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Atlanta Public Schools

Teacher's Curriculum Supplement

Mathematics II: Unit 2
Right Triangle Trigonometry

GE Foundation

This document has been made possible by funding from the GE Foundation Developing Futures grant, in partnership with Atlanta Public Schools. It is derived from the Georgia Department of Education Math II Framework and includes contributions from Georgia teachers. It is intended to serve as a companion to the GA DOE Math II Framework Teacher Edition. Permission to copy for educational purposes is granted and no portion may be reproduced for sale or profit.

Preface

We are pleased to provide this supplement to the Georgia Department of Education's Mathematics II Framework. It has been written in the hope that it will assist teachers in the planning and delivery of the new curriculum, particularly in these first years of implementation. This document should be used with the following considerations.

- The importance of working the tasks used in these lessons cannot be overstated. In planning for the teaching of the Georgia Performance Standards in Mathematics teachers should work the task, read the teacher notes provided in the Georgia Department of Education's Mathematics II Framework Teacher Edition, and *then* examine the lessons provided here.

- This guide provides day-by-day lesson plans. While a detailed scope and sequence and established lessons may help in the implementation of a new and more rigorous curriculum, it is hoped that teachers will assess their students informally on an on-going basis and use the results of these assessments to determine (or modify) what happens in the classroom from one day to the next. Planning based on student need is much more effective than following a pre-determined timeline.

- It is important to remember that the Georgia Performance Standards provide a balance of concepts, skills, and problem solving. Although this document is primarily based on the tasks of the Framework, we have attempted to help teachers achieve this all important balance by embedding necessary skills in the lessons and including skills in specific or suggested homework assignments. The teachers and writers who developed these lessons, however, are not in your classrooms. It is incumbent upon the classroom teacher to assess the skill level of students on every topic addressed in the standards and provide the opportunities needed to master those skills.

- In most of the lesson templates, the sections labeled *Differentiated support/enrichment* have been left blank. This is a result of several factors, the most significant of which was time. It is hoped that as teachers use these lessons, they will contribute their own ideas, not only in the areas of differentiation and enrichment, but in other areas as well. Materials and resources abound that can be used to contribute to the teaching of the standards.

On the topic of differentiation, it is critical to reiterate that many of the strategies used in a standards-based mathematics classroom promote differentiation. These strategies include

- the use of rich tasks with multiple points of entry and more than one path to a solution,
- flexible grouping of students,
- multiple representations of mathematical concepts,
- writing in mathematics,
- monitoring of progress through on-going informal and formative assessments, and
- analysis of student work.

We hope that teachers will incorporate these strategies in each and every lesson. It is hoped that you find this document useful as we strive to raise the mathematics achievement of all students in our district and state. Comments, questions, and suggestions for inclusions related to the document may be emailed to Dr. Dottie Whitlow, Executive Director, Mathematics and Science Department, Atlanta Public Schools, dwhitlow@atlantapublicschools.us

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Explanation of the Terms and Categories Used in the Lesson Template

Task: This section gives the suggested number of days needed to teach the concepts addressed in a task, the task name, and the problem numbers of the task as listed in the Georgia Department of Education's Mathematics II Framework Teacher Edition (GaDOE TE).

In some cases new tasks or activities have been developed. These activities have been named by the writers.

Standard(s): Although each task addresses many Math II standards and uses mathematics learned in earlier grades, in this section, only the key standards addressed in the lesson are listed.

New Vocabulary: Vocabulary is listed here the *first* time it is used. It is strongly recommended that teachers, particularly those teaching Math Support, use interactive word walls. Vocabulary listed in this section should be included on the word walls and previewed in Math Support.

Mathematical concepts/skills: Major concepts addressed in the lesson are listed in this section whether they are Math II concepts or were addressed in earlier grades or courses.

Prior knowledge: Prior knowledge includes only those topics studied in previous grades or courses. It does not include Math II content taught in previous lessons.

Essential Question(s): Essential questions may be daily and/or unit questions.

Suggested materials: This is an attempt to list all materials that will be needed for the lesson, including consumable items, such as graph paper; and tools, such as graphing calculators and compasses. This list does not include those items that should always be present in a standards-based mathematics classroom such as markers, chart paper, and rulers.

Warm-up: A suggested warm-up is included with every lesson. Warm-ups should be brief and should focus student thinking on the concepts that are to be addressed in the lesson.

Opening: Openings should set the stage for the mathematics to be done during the work time. The amount of class time used for an opening will vary from day-to-day but this should not be the longest part of the lesson.

Worktime: The problem numbers have been listed and the work that students are to do during the worktime has been described. A student version of the day's activity follows the lesson template in every case. In order to address all of the standards in Math II, some of the problems in some of the original GaDOE tasks have been omitted and less time consuming activities have been substituted for those problems. In many instances, in the student versions of the tasks, the writing of the original tasks has been simplified. In order to preserve all vocabulary, content, and meaning it is important that teachers work the original tasks as well as the student versions included here.

Teachers are expected to both facilitate and provide some direct instruction, when necessary, during the work time. Suggestions related to student misconceptions, difficult concepts, and deeper meaning have been included in this section. However, the teacher notes in the GaDOE Math II Framework are exceptional. In most cases, there is no need to repeat the information provided there. Again, it is imperative that teachers work the tasks and read the teacher notes that are provided in GaDOE support materials.

Questioning is extremely important in every part of a standards-based lesson. We included suggestions for questions in some cases but did not focus on providing good questions as extensively as we would have liked. Developing good questions related to a specific lesson should be a focus of collaborative planning time.

Closing: The closing may be the most important part of the lesson. This is where the mathematics is formalized. Even when a lesson must be continued to the next day, teachers should stop, leaving enough time to "close", summarizing and formalizing what students have done to that point. As much as possible students should assist in presenting the mathematics learned in the lesson. The teacher notes are all important in determining what mathematics should be included in the closing.

Homework: In some cases, homework suggestions are provided. Teachers should use their resources, including the textbook, to assign homework that addresses the needs of their students.

Homework should be focused on the skills and concepts presented in class, relatively short (30 to 45 minutes), and include a balance of skills and thought-provoking problems.

Differentiated support/enrichment: On the topic of differentiation, it is critical to reiterate that many of the strategies used in a standards-based mathematics classroom promote differentiation. These strategies include

- the use of rich tasks with multiple points of entry and more than one path to a solution,
- flexible grouping of students,
- multiple representations of mathematical concepts,
- writing in mathematics,
- monitoring of progress through on-going informal and formative assessments, and
- analysis of student work.

Check for understanding: A check for understanding is a short, focused assessment-a ticket out the door, for example. There are many good resources for these items, including the GaDOE culminating task at the end of each unit and the *Mathematics II End-of-Course Study Guide*. Both resources can be found on-line at www.georiastandards.org, along with other GaDOE materials related to the standards. Problem numbers from the GaDOE culminating task have been listed with the appropriate lessons in this document.

Resources/materials for Math Support: Again, in some cases, we have provided materials and/or suggestions for Math Support. This section should be personalized to your students, class, and/or school, based on your resources.

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Unit 2

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Unit 1 Timeline

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Task 4: Find That Side or Angle	1 day
Task 5: Finding the Overhang	1 day

Task Notes

The importance of working the tasks used in these lessons cannot be overstated. In planning for the teaching of the Georgia Performance Standards in Mathematics, teachers should work the task, read the teacher notes provided in the Georgia Department of Education's Mathematics II Framework Teacher Edition, and *then* examine the lessons provided here.

Task 1 of the Georgia Department of Education's Mathematics II Framework, *Eratosthenes Find the Circumference of the Earth*, has been omitted in this document. The mathematical content of the task has been incorporated in other tasks.

Task 1: Discovering Special Triangles
The big ideas presented in this task include:

- The lengths of the legs of any 30°- 60°- 90° triangle are always in the ratio $a : a\sqrt{3} : 2a$ and these relationships can be derived by halving an equilateral triangle using an altitude.
- The lengths of the legs of any 45°- 45°- 90° triangle are always in the ratio $a : a : a\sqrt{2}$ and these relationships can be derived by halving a square using a diagonal.
- Special right triangles can be used to solve a variety of real-world problems.

Some items from the original GaDOE task have been re-written to provide a more open-ended investigation. However, all items of the original task are addressed and GaDOE Teacher Notes are appropriate.

Task 2: Finding Right Triangles in Your Environment
The big ideas presented in this task include:

- Some right triangles can be constructed with compass and straightedge.
- Similar right triangles can be used to solve real-world problems.
- Ratios of side lengths in similar right triangles depend on the acute angles of the triangles, not on the lengths of the sides.

Item 1 of the GaDOE task was omitted here. However, the intent of the problem is addressed in other items. Problems have been revised for clarity and to reduce the amount of time needed to complete the task. GaDOE teacher notes are still applicable.

