations. Calculating means of random samples from the same population is another.

7. If a distribution were perfectly normal, what would you know about its measures of center?

8. Draw a smooth curve over the data values in your class distribution of the measures taken of a tennis ball. Does your distribution look somewhat normal? Are there any outliers in your class data? Can you determine reasons for any outliers that exist?

9. Approximate the mean of your distribution by drawing an arrow on the horizontal axis where you think the mean should be.

10. At the peak of a normal distribution, the curve is concave down. On each side of the curve, there is an inflection point – the point where the curve changes from being concave down to being concave up. On your plot, with your finger, trace one side of the curve to estimate the inflection point. Make a small mark on the axis below this point. **For a perfectly normal distribution, the distance between the mean and the point of inflection is one standard deviation.** What is this distance on your class plot? If your distribution is approximately normal, this is an estimation of the standard deviation of your data.

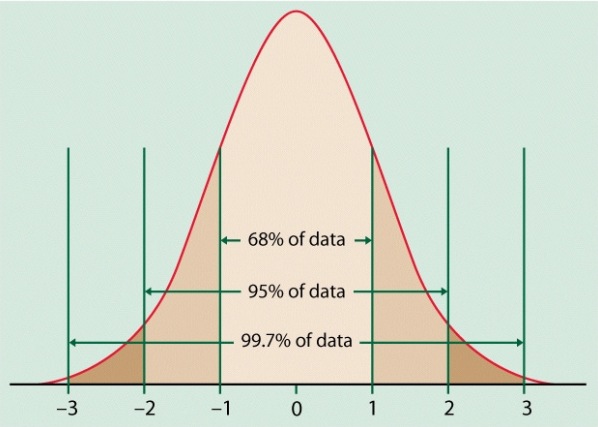
11. Using your estimate for the standard deviation obtained in *Problem 9*; make a mark on your plot that is one standard deviation on the opposite side of the mean from the mark drawn in *Problem 9*. Count the number of data values between your marks which represent one standard deviation below the mean and one standard deviation above the mean. How many values are there? What percent of the total number of data values do they represent?

12. Starting at the mean and using the approximate standard deviation that you found in *Problem 9*, mark 2 standard deviations to the right and 2 to the left on the horizontal axis. Count the number of data values between your marks. How many values are there? What percent of the total number of data values do they represent?

13. Starting at the mean and using the distance that you found in *Problem 9* as one standard deviation, mark 3 standard deviations to the right and 3 to the left on the horizontal axis. Count the number of data values between your marks. How many values are there? What percent of the total number of data values do they represent?

The Empirical Rule

The Empirical Rule tells us that if a distribution is normal, then approximately:
68% of the data will fall within one standard deviation of the mean,
95% of the data will fall within two standard deviations of the mean, and
99.7% of the data will fall within three standard deviations of the mean.

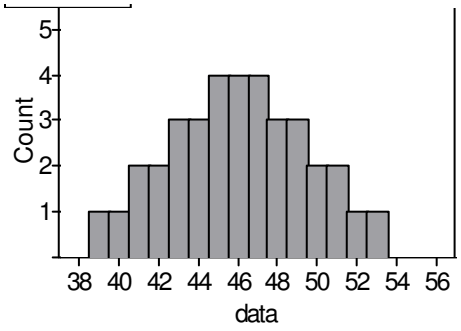


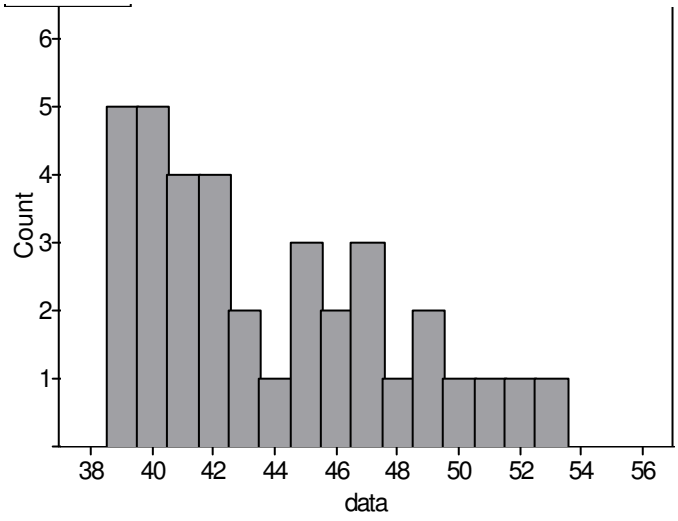
14. Examine the percents of data values found in Problems 10 – 12. Based on your estimates of mean and standard deviation and on the Empirical Rule above, do you think the class distribution is approximately normal? Justify your decision.

15. Now calculate the actual mean and standard deviation of the class data. How close were your approximations to these values? Using these values, what percent of the class data falls within one standard deviation of the mean? Two standard deviations? Three standard deviations? Do you feel the distribution is approximately normal based on this information? Justify your response.

Mathematics II
Normal Distributions
Day 1 Homework

Given the dotplots below, calculate the mean and standard deviation. Use these values to determine whether the distribution could be normal.





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Teacher's Curriculum Supplement

Mathematics II: Unit 4
Statistics

Task 5: Does a Sample Really Represent a Population?