This task does not directly address a particular element of the standards but builds the conceptual understanding needed to address standard MM2G2.

Task 3: Create Your Own Triangles

The big ideas presented in this task include:

- Trigonometric functions (sine, cosine, and tangent) of acute angles are defined as the ratios of side lengths of similar right triangles.
- Trigonometric ratios are dependent only on angle measure.
- For each pair of complementary angles in a right triangle, the sine of an angle is the cosine of its complement; the tangent of an angle is the reciprocal of the tangent of its complement; and the tangent of any angle θ is equal to the sine of θ divided by the cosine of θ.
- The sine, cosine, and tangent of a given angle are nonlinear functions of that angle.

In Task 3 we have combined two tasks of the original GaDOE Framework, *Create Your Own Triangles* and *Discovering Trigonometric Ratio Relationships.* Task names and problem numbers of the original tasks are indicated in the lesson plans so that GaDOE Teacher Notes may be easily utilized.

Task 4: Find That Side or Angle

The big ideas presented in this task include:

- Calculators give approximations of trigonometric functions of acute angles.
- Calculators can be used to find angle measures given trigonometric functions of the angles.
- Trigonometric functions can be used to finding missing parts of right triangles.
- Trigonometric functions can be used to solve real-world problems.

This task is a collection of fairly traditional application problems. It provides students with opportunities to apply the mathematics they have learned in the unit to this point.
All parts of the original GaDOE task are addressed in these plans.

Task 5: Finding the Overhang

The big ideas presented in this task include:

- Trigonometric functions can be used to solve real-world problems.

The house described in this task was actually built by a gentleman in Montana seeking to minimize the effect of his home on the environment surrounding it, and in the process, to reduce his energy cost. Materials used were chosen for their ability to reduce the amount of energy needed to maintain the home.

In seeking to maximize the energy of the sun in the winter and minimize it in the summer, the builder approached the faculty at Georgia Tech for help in deciding how to construct the roof. The questions asked of students in the task are the same questions explored at Tech. It is good for students to realize that the mathematics they are learning is used in real, uncontrived situations everyday to solve problems.

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Mathematics II: Unit 2
Right Triangle Trigonometry

Task 1: Discovering Special Triangles

Mathematics II
Task 2: ***Discovering Special Triangles*** **Day 1/2**
(GaDOE TE # 1 – 6)

MM2G1. Students will identify and use special right triangles.

a. Determine the lengths of sides of 30°- 60°- 90° triangles.

New vocabulary: special right triangles, 30°- 60°- 90° Triangle Theorem

Mathematical concepts/skills:

- using properties of equilateral and isosceles triangles, similar triangles, congruence postulates, and the Pythagorean Theorem to find the measures of triangles created by an altitude of an equilateral triangle
- determining the relationships among side lengths of 30° - 60° - 90° triangles
- given one side of a 30° - 60° - 90° triangle, find the measures of the remaining two sides
- solving problems involving 30° - 60° - 90° triangles

Prior knowledge:

- properties of similar triangles
- Pythagorean Theorem
- properties of isosceles and equilateral triangles
- triangle congruence postulates

Essential question(s): What are the relationships among the side lengths of a 30° - 60° - 90° triangle? How can I use these relationships to solve real-world problems?

Suggested materials:

Warm-up: Post the following.

Sketch an equilateral triangle and an altitude of the triangle. What do you know about the two smaller triangles formed by the altitude?

Opening: Discuss the warm-up. Allow students to share their observations. The discussion may include the following:

- because an equilateral triangle is also isosceles, base angles are congruent;
- the altitude bisects both the vertex angle and the base;
- the smaller triangles are right triangles;
- the two smaller triangles formed by the altitude can be proved congruent by SSS, SAS, ASA, or HL;
- angles of the smaller triangles measure 30°, 60°, and 90°.

Worktime: Students should work problems 1 – 6 of the student task. Problem 6 may be finished for homework, if necessary. Although the student task in this supplement has been revised to create more open-ended items, GaDOE Teacher Notes are appropriate.

Closing: Students should present their work. Problem 5 should be discussed in-depth. All students should understand that for any 30° - 60° - 90° triangle, if the leg opposite the 30° angle has length a, the leg opposite the 60° angle has length $a\sqrt{3}$, and the hypotenuse has length $2a$.

Homework: Problems 6 of the student task should be completed for homework, if not finished in class. Additional homework problems follow the student task. These problems give students opportunities to apply the concepts learned in this lesson.

Differentiated support/enrichment: Construct a 30° - 60° - 90° triangle with a straight edge and a compass. Explain your thinking.

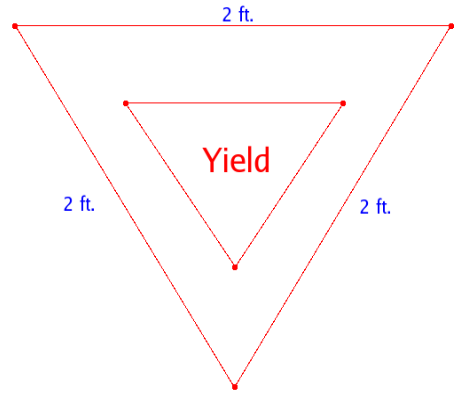
Check for Understanding: Explain how you would find the lengths of the remaining 2 sides of any 30° - 60° - 90° triangle, given the length of one side.

Resources/materials for Math Support: Students should preview;
- properties of equilateral and isosceles triangles;
- proving triangles congruent;
- proving similar triangles;
- using the Pythagorean Theorem to find the side lengths of right triangles; and
- simplifying and computing with square roots.

The activity below is excellent for helping students discover or reinforce the concepts in this lesson:

On square dot paper, have students draw right triangles in which the hypotenuse is twice as long as one of the legs. Then have them find the lengths of the third sides and measure the angles. Allow students to discuss their findings.

Mathematics II
Discovering Special Triangles
Day 1 Student Task



1. Adam is a quality assurance specialist in a plant that produces Yield signs for his state. To check the uniformity of signs as they roll of the assembly line, he checks the heights (altitude) of the signs. Adam knows that all yield signs have the shape of an equilateral triangle. Why is it sufficient for him to check just the heights (altitudes) of the signs to verify uniformity?

2. A Yield sign created in Adam's plant is pictured to the right. It has the shape of an equilateral triangle with a side length of 2 feet. If you draw the altitude of the triangular sign, you split the Yield sign in half vertically. Use what you know about triangles to find the angle measures and the side lengths of the two smaller triangles formed by the altitude.

3. Adam also needs to know the altitude of the smaller triangle within the sign. Each side of this smaller equilateral triangle is 1 ft. long. What is the altitude of this equilateral triangle? Find the angle measures and the side lengths of the two smaller triangles formed by this altitude. Show how you know.

4. Now that we have found the altitudes of both equilateral triangles, we look for patterns in the data. Fill in the first two rows of the chart below, and write down any observations you make. Then fill in the remaining rows.

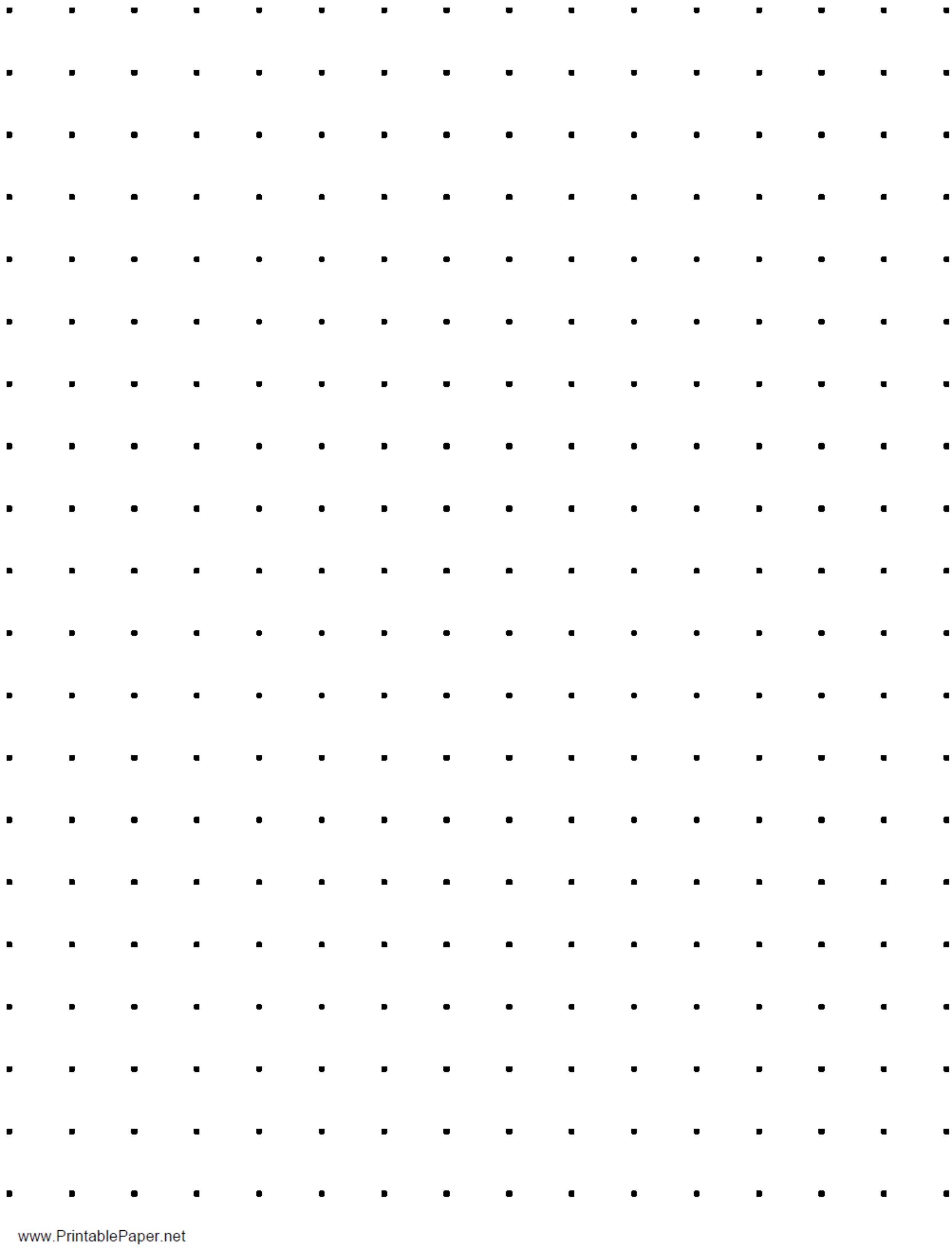
Side Length of Equilateral Triangle	**Each 30°- 60°- 90° right triangle formed by drawing altitude**		
	Hypotenuse Length	**Shorter Leg Length**	**Longer Leg Length**
2			
1			
4			
5			
6			

5. What is true about the lengths of the sides of any 30°-60°-90° right triangle? Why does this relationship among the side lengths remain constant for every 30° - 60° - 90° triangle?

6. The relationship you described in *Item 5* is often referred to as the *30° - 60° - 90° Triangle Theorem.* Write a ratio to represent this relationship.

7. Use your answer for *Item 5* to help you complete the table below. Leave your answers in simple radical form. (You do not need to rationalize denominators.)

30°-60°-90° triangle	Δ #1	Δ #2	Δ #3	Δ #4	Δ #5	Δ #6	Δ #7	Δ #8
hypotenuse length	*11*				$3\sqrt{5}$			
shorter leg length		π		$\frac{12}{5}$		$\sqrt{3}$		$\frac{\sqrt{2}}{2}$
longer leg length			$\frac{\sqrt{3}}{7}$				*4*	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Mathematics II
Discovering Special Triangles
Day 1 Homework

1. Perform the following operations and write each expression in simple radical form.

   a. $2\sqrt{12}+3\sqrt{18}$ b. $\sqrt{6}+\sqrt{3}$ c. $\frac{4}{\sqrt{3}}\bullet\sqrt{6}$

2. Find the perimeter of an equilateral triangle with a median that measures 5 meters. Show how you know.

3. Find the area of an equilateral triangle with side length 8 inches. Show how you know.

4. A rhombus contains an angle of 60° and the side lengths are 10 inches. Find the length of each diagonal.

Mathematics II
Task 2: *Discovering Special Triangles* **Day 2/2**
(GaDOE TE # 7 - 10)

MM2G1. Students will identify and use special right triangles.

b. Determine the lengths of sides of 45°- 45°- 90° triangles.

New vocabulary: special right triangles, $45^o – 45^o – 90^o$ Triangle Theorem

Mathematical concepts/skills:

- using properties of quadrilaterals, isosceles triangles, similar triangles, congruence postulates, and the Pythagorean Theorem to find the measures of triangles created by the diagonal of a square
- determining the relationships among side lengths of 45° - 45° - 90° triangles
- given one side of a 45° - 45° - 90° triangle, find the measures of the remaining two sides
- solving problems involving 45° - 45° - 90° triangles

Prior knowledge:

- properties of quadrilaterals
- properties of similar triangles
- Pythagorean Theorem
- properties of isosceles and equilateral triangles
- triangle congruence postulates

Essential question(s): What are the relationships among the side lengths of a 45° - 45° - 90° triangle? How can I use these relationships to solve real-world problems?

Suggested materials:

Warm-up: Have students compare homework with a partner. Ask them to be prepared to ask questions about any problems they still do not understand.

Opening: Answer any remaining questions about problems 1 – 3 of the homework. Have students share several different methods for solving problem 4. This problem provides an opportunity to discuss properties of quadrilaterals and the congruence postulates studied in Math I and will set the stage for today's lesson.
Along with the congruence postulates, properties discussed may include:

- A rhombus is a parallelogram.
- All sides of a rhombus are congruent.
- The diagonals of a rhombus are perpendicular.
- Diagonals of a rhombus bisect its angles.

Read the scenario at the beginning of today's task. Students should understand that, in this situation, the second baseman and the catcher are at the vertices of the square.

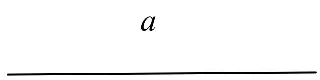
Worktime: Students should work problems 8 - 11 of the student task. Although the student task in this supplement has been revised extensively to provide for more open-ended investigation, the GaDOE teacher Notes are still appropriate.

Note: In problem 11, students are not expected to rationalize denominators.

Closing: Students should present their work. Problem 10 should be discussed in-depth. All students should understand that for any 45° - 45° - 90° triangle, if the legs have length a, the hypotenuse has length $a\sqrt{2}$.

Homework: Problem 11 of the student task should be completed for homework, if not finished in class. Additional homework problems follow the student task. These problems give students opportunities to apply the concepts learned in this lesson.

Differentiated support/enrichment: Given the segment with length a below, use a compass and straightedge to construct a segment of length $a\sqrt{2}$. Describe the steps you took.



Check for Understanding: Explain how you would find the length of the leg of an isosceles right triangle, given the length of the hypotenuse.

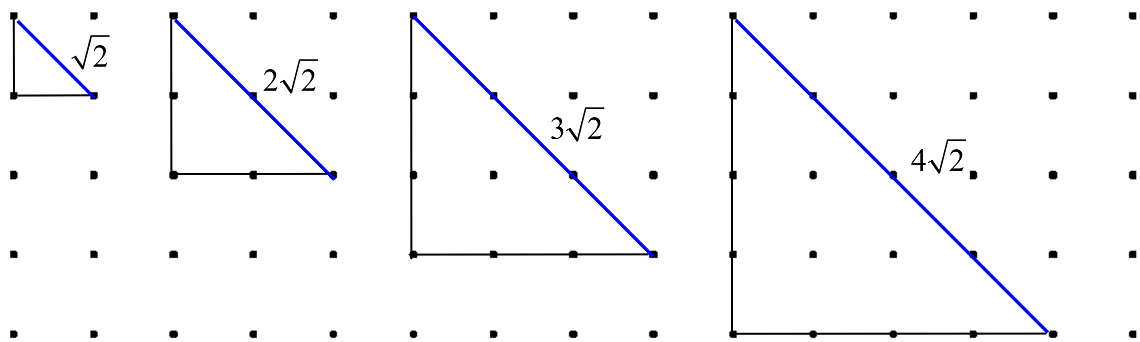
Resources/materials for Math Support: Students should preview;

- properties of quadrilaterals;
- properties of equilateral and isosceles triangles;
- proving triangles congruent;
- proving similar triangles;
- using the Pythagorean Theorem to find the side lengths of right triangles; and
- simplifying and computing with square roots.

The activity below is excellent for helping students discover or re-enforce the concepts in this lesson:

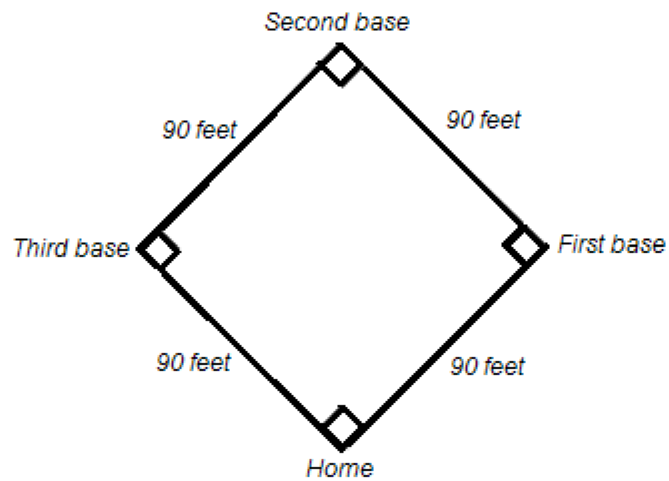
On square dot paper, have students draw right triangles in which the two legs are congruent. Then have them find the lengths of the third sides and measure the angles. Allow students to discuss their findings.