Mathematics II
Task 5: ***Does a Sample Really Represent a Population?*** **Day 1/2**

Standard(s): MM2D1: Using sample data, students will make informal inferences about population means and standard deviations

b. Understand and calculate the means and standard deviations of sets of data.
c. Use means and standard deviations to compare data sets.
d. Compare the means and standard deviations of random samples with the corresponding population parameters, including those population parameters for normal distributions. Observe that the different sample means vary from one sample to the next. Observe that the distribution of the sample means has less variability than the population distribution.

New Vocabulary: population distribution, data distribution, sampling distribution, sample standard deviation

Mathematical concepts/skills:

- using simulations to create random samples
- using appropriate technology for analysis of data
- comparing the means and standard deviations of random samples with the corresponding population parameters
- observing that the different sample means vary from one sample to the next
- observing that the distribution of the sample means has less variability than the population distribution
- observe that the shape of the sampling distribution becomes more normal as n increases despite the shape of the population distribution

Prior knowledge:

- using simulations to choose random samples
- calculation of the mean

Essential question(s): Can I really use a sample to obtain useful information about a population?

Suggested materials:

- 1 standard deck of playing cards per student
- small post-it notes
- graphing calculators

Warm-up: Post the following:

In today's task, we will examine the card values in a regular deck of playing cards (52 cards – no jokers). Let aces represent 1, jacks represent 11, queens 12, and kings 13.

a. Make a dotplot of the population distribution. How would you describe the shape of this distribution?
b. What is the mean of the population?
c. What is the standard deviation?

Opening: Discuss the warm-up. Have students read the first paragraph of the task and then ask why it is necessary to use samples to answer questions related to populations. Ask students to share situations in which we use samples to answer questions about populations.

Worktime: After completing the warm-up (*Problem 1* of the task), students should complete problem 2 of the task. (Note on *Problem 2b*: Dealing with more than 60-70 data points is extremely tedious. If you have 30 students, each student will need to repeat step *a* only once, generating 2 samples of size 5.)

Once students have created the class histogram and have had time to answer *Problems 2d – 2f*, have a whole-class discussion of these questions, including students' explanations of their decisions.

For *Problems 2g* and *2f*, students should use graphing calculators to enter the sample means and their frequencies into lists and then calculate the mean and standard deviation of the sample means. To make this work more efficiently, you may want to have a student who finishes calculating and posting sample means quickly, begin to compile this data.

Closing: Discuss *Problems 2g* and *2h*. Be sure to discuss new vocabulary including distribution of sample means, mean of the sample means, and sample standard deviation. Big ideas that should be included in a closing discussion are:

- The distribution of sample means is approximately normal even though the original population distribution was uniform.
- The mean of the sample means is a good approximation of the population mean.
- The distribution of sample means has less variability than the population.

Homework: The homework that follows this student task is adapted from the GaDOE TE page 98.

Differentiated support/enrichment:

Check for Understanding:

Resources/materials for Math Support: Students should preview:

- new vocabulary listed for this lesson; and
- calculator operations including generating random integers, entering data into lists, creating plots, and calculating 1-variable statistics.

Mathematics II
Does a Sample Really Represent a Population?
Day 1 Student Task

We usually do not know much about a population when we take a sample. In fact, we may be sampling to learn about the population. We want to be able to find information from the sample and then use that information to make an inference about the population.

Let's start by considering characteristics (parameters) of a familiar population (a deck of playing cards). Then we will pick random samples to form a sampling distribution. We can compare the characteristics of our sampling distribution to the population to see if we should have any confidence in a sample as being representative of the population.

1. We will examine the card values in a regular deck of playing cards (52 cards – no jokers). Let aces represent 1, jacks represent 11, queens 12, and kings 13.

    a. Make a dotplot of the population distribution. How would you describe the shape of this distribution?

    b. What is the mean of the population?

    c. What is the standard deviation?

    d. Record your results on the Summary Table below.

2. Shuffle the cards.

    a. Take a random sample of size 5 from your deck and compute the mean. Record your results below.

    b. Replace the cards and repeat step *a* five times.

Sample #	1	2	3	4	5	6
Mean of Sample						

    c. Write each mean on a post-it note and add it to the class histogram.

    d. Do you think the **mean of the sample means** will be larger than, smaller than, or the same as the mean of the population of all the cards? _____ Explain your thinking.

    e. Estimate the standard deviation of the sample means in the histogram._____ Explain your thinking.

f. How would you describe the shape of the **distribution of sample means**? Justify your response.

g. Copy the histogram and calculate the mean and standard deviation. Record results in the Summary Table below.

h. If you were a researcher, you would probably obtain a sample once. If you were to randomly choose 5 cards, what is the probability that the sample mean of those 5 cards is the same as the population mean? (Hint: count the post-its with the mean and the total number of post-its.) _____

Distribution	Shape of the distribution	Mean of the distribution	Standard Deviation of the distribution	Probability that the mean of one sample is the same as the mean of the distribution
individual cards in the population				
the means for sample size = 5				
the means for sample size = 15				

Mathematics II
Task 5: ***Does a Sample Really Represent a Population?*** **Day 2/2**

Standard(s): MM2D1: Using sample data, students will make informal inferences about population means and standard deviations

b. Understand and calculate the means and standard deviations of sets of data.
c. Use means and standard deviations to compare data sets.
d. Compare the means and standard deviations of random samples with the corresponding population parameters, including those population parameters for normal distributions. Observe that the different sample means vary from one sample to the next. Observe that the distribution of the sample means has less variability than the population distribution.