You can also demonstrate the Isosceles Right Triangle Conjecture for integers on square dot paper, as shown below.



Mathematics II
Discovering Special Triangles
Day 2 Student Task

A baseball diamond is, geometrically speaking, a square turned sideways. Each side of the diamond measures 90 feet. (See the diagram to the right.) A player is trying to slide into home plate. The ball is at second base. Assume that the second baseman and catcher are standing on their bases at the corners of the diamond.



8. On the sketch, draw the diagonal whose length represents the distance the second baseman would need to throw the ball in order to reach the catcher.

   a. Describe the two triangles created by this diagonal.

   b. Find the distance the catcher needs to throw the ball. Give your answer in both simple radical form and to the nearest tenth of a foot.

   c. Give the angle measures and the lengths (in radical form) of the sides of the triangles you described in part *a*.

9. Without moving from his position, the catcher tags the runner out and then throws the ball back to the pitcher, who is standing at the center of the baseball diamond.

   a. How far was the catcher's throw to the pitcher? Explain your thinking.

   b. Sketch and describe the triangle between home base, the center of the field, and first base.

   c. How does the triangle in part b compare to the triangles you described in problem 7?

   d. In the previous lesson, you studied *special right triangles* with angle measures 30, 60, and 90 degrees. You also learned that there was a special relationship among the lengths of the legs of these triangles. Right triangles with angle measures of 45, 45, and 90 degrees are also considered *special right triangles*. Now that you have found the side lengths of two 45°- 45°- 90° triangles, see if you can observe a pattern in the lengths of the sides of these triangles. Fill in the table below using exact values in simple radical form.

In each 45°- 45°- 90° right triangle		
Leg Length	Other Leg Length	Hypotenuse Length
90 ft.		
$45\sqrt{2}$ ft.		

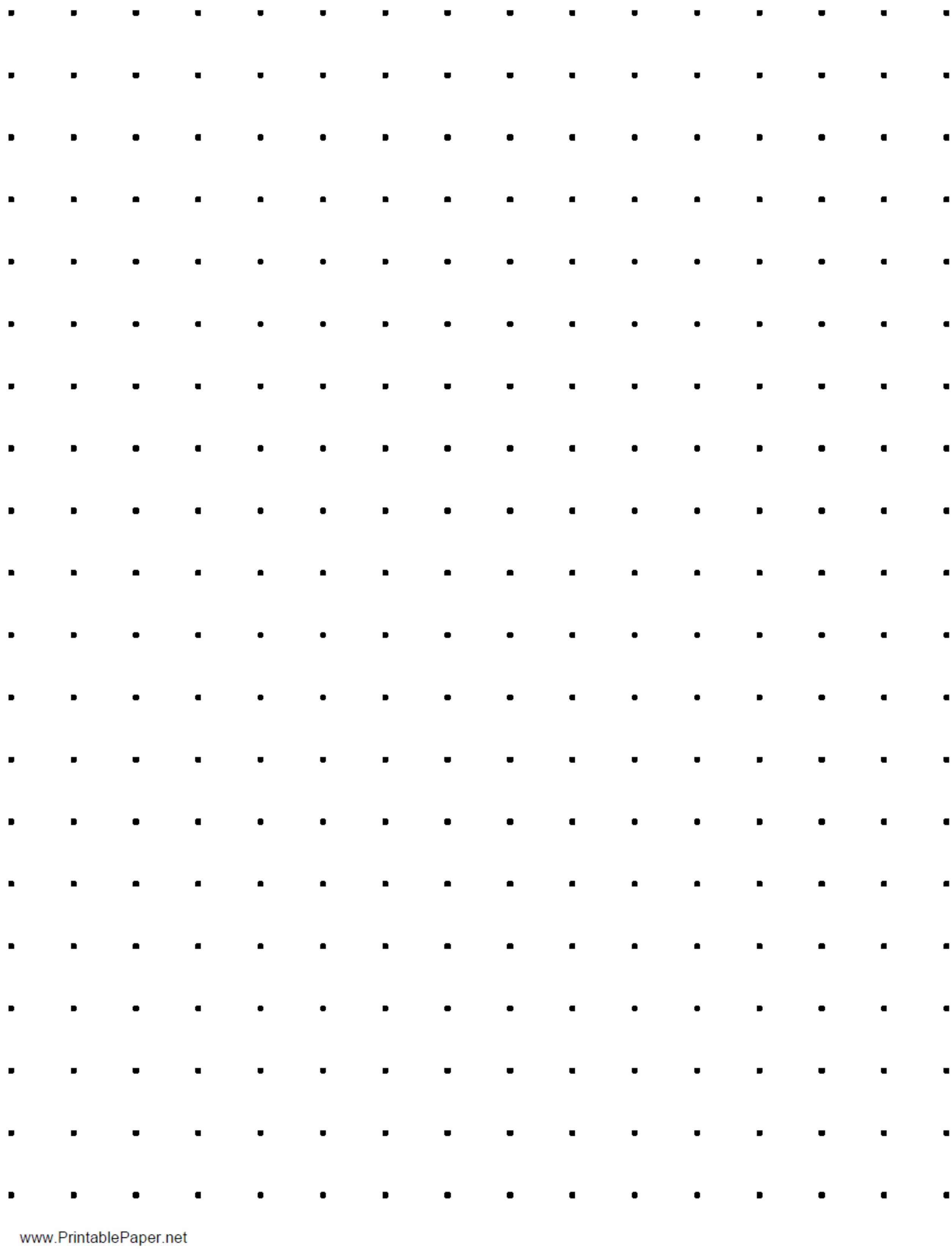
e. Show, by direct calculation, that the entries in the second row are related in the same way as the entries in the second row.

10. What is true about the lengths of the sides of any 45°- 45°- 90° right triangle? How do you know? Write a ratio that illustrates your answer. This ratio is often referred to as the *Isosceles Right Triangle Conjecture.*

11. Use your answer for *Item 10* as you complete the table below. Leave your answers in simple radical form.

12.

45°-45°-90° triangle	Δ#1	Δ#2	Δ#3	Δ#4	Δ#5	Δ#6	Δ#7	Δ#8
hypotenuse length			11				$3\sqrt{5}$	
one leg length		π		$\sqrt{2}/2$		$\sqrt{3}$		$12/5$
other leg length	4				$\sqrt{3}/7$			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Mathematics II
Discovering Special Triangles
Day 2 Homework

1. Find the perimeter of a square with diagonal $18\sqrt{2}$.

2. The lengths of the bases of an isosceles trapezoid are 7 and 15 inches. Each leg makes an angle of 45° with the longer base. Find the length of the altitude of the trapezoid.

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Mathematics II: Unit 2
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Task 2: Finding Right Triangles in Your Environment

Mathematics II
Task 2: *Finding Right Triangles in Your Environment* **Day 1/1**
(GaDOE TE # 2 - 4)

MM2G2. Students will define and apply sine, cosine, and tangent ratios to right triangles.

New vocabulary:

Mathematical concepts/skills:

- using similar triangles to solve problems
- constructing right triangles using compass and straightedge
- measurement error
- ratios of corresponding sides in a right triangle depend on the measures of the acute angles of the triangles, not on the lengths of the sides

Prior knowledge:

- basic constructions using compass and straightedge
- use of protractors to measure angles
- using similar triangles to solve problems

Essential question(s): How can I use similar right triangles to solve real-world problems?

Suggested materials:

- heavy cardstock, cardboard, un-ruled index cards, or other heavy paper
- compass and straightedge
- miras
- patty paper
- protractors
- centimeter rulers
- Geometer's sketchpad or other dynamic software

Warm-up: Give each student a piece of heavy paper-cardstock, unlined index cards, etc., a compass, and a straightedge. Post the following:

Use a compass and a straightedge to construct an equilateral triangle.

Opening: Students will be asked, in this task and the next, to construct several different triangles. The warm-up is designed to remind students of some of the constructions they have learned and to begin the construction that they will be asked to complete in *Item 1b* of the student task. Allow students to share methods they used for constructing an equilateral triangle.

Worktime: Students should work problems 1 - 3 of the student task. Although the student task in this supplement has been revised, GaDOE teacher Notes are still appropriate.

In *Item 1b*, students are asked to construct a 15° - 75° - 90° triangle. Allow them to use the equilateral triangle constructed in the warm-up, as a starting place for this problem. Guiding questions that might be asked include;

- How might we construct a 75° angle?
- What methods did we use in the previous lesson that might help us with this?
- How might an equilateral triangle help you to construct a 75° angle?