New Vocabulary: population distribution, data distribution, sampling distribution, sample standard deviation

Mathematical concepts/skills:

- using simulations to choose random samples
- using appropriate technology for analysis of data
- comparing the means and standard deviations of random samples with the corresponding population parameters
- observing that the different sample means vary from one sample to the next
- observing that the distribution of the sample means has less variability than the population distribution
- observing that there is less variability among sample means as sample size increases
- observe that the shape of the sampling distribution becomes more normal as n increases despite the shape of the population distribution
- estimating the mean and standard deviation of a population using the mean and standard deviation of a distribution of sample means

Prior knowledge:

- using simulations to choose random samples
- calculation of the mean

Essential question(s): Can I really use a sample to obtain information about a population?

Suggested materials:

- small post-it notes
- graphing calculators

Warm-up: Post the following:

> *On your calculator, enter:*
> - *Math*
> - *PRB*
> - *randInt(1, 13, 15)*
> - *ENTER*
>
> *Discuss with your partner how the list of numbers you have generated relates to the population you began discussing in the previous lesson.*

Opening: Discuss the warm-up. Hopefully students will realize the set of 15 numbers generated by their calculator could be used to represent a sample of 15 cards chosen from the deck of 52 cards considered as the population for this task.

Worktime: Students should complete problems 3 – 5 of the task.

In *Problem 3a* be sure students store their generated values in a list and compute means and standard deviations using their calculators as they go. This will make the task much more efficient.

(A warning about your calculators: Number generation on calculators is not truly random. The calculators are seeded to a specific number and often different students will obtain the same lists of "random" numbers. To prevent this from happening, seed your calculators *before* students start this activity. Most schools number their calculators. A good way to seed is to do so with the number of the calculator. Simply press (or have the students press) the *calculator number* + **rand** and then **ENTER**. (i.e. 23+ rand, ENTER). This will seed the calculators to a different numbers.

(Note on *Problem 3b*: Again, dealing with more than 60-70 data points is extremely tedious. If you have 30 students, each student will need to repeat step *a* only once, generating 2 samples of size 15.)

Once students have created the class histogram and have had time to answer *Problems 3d – 3f*, have a whole-class discussion of these questions, including students' explanations of their decisions.

For *Problems 3g* and *3f*, students should use graphing calculators to enter the sample means and their frequencies into lists and then calculate the mean and standard deviation of the sample means. To make this work more efficiently, you may want to have a student who finishes calculating and posting sample means quickly, begin to compile this data.

As you monitor student work, make sure students have the correct mean and standard deviation for the class data before allowing them to move on to problem 4.

Closing: Discuss *Problems 4* and *5* thoroughly. Big ideas that should be included in a closing discussion are:

- The distribution of sample means is approximately normal even though the original population distribution was uniform.
- The distribution of sample means has less variability than the population.
- There is less variability among sample means as sample size increases.
- The mean of the sample means is a good approximation of the population mean.
- If the standard deviation of the sample means is $s_{\bar{x}}$, then the standard deviation of the population can be estimated by $s_{\bar{x}}\sqrt{n}$, where *n* is the sample size.

Homework: *Problems 4 - 6* of the homework that follows this student task are adapted from the GaDOE TE page 101.

Differentiated support/enrichment:

Check for Understanding:

Resources/materials for Math Support: Students should preview:

- new vocabulary listed for this task
- calculator operations including generating random integers, entering data into lists, creating plots, and calculating 1-variable statistics
- creating sample distributions and analyzing those distributions in relation to the concepts explored in this task

Mathematics II
Does a Sample Really Represent a Population?
Day 2 Student Task

3. We want to repeat the simulation in *Problem 2* with larger samples, but let's use technology.

   a. With your calculator enter: math, PRB, randInt(1, 13, 15) sto L1. Press enter to produce 15 random numbers. (It is okay if you have repeats.) Now press stat, CALC, 1-Var Stats. Press enter to get the mean of this sample. Record the mean below.

   b. Repeat step *a* five times and record the means in the table below.

Sample #	1	2	3	4	5	6
Mean of Sample						

   c. Write each mean on a post-it note and add it to the class histogram for a sample size of 15.

   d. How would you describe the shape of this distribution of sample means? Justify your response.

   e. Do you think the mean of these sample means will be larger than, smaller than, or the same as the mean of the population of all the cards? _____
   Explain your thinking.

   f. Estimate the standard deviation of the sample means._____ Explain your thinking.

   g. Copy the histogram and calculate the mean and standard deviation. Record results in the Summary Table following *Problem 2*.

   h. If you were to randomly choose 15 cards, what is the probability that the sample mean of those 15 cards is the same as the population mean? (Hint: Count the post-its with the same mean as the population mean and the total number of post-its.)

4. We began our task by stating that we often use sampling to learn about a population. Our goal in conducting the simulations in Problems 2 and 3 is to verify that the techniques used by statisticians, at least in this case, are reasonable. The questions in this problem are intended to help you analyze the results of your sampling against a population with known parameters. Use the results recorded in your Summary Table to help you answer these questions.

    a. How do the mean and standard deviation of the first sample compare with the mean and standard deviation of the population?

    b. How do the mean and standard deviation of the second sample compare with the mean and standard deviation of the population?

    c. How do the mean and standard deviation of the second sample compare with the mean and standard deviation of the first sample?

    d. What can you conclude from this information?

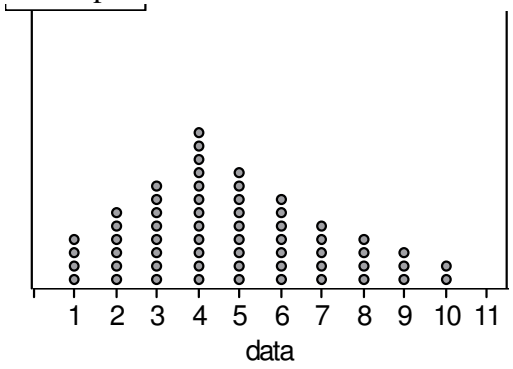
5. Statisticians use the mean and standard deviation of sample means to estimate the mean and standard deviation of a population when these values are unknown. If the mean of the sample means is $\bar{X}$, then the mean of the population is estimated using $\bar{X}$. If the standard deviation of the sample means is $S_{\bar{x}}$, then the standard deviation of the population can be estimated by $S_{\bar{x}}\sqrt{n}$, where *n* is the sample size.

    a. Use the mean and standard deviation for the means of sample size 5 to estimate the mean and standard deviation of our original population. How do the estimates compare to the actual population parameters?

    b. Use the mean and standard deviation for the means of sample size 15 to estimate the mean and standard deviation of our original population. How do the estimates compare to the original population parameters?

Mathematics II
Does a Sample Really Represent a Population?
Day 2 Homework

Use the dotplot shown below for problems 1 – 3.



1. In this problem you will use the dotplot below to generate 5 samples of size 2.
   a. On your calculator, type randInt(1,60,2) to select 2 points.

   b. Locate the associated 2 points on the dotplot below by counting from left to right. What are they?_____ Calculate the average of these 2 points. _____

   c. Repeat this process four more times and record the results below:

      randint(1,60,2)=_____ mean of the 2 associate values on the dotplot :_____

      randInt(1,60,2)=_____ mean of the 2 associate values on the dotplot :_____

      randInt(1,60,2)=_____ mean of the 2 associate values on the dotplot :_____

      randInt(1,60,2)=_____ mean of the 2 associate values on the dotplot :_____

   d. What is the mean of these 5 sample means of size 2?_____ What is the standard deviation of these 5 sample means?_____

2. You will now generate 5 samples of size 5.
   a. On your calculator, type randInt(1,60,5) to select five points. Locate the associated five points on the dotplot below by counting from left to right.
   What are they?_____ Calculate the average of these 5 points. _____

   b. Repeat this process four more times and record the results below:

   randInt(1,60,5)=_____mean of the 5 associate values on the dotplot :_____

   randInt(1,60,5)=_____ mean of the 5 associate values on the dotplot :_____

   randInt(1,60,5)=_____ mean of the 5 associate values on the dotplot :_____

   randInt(1,60,5)=_____mean of the 5 associate values on the dotplot:_____

   c. What is the mean of these 5 sample means?_____ What is the standard deviation of these 5 sample means?_____

3. You will now generate two samples of size 30.
   a. On your calculator, type randInt(1,60,30) to select 30 points. Locate the associated 30 points on the dotplot below. What are they?_____
   Calculate the average of these 30 points. _____

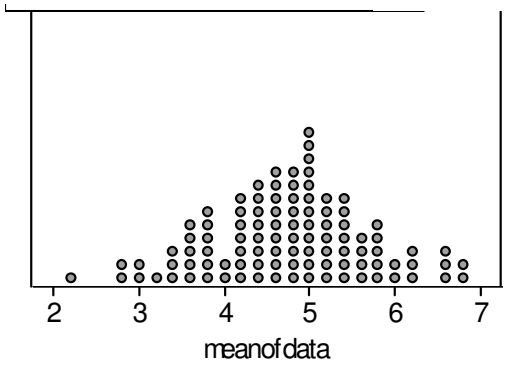
   b. Repeat this process one more time and record the results below:
   randInt(1,60,30)=_____
   mean of the 30 associate values on the dotplot :_____

   c. Find a partner in your class. What are his/her sample means of size 30?_____, _____

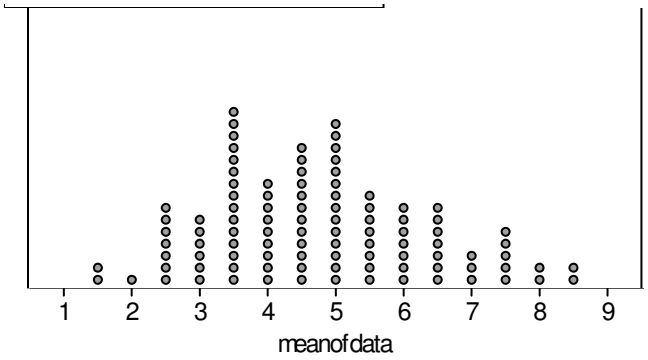
   d. What is the mean of these 4 sample means?_____ What is the standard deviation of these 4 sample means?_____

4. Below are three dotplots. Which dotplot could represent the distribution of the mean for samples of size 2, 5, and 30? Why?