(See GaDOE Teacher Notes)

Measurement error will cause student solutions to differ slightly even when constructions are performed correctly and measurements are fairly precise. This should be discussed with the whole group. (See Teacher Notes.)

(Note: Be sure students save their constructions as they will be used in the next several lessons.)

In Item 2, students are to find right triangles in their environment that they *can measure*. These triangles may be found in their classrooms, the halls, or any other areas of the building or grounds that are appropriate for them to work. You may also want to bring in magazines or other publications that they may use as well. Measuring parts of triangles found in pictures may be easier that measuring angles in real-life objects.

Closing: The major idea addressed here is that ratios of side lengths of similar right triangles depend on the measures of the acute angles of the triangles, not on the lengths of the legs. This fact builds the understanding necessary to begin to investigate right triangle trigonometry. Allow students to share some of their problems.

Homework: Have four students post the problems that they wrote in class today. Ask all students to copy *three* of the four problems that they did *not* work in class and complete them for homework.

Differentiated support/enrichment: You may want to allow students who struggle with constructions to use miras, patty paper, and/or Geometer's Sketchpad to construct the triangles in both the *warm-up* and *Item 2*. If using Sketchpad, print the constructions, cut them out, and paste onto cardstock.

Check for Understanding:

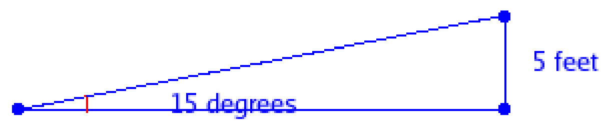
Resources/materials for Math Support: Students should preview:

- basic constructions, including copying a segment; copying an angle; bisecting a segment; bisecting an angle; and constructing the perpendicular bisector of a line segment; and
- solving simple proportions.

Mathematics II
Finding Right Triangles in Your Environment
Day 1 Student Task

1. An older building in your school district sits on the side of a hill and is accessible from ground level on both the first and second floors. However, access at the second floor requires use of several stairs. Amanda and Tom have been given the task of designing a ramp so that people who cannot use stairs can get into the building on the second floor level. The rise has to be 5 feet, and the angle of the ramp has to be 15 degrees.

    a. Tom and Amanda need to determine how long that ramp should be. One way to do this is to use a compass and straightedge to construct a 15°- 75°- 90° triangle on your paper. Such a triangle must be similar to the triangle defining the ramp. ***Explain why the triangles are similar.***



The diagram is not to scale.

    b. Construct a 15°- 75°- 90° triangle on cardstock, cardboard, or other heavy paper using straightedge and compass or other geometry tools provided by your teacher. Use a protractor to verify the angle measurements.

    c. Use similarity of the ramp triangle and measurements from your constructed triangle to find the length of the ramp. (Save the triangle and its measurements. You'll need them in the next Learning Task also.)

2. Your next job is to make up your own problem similar to the ramp problem that you solved in *Item 1*. Find an existing right triangle in your environment that you can measure. Measure the angles and sides of this existing triangle, and use these measurements to create your problem. Look again at the diagram of the ramp in *Item 1a*. What information was given and what information were you asked to find? Be sure to provide, and ask for, the same type of information in your problem. Include a sketch of your triangle that contains the given information.

3. Exchange triangle problems with your partner or another student in your class. Work your partner's problem and compare your results to his/her solution. Are they the same? If not, examine the problem together until you reach consensus on the solution.

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Mathematics II: Unit 2
Right Triangle Trigonometry

Task 3: Create Your Own Triangles

Mathematics II
Task 3: *Create Your Own Triangles* **Day 1/2**
(GaDOE TE #1 - 2)

MM2G2. Students will define and apply sine, cosine, and tangent ratios to right triangles.

a. Discover the relationship of the trigonometric ratios for similar triangles.

New vocabulary: opposite side, adjacent side, letters of the Greek alphabet as variables, sine of θ, cosine of θ, tangent of θ

Mathematical concepts/skills:

- drawing right triangles with acute angles of differing degrees
- determining positions of the legs of right triangles in relation to a given acute angle
- understanding and evaluating sines, cosines, and tangents of acute angles as the ratios of side lengths of right triangles
- understanding that the values of the trigonometric functions of a given acute angle are constant and depend strictly on the measure of the angle, not on the side lengths of the right triangle
- understanding that trigonometric values of acute angles are exact and can be written as exact values when side lengths are given in exact form but that values obtained by measuring side lengths will be approximations

Prior knowledge:

- properties of similar triangles
- basic constructions

Essential question(s): How can I find the sine, cosine, and tangent of a given angle θ?

Suggested materials:

- unlined paper
- compass and straightedge
- protractor
- centimeter ruler
- 15° - 75° - 90° triangle on heavy paper from previous lesson

Warm-up: Post the following:

Use the 15° - 75° - 90° triangle that you constructed in the previous task and any of the other tools available at your workspace, to draw a right triangle with an acute angle of 5°.

Opening: Discuss the warm-up. Allow students to share different methods for drawing the triangle. Choose students who opted to construct their right angles and those who opted to 'trace' the right angle using the triangle constructed in the previous lesson. Students cannot construct a 5° angle so they must use the protractor or an alternative method discovered by Archimedes. (See teacher Notes.)

Note: In this task, students are required to 'make' eight triangles in addition to the triangle completed in the previous task. The triangle created in the warm-up is the first of the eight. The original GaDOE task suggests that students construct as many of the triangles as possible and cut the triangles out in order to measure them more accurately. We opted here to allow them to construct their right angles each time or to 'trace' them using the triangle they constructed in the previous lesson. They may use protractors in each case to approximate the acute angles of the right triangles. We suggest that some students be allowed to use Geometer's Sketchpad or other software to draw and measure parts of the triangles so that these measures can be compared with others for accuracy.

We did **not** require that students cut the triangles out. We figured the mistakes they made in cutting would offset measuring mistakes and the triangles would be much easier to manage on larger sheets of paper. It is very important to stress to students that their measurements of both the angles and the sides of the triangles need to be as precise as possible.

Worktime: Students should complete problems 1 and 2 of the student task.

Students should work in groups of three to draw and measure the sides of the seven remaining triangles. They should share the work with each student drawing and measuring at least two triangles.

When most groups have had time to draw and measure their triangles, stop and have a mini-lesson on labeling right triangles and defining the three trigonometric functions addressed in this lesson.

This may be the first time some students have seen a Greek letter used to represent the measure of an angle, or the legs of a right triangle described by their position relative to a given acute angle. Make sure all students understand these concepts. (See GaDOE Teacher Notes, page 10.)

After the mini-lesson, have each group of students compile their data to complete Table 1 of the task. Each student should complete their own copy of the table.

Closing: Have different groups of students share values for a particular function, for example the sine 35°. (You might want to let them pick the value to be examined or choose one randomly.) Questions might include:

- How close are your values?
- How might we compare them?
- Are some closer than others?
- Are any of the values very different from the others?
- Do you think the values should be the same or different? Why?

By the end of the closing all students should:

- understand and be able to evaluate the sine, cosine, and tangent of a given acute angle using ratios of the side lengths of right triangles;
- realize that the value of any one function, sin 35° for example, is always the same because it is defined as the ratio of the side lengths of similar triangles;
- realize that values may differ from group to group due to measurement error.

Students may not realize at this point that values of trigonometric functions are exact but will learn that they are in the next lesson.

Homework: Students should complete any part of Table 1 not yet finished.

Differentiated support/enrichment: See notes on Angle Trisection on page 26 of the GaDOE TE.

Check for Understanding:

Resources/materials for Math Support: Students should preview:

- measuring angles with a protractor;
- using letters of the Greek alphabet as variables;
- identifying the opposite and adjacent legs of a right triangle relative to a given angle; and
- definitions of sine θ, cosine θ, and tangent θ.

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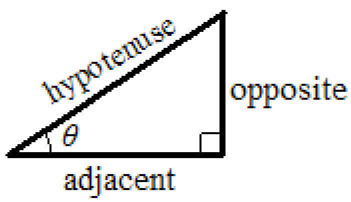
In helping students understand positions of the legs of a triangle relative to a given angle, have 3 students 'form' a triangle by standing with arms outstretched and hands touching. Ask questions like, "*Who is opposite Mary? Who is adjacent to Mary?* ".

Mathematics II
Create Your Own Triangles
Day 1 Student Task

1. Use the tools and instructions provided by your teacher to draw nine right triangles. The first triangle should have an acute angle of 5°, the next should have an acute angle of 10°, and so forth, all the way up to 45°. Note that you should have already constructed a right triangle with an angle of 15° that you saved from the previous lesson. Use it or draw a new one so that you have all nine triangles.

   As you make your triangles, label both acute angles with their measurements in degrees and label all three sides with their measurements in centimeters to the nearest tenth of a centimeter.

In the figure below, we have denoted the measure of one of the acute angles of a right triangle using the Greek letter $\boldsymbol{\theta}$. The legs of the triangle are described by their position relative to the vertex of angle θ as the leg **opposite** θ, and the leg **adjacent** to θ.



Given what we have learned from previous tasks about the ratios of side lengths of right triangles, we are now ready to define some very important new *functions*. **For any acute angle in a right triangle, we denote the measure of the angle using a variable (in this case θ)** and define three numbers related to θ as follows:

$$\textbf{sine of } \theta = \frac{\text{length of leg opposite the vertex of the angle}}{\text{length of hypotenuse}}$$

$$\textbf{cosine of } \theta = \frac{\text{length of leg adjacent to the vertex of the angle}}{\text{length of hypotenuse}}$$

$$\textbf{tangent of } \theta = \frac{\text{length of leg opposite the vertex of the angle}}{\text{length of leg adjacent to the vertex of the angle}}$$

Since the terms "opposite," "adjacent," and "hypotenuse" are used as shorthand for the lengths of the sides of a triangle in relation to an angle θ, we can give abbreviated versions of the above definitions:

$$\text{sine of } \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{cosine of } \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{tangent of } \theta = \frac{\text{opposite}}{\text{adjacent}}$$

Abbreviating even further, we can use the notation that is standard in higher mathematics.

sine of θ is denoted by $\sin(\theta)$

cosine of θ is denoted by $\cos(\theta)$

tangent of θ is denoted by $\tan(\theta)$

2. Using the measurements from the triangles that you created in *Item 1*, complete the row corresponding to each acute angle. The first three columns ask for the lengths of the sides of the triangle depending upon their position relative to the angle listed. The last three columns ask for the sine of the angle, the cosine of the angle, and the tangent of the angle. (Write your table entries in the last three columns as fractions. Do not convert to decimals.)

TABLE 1

angle measure	***opposite***	***adjacent***	***hypotenuse***	***sine (opp/hyp)***	***cosine (adj/hyp)***	***tangent (opp/adj)***
5°						
10°						
15°						
20°						
25°						
30°						
35°						
40°						
45°						
50°						
55°						
60°						
65°						
70°						
75°						
80°						
85°						

Mathematics II
Task 3: ***Create Your Own Triangles*** **Day 2/2**
(GaDOE TE Create Your Own Triangles #3 – 8
Discovering Trigonometric Ratio Relationships #1 - 4)

MM2G2. Students will define and apply sine, cosine, and tangent ratios to right triangles.

a. Discover the relationship of the trigonometric ratios for similar triangles.
b. Explain the relationship between the trigonometric ratios of complementary angles.

New vocabulary: trigonometric identities

Mathematical concepts/skills:

- using measures of special right triangles to determine exact values of trigonometric ratios for angles of 30^o, 60^o and 45^o
- values of the trigonometric functions of a given acute angle are constant and depend strictly on the measure of the angle, not on the side lengths of the right triangle
- trigonometric values of acute angles are exact and can be written as exact values when side lengths are given in exact form but that values obtained by measuring will be approximations
- examining the trigonometric ratios as non-linear functions
- deriving trigonometric identities

Prior knowledge:

- properties of similar triangles
- Pythagorean Theorem
- complementary angles

Essential question(s): What special relationships exist among the trigonometric ratios?

Suggested materials:

Warm-up: Post the following:

Sketch a 30° – 60° – 90° triangle and label the sides with any appropriate measures.

Opening: Allow students to share their sketches of 30° – 60° – 90° triangles. Be sure to choose students who used different measures for the lengths of the sides of their triangles.

Worktime: Students should complete problems 3 - 9 of the student task. As students begin work, monitor carefully to be sure they remember or are referring back to the definitions of the trigonometric functions learned in the previous lesson.

Note: This student task combines problems 3 – 8 of the original DOE task *Create Your Own Triangles*, and problems 1 – 4 of *Discovering Trigonometric Ratio Relationships.* However, GaDOE Teacher Notes are still appropriate and should be utilized in teaching this lesson.

Closing: Allow students to share their work.

By the end of the closing all students should understand that:

- values of trigonometric functions depend on the value of the angle, not on the lengths of the sides of the right triangles;
- values of trigonometric functions are exact;
- the sine, cosine, and tangent of a given angle are nonlinear functions of that angle;
- for each pair of complementary angles in a right triangle, the sine of one angle is the cosine of its complement;
- for each pair of complementary angles in a right triangle, the tangent of one angle is the reciprocal of the tangent of its complement; and
- $\tan(\theta) = \sin(\theta)/\cos(\theta)$

Homework: See homework following the student task.

Differentiated support/enrichment:

Check for Understanding:

Resources/materials for Math Support: Students should preview:

- definition of a function;
- vertical line test;
- definition of a linear function;
- simplifying complex fractions; and
- complementary angles.

Mathematics II
Create Your Own Triangles
Day 2 Student Task

3. Earlier in this unit, you discovered a special relationship among the side lengths of any 30° - 60° - 90° triangle.

   a. Sketch a 30° - 60° - 90° triangle and label its sides with any appropriate lengths, using exact values in simple radical form.

   b. Use your sketch and the definitions you have learned today to find the following values;

      sin (30°) cos (30°) tan (30°)

   c. Should the sine of 30° always be the same no matter what 30° - 60° - 90° triangle you use? Why or why not? What about the cosine of 30°? The tangent of 30°?

   d. How do the values you found in part *b* above, compare to the corresponding values for 30° that you listed in Table 1? Explain how this does or does not contradict your thinking in part c.

4. You have also discovered a special relationship among the side lengths of any 45° - 45° - 90° triangle.

   a. Sketch a 45° - 45° - 90° triangle and label its sides with any appropriate lengths, using exact values in simple radical form.

   b. Use your sketch to find;

      sin (45°) cos (45°) tan (45°)

   c. How do the values you found in part *b* above, compare to the corresponding values for 45° that you listed in Table 1? Are they the same? Which values do you think are more exact? Why?

5. In any right triangle with an angle of 80°, approximately what is the ratio of the length of the side opposite 80° to the length of the hypotenuse? Explain.

6. Explain why the trigonometric ratios of sine, cosine, and tangent define ***functions* of *θ*, where 0° < *θ* < 90°.**

7. Are the functions sine, cosine, and tangent linear functions? Why or why not?

There are some special relationships that exist among the trigonometric functions that you have learned so far. We will investigate two of those relationships in the remaining items of this task.

8. Look back at the definitions of the sine, cosine, and tangent functions.

   a. Write an equation relating the sine, cosine, and tangent that is always true. Explain how you know the equation is always true. Statements that are always true are referred to as **identities.** In higher mathematics, there are many **trigonometric identities**.

   b. Choose one angle from Table 1 above, and verify that your identity is true for that angle.

9. The acute angles of a right triangle are complementary. Why?

   a. Identify two pairs of complementary angles from Table 1 and copy the appropriate information for those angles into the table below.

	sine	cosine	tangent
Angle			
Complement			
Angle			
Complement			

   What relationships among the values do you notice? Do you think these relationships hold true for all pairs of complementary angles in right triangles? Explain your reasoning.

   b. If θ is the degree measure of an acute angle, what is the measure of its complement? Write an algebraic expression that always gives the complement of θ.

   c. State the relationships you described in part *a* as **trigonometric identities** involving sines, cosines, and/or tangents of θ and its complement.

Mathematics II
Creating Your Own Triangles
Day 2 Homework

Determine whether each statement listed below is **always true**, **sometimes true**, or **never true**. In each case, justify your decision using definitions, properties, or theorems; or by giving counterexamples. Illustrate your argument with a sketch, when possible.

1. Similar triangles are congruent.

2. The sine of an angle is equal to the cosine of that angle.

3. If two angles of a triangle measure 30° and 60°, the length of the side opposite the 30° angle is half the length of the side opposite the 60° angle.

4. The sine of an acute angle is equal to the cosine of its complementary angle.

5. As the measure of an acute angle increases, the tangent of the angle deceases.

6. In triangle *DEF*, if the measure of angle *F* is 90°, *DE* = 15, and *DF* = 12, the tangent of *E* is less than 1.

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Mathematics II: Unit 2
Right Triangle Trigonometry

Task 4: Find That Side or Angle

Mathematics II
Find That Side or Angle **Day 1/1**
(GaDOE TE # 1 - 8)

MM2G1. Students will identify and use special right triangles.
a. Determine the lengths of sides of 30°- 60°- 90° triangles.
b. Determine the lengths of sides of 45°- 45°- 90° triangles.

MM2G2. Students will define and apply sine, cosine, and tangent ratios to right triangles.
c. Solve application problems using the trigonometric ratios.