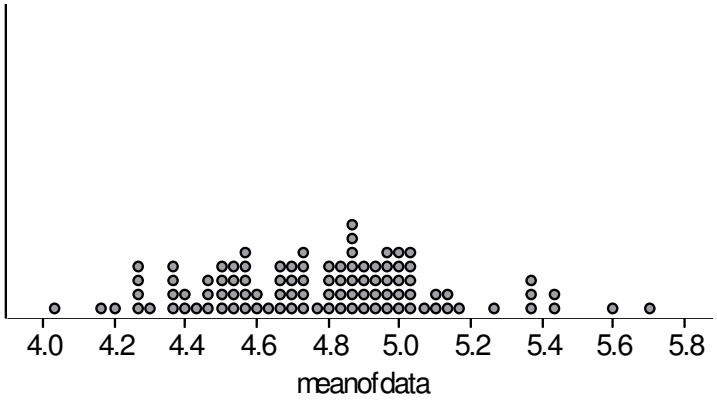
PLOT 1



PLOT 2



PLOT 3



In today's lesson, you learned that the standard deviation of a population can be estimated by the formula $\sigma = S_{\bar{x}}\sqrt{n}$, where $S_{\bar{x}}$ represents the standard deviation of sample means and *n* represents the sample size.

5. How might you use this formula to estimate the standard deviation of a sampling distribution?

6. How might you use this formula to explain the following concepts related to sampling distributions?

    a. A distribution of sample means from a given population has less variability than the population distribution.

    b. There is less variability among sample means as sample size increases.

ATLANTA PUBLIC SCHOOLS
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Teacher's Curriculum Supplement

Mathematics II: Unit 4
Statistics

Task 6: Your Class, Celebrities, and the Golden Ratio

Mathematics II
Task 6: *Your Class, Celebrities, and the Golden Ratio* **Day 1/2**
(Adapted from GaDOE SE Task 6)

Standard(s): MM2D1: Using sample data, students will make informal inferences about population means and standard deviations

e. Pose a question and collect sample data from at least two different populations.
f. Understand and calculate the means and standard deviations of sets of data.
g. Use means and standard deviations to compare data sets.

New Vocabulary: golden ratio

Mathematical concepts/skills:

- measuring using centimeter rulers
- collecting data
- describing data sets using center, shape, and spread
- using appropriate technology for analysis of data

Prior knowledge:

- measuring using centimeter rulers
- describing distributions using shape, center (mean and median), and spread (range, mean absolute deviation, interquartile range)
- comparing data sets using dotplots and boxplots

Essential question(s): How can I describe a distribution using center, shape, and spread? What questions might be answered by a given set of data?

Suggested materials:

- centimeter rulers
- small post-it notes
- graphing calculators

Warm-up: The video posted at the site below gives interesting and current examples of the prevalence of the Golden Ratio in nature, art, architecture and many other areas. You may want to allow students to watch the video and then ask them to read the scenario at the beginning of the student task.

http://www.youtube.com/watch?v=2zWivbG0RIo&feature=fvw

Opening: Discuss the video or other methods of introducing the Golden Ratio. Have students use their calculators to approximate the Ratio by evaluating $\frac{1+\sqrt{5}}{2}$. You may also want them to evaluate successive ratios of terms in Fibonacci sequence as described in the video.

Discuss the introduction of the task, making sure to stress the fact that "beauty" is defined in as many different ways as there are people. Our focus is on "typical" dimensions of the face and comparing groups using those dimensions.

Read problem 1. Make sure students understand that they should work in pairs, measure to the nearest millimeter, and use the template given in the task as a guide for measuring.

Worktime: Students should complete problems 1 – 4.

The use of correct vocabulary is extremely important throughout this unit. The distributions created by pooling the class data for each ratio are **data** distributions (data collected from **one** sample) as opposed to a **population** distribution (data collected from an entire population) or a **sampling** distribution (a distribution of statistics calculated from samples taken from a given population) .

As you monitor student work, ask guiding questions to ensure that students are addressing the following concepts as they describe each distribution:

- Shape: Students should discuss the overall shape of the distribution by first examining the dotplot. Existence of clusters, gaps, and outliers should be noted. (**Note**: This data should be somewhat normal. The more data values you have, the more normal the distribution should be. If you have a small class, you may want to pool this data with that of another teacher or another of your classes.)

- Center: Students should discuss both the mean and the median. If the distribution is approximately normal, these two statistics will be close to the same value. If the distribution is skewed, a discussion of which statistic gives the most *representative* ratio for the class would be appropriate.

- Spread or variability: At this point students should be able to talk about range, the IQR, the mean absolute deviation, and the standard deviation of a distribution. They should be able to verbalize what each of these measures tells us about a distribution. If the distribution appears to be somewhat normal, an investigation based on the Empirical Rule is appropriate.

- **Closing:** For each distribution discuss the bulleted information outlined above. Allow students to respond to question four based on your discussion. See teacher notes for GaDOE TE Task 6.

Homework: Tell students that in the next lesson they will collect the same data for celebrities that they collected for their class in today's lesson. Ask them to find, or print from their computers, a picture of the celebrity of their choice. The picture must contain enough of the celebrity's face that appropriate measurements can be taken. They should bring their picture to the next class.