New vocabulary: angle of elevation, angle of depression

Mathematical concepts/skills:
- finding side lengths of special right triangles using known ratios of sides of special right triangles and trigonometric ratios of angles of special right triangles
- finding side lengths of general right triangles using trigonometric ratios
- finding acute angles given trigonometric ratios for the angle
- solving real-world problems using trigonometric ratios and special right triangles

Prior knowledge:
- Pythagorean Theorem

Essential question(s): How can I use trigonometric ratios and special right triangles to solve real-world problems?

Suggested materials: graphing calculator

Warm-up: Ask students to read *Item 1* of the student task and draw a sketch that represents the situation.

Opening: Have a student read *Item 1* of the task and ask another student to share their sketch. Discuss the picture, making sure that all students understand the situation.

Worktime: Student should complete *Items 1 – 8* of the task. After students have had time to complete problems 1 and 2 of the task, discuss the problems. Then have a short mini-lesson on using the graphing calculator to find values of trigonometric functions of acute angles given in degrees. (See GaDOE teacher Notes.) Ask students how they think they might find the measure of an angle θ if they know the sin (θ). Discuss using the 2^{nd} function key to find an angle when the trigonometric function of the angle is known. *Do not discuss inverse functions in detail. This topic will be addressed in unit 5.*

As you monitor student work, make sure that all students understand what is meant by an angle of elevation and an angle of depression.

Closing: Allow students to share their work. (See GaDOE teacher notes.)

Homework: In preparation for the next task, have students look up their local latitude and write short paragraphs defining the following terms:

- autumnal equinox
- vernal equinox
- summer solstice
- winter solstice

Differentiated support/enrichment:

Check for Understanding:

Resources/materials for Math Support: Students should preview:

- use of the graphing calculator to find values of sine, cosine, and tangent for acute angles given in degree measure
- use of the graphing calculator to find angles in degree measure given sine, cosine, and/or tangent of the angle
- problems involving angles of elevation and angles of depression
- application problems similar to those included in the task

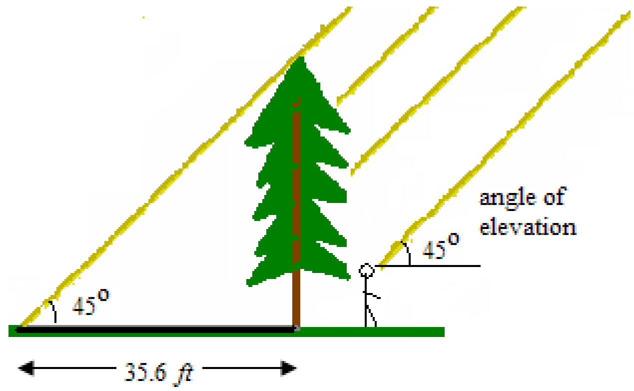
Mathematics II
Find That side of Angle
Day 1 Student Task

1. A ladder is leaning against the outside wall of a building. The ladder is exactly10 feet long and makes an angle of 60° with the ground. Sketch a picture of the situation described.

   If the ground is level, what angle does the ladder make with the side of the building? Label your picture with the information that you know so far.

   Use *two different methods* discussed in this unit to find how far up the building the ladder reaches. Give an exact value and then approximate to the nearest inch.

2. One afternoon, a tree casts a shadow that is 35.6 feet long. At that time, the **angle of elevation** of the sun is 45°, as shown in the figure at the right. We speak of the angle of elevation of the sun because we must raise, or elevate, our eyes from looking straight ahead (looking in the horizontal direction) to see the position of the sun above us. (Due to the distance from the sun to the Earth, rays of sunlight are all parallel, as shown in the figure.)

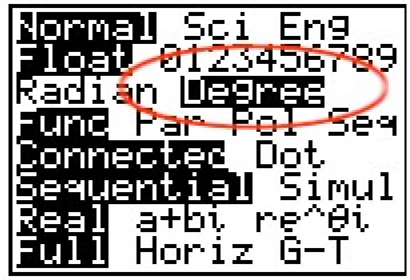


   a. How tall is the tree?

   b. What is tan (45°)?

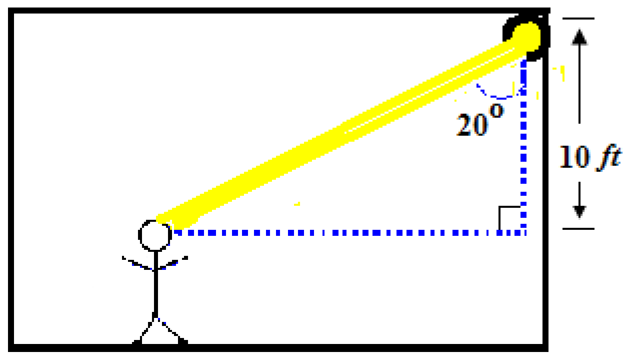
   c. What's the connection between *part a* and *part b?*

The first two problems in this task involve trigonometric ratios in special right triangles, where all the ratios are known. However, there are many applications that do not involve special right triangles. Graphing calculators include keys to give values for the sine, cosine, and tangent functions that are accurate approximations for all trigonometric ratios of degree measures greater than 0° and less than 90°. You should use calculator values for trigonometric functions, as needed, for the remainder of this task.



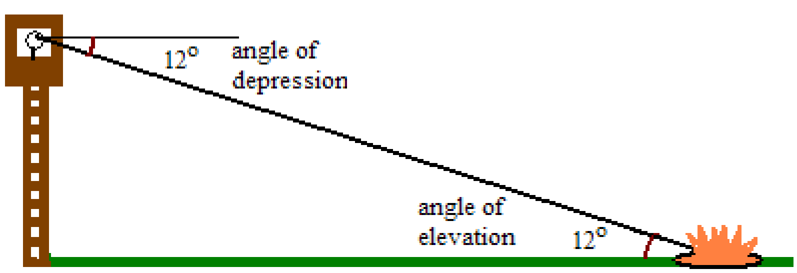
In higher mathematics, it is standard to measure angles in radians. Learning more about this method of angle measure is a topic for Mathematics IV. The issue concerns you now because you need to make sure that your calculator is in **degree mode** (and not radian mode) before you use it for finding values of trigonometric ratios. If you are using any of the TI-83/84 calculators, press the MODE button, then use the arrow keys to highlight "Degree" and press enter. The graphic at the right shows how the screen will look when you have selected degree mode. To check that you have the calculator set correctly, check by pressing the TAN key, 45, and then ENTER. As you know, the answer should be 1. If you are using any other type of calculator, find out how to set it in degree mode, do so, and check as suggested above. Once you are sure that your calculator is in degree mode, you are ready to proceed to the remaining items of the task.

3. The main character in a play is delivering a monologue, and the lighting technician needs to shine a spotlight onto the actor's face. The light being directed is attached to a ceiling that is 10 feet above the actor's face. When the spotlight is positioned so that it shines on the actor's face, the light beam makes an angle of 20° with a vertical line down from the spotlight. How far is it from the spotlight to the actor's face?



   How much further away would the actor be if the spotlight beam made an angle of 32° with the vertical?

4. A forest ranger is on a fire lookout tower in a national forest. His observation position is 214.7 feet above the ground when he spots an illegal campfire. The **angle of depression** of the line of site to the campfire is 12°. (See the figure below.)

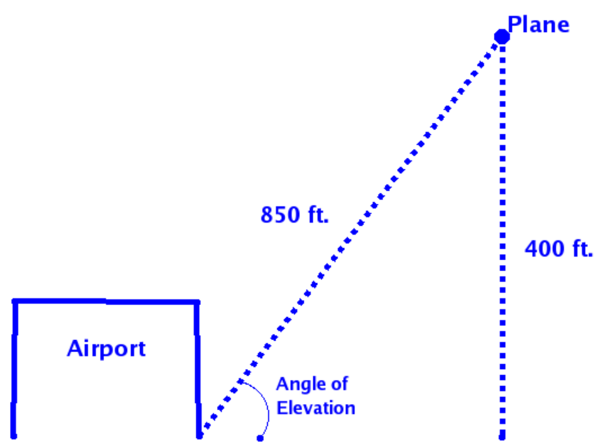


Note that an angle of depression is measured down from the horizontal because, to look down at something, you need to lower, or depress, your line of sight from the horizontal. We observe that the line of sight makes a transversal across two horizontal lines, one at the level of the viewer (such as the level of the forest ranger) and one at the level of the object being viewed (such as the level of the campfire). Thus, the angle of depression is the angle looking down from the fire lookout tower to the campfire, and the angle of elevation is the angle looking up from the campfire to the tower. The type of angle that is used in describing a situation depends on the location of the observer.

a. What is the relationship between the angle of depression and the corresponding angle of depression. Why?

b. Assuming that the ground is level, how far is it from the base of the tower to the campfire?

5. An airport is tracking the path of one of its incoming flights. If the distance to the plane is 850 ft. (from the ground) and the altitude of the plane is 400 ft.



a. What is the sine of the angle of elevation from the ground at the airport to the plane (see figure at the right)?

b. What are the cosine and tangent of the angle of elevation?

c. Now, use your calculator to find the measure of the angle itself. Pressing "2nd" followed by one of the trigonometric function keys finds the degree measure corresponding to a given ratio. Press 2nd, SIN, followed by the sine of the angle from *part a* . What value do you get?

d. Press 2nd, COS, followed by the cosine of the angle from *part b*. What value do you get?

e. Press 2nd, TAN, followed by the tangent of the angle from *part b*. What value do you get?

Did you notice that, for each of the calculations in *parts a – c,* the name of the trigonometric ratio is written with an exponent of -1? These expressions are used to indicate that we are starting with a trigonometric ratio (sine, cosine, or tangent, respectively) and going backwards to find the angle that gives that ratio. You'll learn more about this notation in Unit 5. For now, just remember that it signals that you are going backwards from a ratio to the angle that gives the ratio.

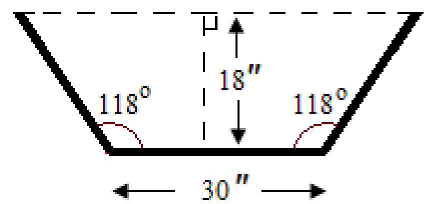
f. Why did you get the same answer each time in *parts c - e*?

g. To the nearest hundredth of a degree, what is the measure of the angle of elevation?

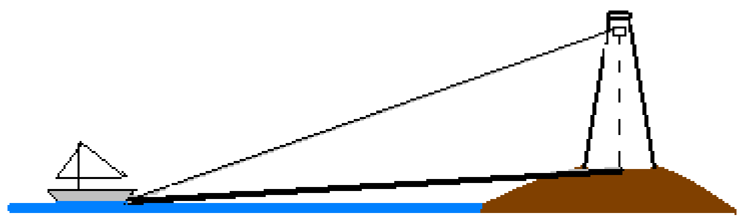
h. Look back at Table 1 from the *Create Your Own Triangles Task*. Is your answer to *part g* consistent with the table entries for sine, cosine, and tangent?

6. The top of a billboard is 40 feet above the ground. Sketch an illustration and determine the angle of elevation of the sun when the billboard casts a 30-foot shadow on level ground?

7. The ends of a hay trough for feeding livestock have the shapes of congruent isosceles trapezoids as shown in the figure at the right. The trough is 18 inches deep, its base is 30 inches wide, and the sides make an angle of 118° with the base. How much wider is the opening across the top of the trough than the base?



8. An observer in a lighthouse sees a sailboat out at sea. The angle of depression from the observer to the sailboat is 6°. The base of the lighthouse is 50 feet above sea level and the observer's viewing level is 84 feet above the base. (See the figure at the right, which is not drawn to scale.)



a. What is the distance from the sailboat to the observer?

b. To the nearest degree, what is the angle of elevation from the sailboat to the base of the lighthouse?

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Mathematics II: Unit 2
Right Triangle Trigonometry

Task 5: Finding the Overhang

Mathematics II
Task 5: *Finding the Overhang* **Day 1/1**
(GaDOE TE Culminating Task)

MM2G2. Students will define and apply sine, cosine, and tangent ratios to right triangles.

c. Solve application problems using the trigonometric ratios.

New vocabulary: autumnal equinox, vernal equinox, summer solstice, winter solstice

Mathematical concepts/skills:

- converting a verbal description to a scale drawing
- determining angles using the Triangle Sum Theorem
- finding side lengths using the Pythagorean Theorem
- using trigonometric ratios to find measures of triangles

Prior knowledge:

- converting a verbal description to a scale drawing
- determining angles using the Triangle Sum Theorem
- finding side lengths using the Pythagorean Theorem

Essential question(s): How can you construct a house to maximize the energy of the sun?

Suggested materials: rulers, protractors, compass, graph paper, calculator

Warm-up: Have students compare their discussions of the terms assigned for homework: autumnal equinox, vernal equinox, summer solstice, and winter solstice.

Opening: Discuss the terms compared during the warm-up. Ask students what they found to be the latitude of their area. Share with students the derivation of this task, discussed in the task notes and re-copied here.

The house described in this task was actually built by a gentleman in Montana seeking to minimize the effect of his home on the environment surrounding it, and in the process, to reduce his energy cost. Materials used were chosen for their ability to reduce the amount of energy needed to maintain the home.

In seeking to maximize the energy of the sun in the winter and minimize it in the summer, the builder approached the faculty at Georgia Tech for help in deciding how to construct the roof. The questions asked of students in the task are the same questions explored at Tech. It is good for students to realize that the mathematics they are learning is used in real, uncontrived situations everyday to solve problems.

Worktime: Allow students to work in groups of 2, 3 or 4. Monitor student work in order to ask guiding questions but do not tell students how to do the task. Students may need to struggle for some time in order to understand the picture and what is really being asked of them.

Closing: Students should present their work as a project. (See GaDOE Teacher Notes.)

Homework:

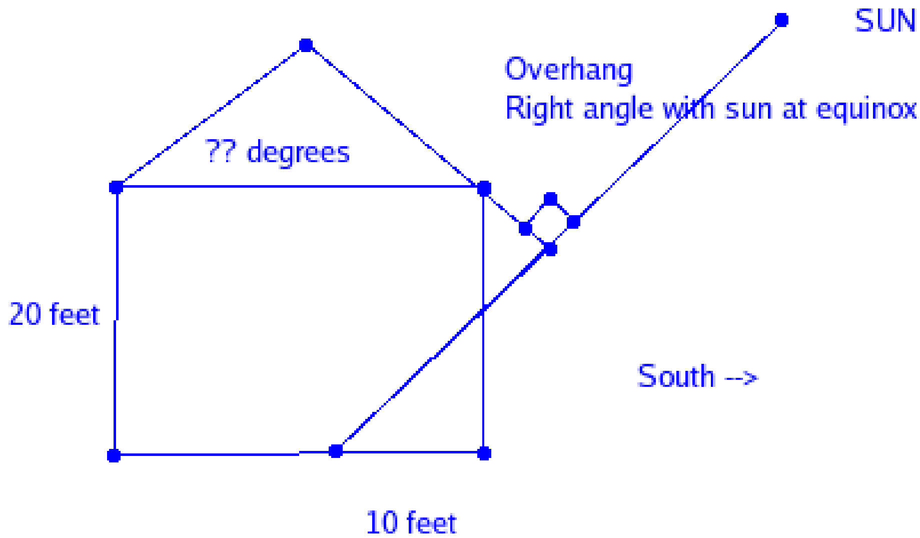
Differentiated support/enrichment:

Check for Understanding:

Resources/materials for Math Support: Students should preview the ideas in this task related to the positions of the sun. Students may need to investigate these concepts using three dimensional figures for the earth and the sun and then transfer the ideas to a two-dimensional perspective.

Mathematics II
Finding the Overhang
Student Task

A group of architects is designing a house to be built in your city. A line drawing of the house is shown below. The back of the house faces due south, and is largely windows.



- At 12 noon local time (this is not the same as 12 noon standard time, but is close to standard time) the sun is as high in the sky as it will be that day. On the **Vernal equinox** and **autumnal equinox** (look up these terms) the sun's highest point is 90 minus the latitude in degrees. (Why?)
- At the summer solstice, the sun (at local noon) is about 23 degrees above the position at the equinoxes, and at the winter solstice, the sun at its highest is about 23 degrees lower than it is at the equinoxes.
- Since you want the winter sun (to warm up the room) and you don't want the summer sun, the house is designed to let in (most of) the winter sun, and little of the summer sun. The picture shows the sun at the **Vernal equinox** or **autumnal equinox**.

The roof is pitched so that, when viewed from floor level inside the house and 10 feet in from the back wall of the house, the sun is right at the end of the overhang. In addition, the line to the sun from a point on the floor (inside the house) that is10 feet back from the outside wall is perpendicular to the roof overhang (as shown in the line drawing).

1. **Draw** the picture to scale at your latitude at the Vernal equinox or autumnal equinox.

2. Keeping the house the same, **draw** the picture to scale at your latitude at the summer and winter solstices. See what happens.

3. **Answer these questions: What is the pitch of the roof, and how long is the overhang?** (Hint: Examine triangle whose vertices are the point 10 feet back on the floor, the point where the line to the sun intersects the back wall and the top of the back wall.) The house is 27 feet from front to back (north to south).

   In finding these answers, you should find all the dimensions of the house that are not already given in the above figure. (Hint: Find as many angles as you can. Only then use Trigonometry to find the missing distances.)