Differentiated support/enrichment:

Check for Understanding:

Resources/materials for Math Support: Students should review/preview the skills and concepts needed to discuss a distribution using shape, center, and spread as described above in the W*orktime*.

Mathematics II
Your Class, Celebrities, and the Golden Ratio
Day 1 Student Task

Euclid of Alexandria (300 B.C.) defined the golden ratio in his book, "Elements." Since then, artists and architects who deem this ratio as being the most aesthetically pleasing ratio have used it as a basis for their art and buildings. It is thought that Leonardo da Vinci may have used the golden rectangle (having sides that are in the golden ratio) when painting the face of the Mona Lisa. The dimensions of Salvador Dali's painting, "Sacrament of the Last Supper," are also equal to the Golden Ratio. The Greeks used the golden ratio in building the Parthenon in Athens.

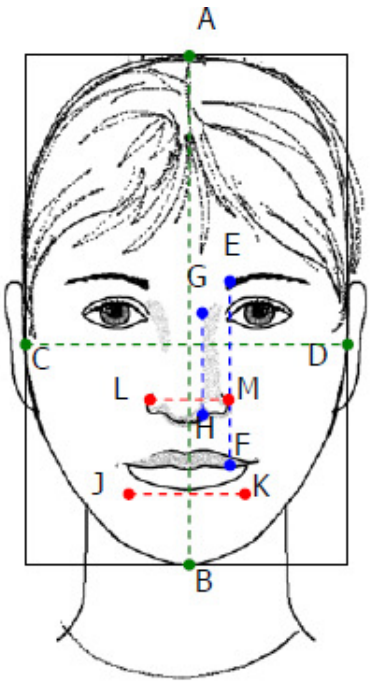
Throughout time, psychologists have tried to determine what humans consider to be beautiful. Gustav Theodor Fechner conducted an experiment during the 1860s and found that students preferred rectangular shapes that had the golden ratio (exactly $\frac{1+\sqrt{5}}{2}$ and approximately 1.62). Since then, similar experiments have had conflicting results.

Some psychologists think that humans who have facial feature ratios closest to the golden ratio are deemed as the most beautiful. Other psychologists think that the people with the most average measurements in their facial features are considered to be the most beautiful. Still others believe that people who are not average (have higher cheek bones, thinner jaw, and larger eyes than normal) are deemed as the most beautiful.

Through the use of statistics, and using our class as a sample, we will investigate some average dimensions of the face and calculate their ratios.

1. With a partner measure the six parts of your face required to calculate the ratios given in the table below. Use the template as a guide for how to measure. You should use centimeters to be as accurate as possible. When you have completed your measurements, use them to calculate the indicated ratios.

Ratios
$\frac{Length\ of\ face}{Width\ of\ face} = \frac{AB}{CD} =$
$\frac{Lips\ to\ eyebrows}{Length\ of\ nose} = \frac{EF}{GH} =$
$\frac{Width\ of\ mouth}{Width\ of\ nose} = \frac{JK}{LM} =$



2. Did you or your partner have ratios close to the golden ratio?

3. Pool the class data for each ratio. (You should round your ratios to the nearest hundredth.) Make a dotplot of the class distribution for each ratio and then describe the distributions using shape, center, and spread (variability).

4. Were any of your class ratios close to being golden? Explain your thinking.

Mathematics II
Task 6: Your Class, Celebrities, and the Golden Ratio **Day 2/2**
(Adapted from GaDOE SE Task 7)

Standard(s): MM2D1: Using sample data, students will make informal inferences about population means and standard deviations

a. Pose a question and collect sample data from at least two different populations.
b. Understand and calculate the means and standard deviations of sets of data.
c. Use means and standard deviations to compare data sets.

New Vocabulary:

Mathematical concepts/skills:

- measuring using centimeter rulers
- collecting data from two different populations
- describing data sets using center, shape, and spread
- comparing two different groups using center, shape, and spread
- using appropriate technology for analysis of data

Prior knowledge:

- measuring using centimeter rulers
- describing distributions using shape, center (mean and median), and spread (range, mean absolute deviation, interquartile range)
- comparing data sets using dotplots and boxplots

Essential question(s): How can I compare two different groups using statistics? What questions might be answered by a given sets of data?

Suggested materials:

- centimeter rulers
- small post-it notes
- graphing calculators

Warm-up: Allow those students who brought in pictures to begin taking measurements of their celebrity's faces. Students who did not bring pictures may choose a picture at the following website or you might assign them a picture that you have already printed.

http://www.angelfire.com/celeb2/celebrityfaces/

Opening: Discuss with your students that for the purposes of this lesson, they will treat their class (or classes, if you pooled data from several different classes) as a *convenience* sample of all Math II students. Students should have learned previously that a convenience sample is not a random sample. Surveys taken at a mall of those people who happen to "walk through the doors" is an example of a convenience sample. The group of celebrities chosen by the students will be

treated as a subjectively chosen sample of all celebrities. Because these samples are not random, we will not be trying to use our data to answer questions related to the populations from which these samples are drawn. Rather, we are simply comparing the two groups based on the data collected. Once you have explained these conditions, ask students what questions might be answered by this data. The class should agree on two or three questions to be answered.

Worktime: Students should complete problems 5 - 10.

As you monitor student work, again ask guiding questions to ensure that students are addressing the following concepts as they describe distributions:

- Shape: Students should discuss the overall shape of the distribution by first examining the dotplot. Existence of clusters, gaps, and outliers should be noted. (**Note**: This data should be somewhat normal. The more data values you have, the more normal the distribution should be. If you have a small class, you may want to pool this data with that of another teacher or another of your classes.)

- Center: Students should discuss both the mean and the median. If the distribution is approximately normal, these two statistics will be close to the same value. If the distribution is skewed, a discussion of which statistic gives the most *representative* ratio for the class would be appropriate.

- Spread or variability: At this point students should be able to talk about range, the IQR, the mean absolute deviation, and the standard deviation of a distribution. They should be able to verbalize what each of these measures tells us about a distribution. If the distribution appears to be somewhat normal, an investigation based on the Empirical Rule is appropriate.

In comparing distributions of the class and the celebrities, boxplots, as well as dotplots, may be useful.

Closing: Discuss problems 8 - 10 thoroughly. See teacher notes for GaDOE TE Task 7. (Note: In this revised version of DOE Task 7, we did not include a chart for organization of summary measures. The chart was omitted in order to allow for more open-ended discussion of the distributions. Students may choose to investigate measures in addition to the mean and the standard deviation depending upon the distributions.)

Homework:

Differentiated support/enrichment:

Check for Understanding:

Resources/materials for Math Support: Students should review/preview the skills and concepts needed to discuss and compare distributions using shape, center, and spread as described above in the W*orktime*.

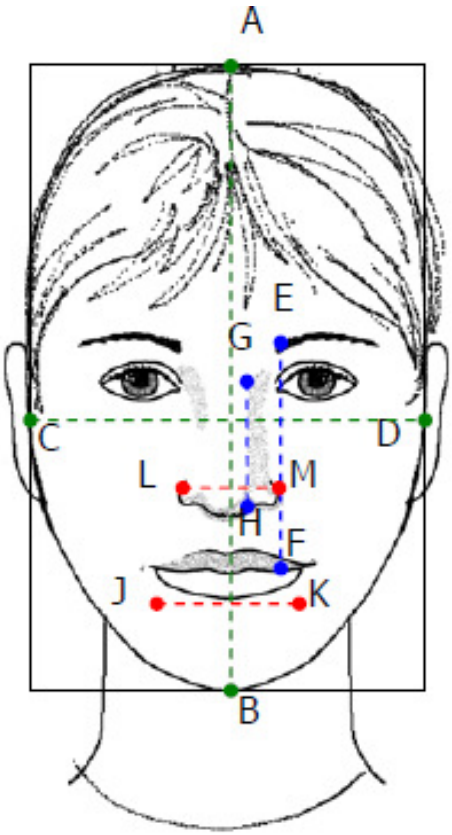
Mathematics II
Your Class, Celebrities, and the Golden Ratio
Day 2 Student Task

In the next part of this task we will collect the same data for celebrities that we collected for your class. We will consider your class to be a *convenience* sample of all Math II students and the celebrities you choose to be a subjective sample of all celebrities.

5. As a class, determine two or more questions that might be answered by data that you are collecting from these two different samples-your class and the famous people you have chosen.

6. Choose a celebrity and again measure the six parts of the face required to calculate the ratios given in the table. Use the template as a guide for how to measure. Use centimeters to be as accurate as possible. When you have completed your measurements use them to calculate the indicated ratios.

Ratios
$\frac{Length\ of\ face}{Width\ of\ face} = \frac{AB}{CD} =$
$\frac{Lips\ to\ eyebrows}{Length\ of\ nose} = \frac{EF}{GH} =$
$\frac{Width\ of\ mouth}{Width\ of\ nose} = \frac{JK}{LM} =$



7. Did the celebrity you chose have any ratios that were close to being golden?

8. Pool the class data for each ratio. (You should round your ratios to the nearest hundredth.) Make a dotplot of the class distribution for each ratio and then describe the distributions using shape, center, and spread.

9. In a paragraph, compare the celebrity data distributions to the class data distributions. Make sure that you discuss center, shape and spread. If there are any discrepancies, discuss possible reasons why they exist.

10. Answer the questions posed by the class in Problem 5.

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Mathematics II: Unit 4
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Resources

TI-83/TI-84 Calculator Commands
from the website: **http://w3.salemstate.edu/~jbelock/MAT%20247/TIcommands.htm**

To enter data into a list:

Press STAT > EDIT, then type all the values in your data set into a single column.

To clear a list:

Go to the spreadsheet by pressing STAT > EDIT. Use the up arrow to place the cursor on the name of the list you want to erase. Press CLEAR, then arrow back down into the list.

To find descriptive statistics:

First, enter the data into a list. Quit to the home screen. Then press STAT > CALC 1:1-var stats and hit ENTER. The command is pasted onto the home screen, and enter the name of the list where your data is.

To make a graphical display:

All the statistical graphs are under 2nd STATPLOT. You must turn on the graph you want, select the appropriate type of graph and enter the correct list. Then press either GRAPH or ZOOM 9: Zoom stat to see your graph.

What if you get an error? Check that the graph is turned on, make sure no other graphs are on (including on the Y = screen), make sure you've entered the correct list, in the case of a scatter plot make sure both the X and Y lists have the same number of values, try hitting ZOOM 9 because sometimes it just needs to be recentered.

To find values from the Normal Distribution:

1. If you know an interval and are looking for area (percentage), use 2nd DISTR 2:normalcdf and hit ENTER. This pastes it onto the home screen. You must then enter as follows:

Normalcdf(*lowerbound*,*upperbound*,μ,σ) then hit ENTER

Remember that μ is the mean of the normal distribution and σ is its standard deviation.

2. 2. If you know the area and are looking for values along the horizontal (i.e. percentiles) use 2nd DISTR 3:invNorm and hit ENTER. This pastes it to the home screen. You must then enter as follows:

InvNorm(*Area to the left of the value you have,*μ,σ) then hit ENTER.

To find the correlation coefficient:

First, your calculator must be set up to display the correlation. (You only have to set it up once, so if you've done it in class, skip this part. Sometimes if you change batteries you have to do it again.) Hit 2nd CATALOG (this is over the 0 button). Go down to DiagnosticOn, hit ENTER then ENTER again. It is now set up to display correlation with the regression line.

Enter the X values in one list and the Y values in another. Go to STAT>CALC 8:LinReg (a+bx) and hit ENTER. It is now pasted to the home screen. You must input the names of the list containing the X values followed by a comma then the list containing the Y values. For example, if my X values are in L1 and Y values are in L2, I would enter LinReg(a+bx) L1,L2

TI-83/TI-84 Calculator Commands
from the website: http://pasles.org/Ti83.html

Calculating mean and standard deviation on the TI-83:

Let's say we have a table of data describing four birds of different species (units of height and weight have been left out here):

height	weight
2	5
3	6
1	5
4	5

You can think of the two columns as representing variables *x* and *y*.

First we have to enter the data. Hit the STAT button and you will see the options EDIT, CALC and TESTS atop the screen. Use the left and right arrows (if necessary) to move the cursor to EDIT, then select 1:Edit...

Now you will see a table with the headings L_1 and L_2. Enter the x values under L_1, the y values under L_2. (If you want to clear pre-existing data first, move the cursor to the top of the column, hit CLEAR and then ENTER.)

(If there were only a single variable, we could enter the data as x-values and leave the 2nd column blank.)

Once all the data is entered, go back to the STAT menu, but this time move the cursor to CALC instead of EDIT. If you can't find your way there, remember: Every TI graphing calculator is equipped with CLEAR, QUIT and/or EXIT commands for getting back out of tough situations.

Once you're in the CALC menu, select *2-Var Stats*. (If we had only entered a single column of data, *1-Var Stats* would be the appropriate choice instead.) Then hit ENTER.

The calculator will display the x-mean (= 2.5), some other stuff, and then the standard deviation (s_x=1.29). Note that s_x is what we called s in class; the calculator refers to it as s_x so we know that this is the standard deviation of the variable x and not that of y (which will be denoted by s_y). This is followed by something called σιγμα $_x$ (which is what you would get as standard deviation if you had used n instead of $n-1$), and finally the sample size (there are n = 4 observations). Use the down arrow to get the corresponding information about y. (Its mean is 5.25, and its standard deviation is s_y =.5).

Now go back to the STAT menu and select CALC, 1-Var stats. See what happens: we get the mean and standard deviation of x, and also its five-number summary! If you want the five-number summary for y, try STAT, CALC, 1-Var stats and then type L_2 (which is the 2nd function on the 2 key) before hitting ENTER.

Valuable Resources for the Statistics Teacher

Books

- Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A Pre-K-12 Curriculum Framework; Franklin, Christine, Gary Kader, etc; American Statistical Association. 2007.

This report may also be assessed online: http://www.amstat.org/education/gaise

- Navigating through Data Analysis in Grades 9-12; Burrill, Gail, Christine Franklin, etc; NCTM Navigation Series, 2003.
- Navigating through Probability in Grades 9-12; Peck, Roxy, etc; NCTM Navigation Series, 2003
- Navigating through Data Analysis in Grades 6-8
- Navigating through Probability in Grades 6-8
- Thinking and Reasoning with Data and Change: 68th NCTM Yearbook; 2006.
- Quantitative Literacy Ser4ies; Pearson Learning
- Data Driven Mathematics; Pearson Learning

Websites

- Causeweb: Consortium for the Advancement of Statistics Education (great resource!) http://www.causeweb.org/
- Ten Websites Every Statistics Instructor Should Bookmark (by Robin Lock) http://it.stlawu.edu/%7Erlock/10sites.html
- ASA Section on Statistical Educatio – the website has links to excellent resources http://www.amstat.org/sections/educ/
- College GAISE report: This report is the next level beyond the Pre-K-12 GAISE report: http://www.amstat.org/education/gaise/

Teachers in Georgia

- The new Georgia Performance Standards: http://www.georgiastandards.org/math.asp

Journals

- *Mathematics Teacher* published by NCTM.
- *Mathematics Teaching in the Middle Schools* published by NCTM
- *Journal of Statistics Education* published by ASA. This is a free online journal. http://www.amstat.org/publications/jse/
- *Teaching Statistics*. An international journal. http://www.rsscse.org.uk/ts/