Atlanta Public Schools
Teacher's Curriculum Supplement
Mathematics I: Unit 5

This document has been made possible by funding from the GE Foundation Developing Futures grant, in partnership with Atlanta Public Schools. It is derived from the Georgia Department of Education Math I Frameworks and includes contributions from Georgia teachers. It is intended to serve as a companion to the GA DOE Math I Frameworks Teacher Edition. Permission to copy for educational purposes is granted and no portion may be reproduced for sale or profit.

Preface

We are pleased to provide this supplement to the Georgia Department of Education's Mathematics I Framework. It has been written in the hope that it will assist teachers in the planning and delivery of the new curriculum, particularly in this first year of implementation. This document should be used with the following considerations.

- The importance of working the tasks used in these lessons cannot be overstated. In planning for the teaching of the Georgia Performance Standards in Mathematics teachers should work the task, read the teacher notes provided in the Georgia Department of Education's Mathematics I Framework Teacher Edition, and *then* examine the lessons provided here.

- This guide provides day-by-day lesson plans. While a detailed scope and sequence and established lessons may help in the implementation of a new and more rigorous curriculum, it is hoped that teachers will assess their students informally on an on-going basis and use the results of these assessments to determine (or modify) what happens in the classroom from one day to the next. Planning based on student need is much more effective than following a pre-determined timeline.

- It is important to remember that the Georgia Performance Standards provide a balance of concepts, skills, and problem solving. Although this document is primarily based on the tasks of the Framework, we have attempted to help teachers achieve this all important balance by embedding necessary skills in the lessons and including skills in specific or suggested homework assignments. The teachers and writers who developed these lessons, however, are not in your classrooms. It is incumbent upon the classroom teacher to assess the skill level of students on every topic addressed in the standards and provide the opportunities needed to master those skills.

- In most of the lesson templates, the sections labeled *Differentiated support/enrichment* have been left blank. This is a result of several factors, the most significant of which was time. It is also hoped that as teachers use these lessons, they will contribute their own ideas, not only in the areas of differentiation and enrichment, but in other areas as well. Materials and resources abound that can be used to contribute to the teaching of the standards.

On the topic of differentiation, it is critical to reiterate that many of the strategies used in a standards-based mathematics classroom promote differentiation. These strategies include

- the use of rich tasks with multiple points of entry and more than one path to a solution
- flexible grouping of students
- multiple representations of mathematical concepts
- writing in mathematics
- monitoring of progress through on-going informal and formative assessments
- and analysis of student work.

We hope that teachers will incorporate these strategies in each and every lesson.
It is hoped that you find this document useful as we strive to raise the mathematics achievement of all students in our district and state. Comments, questions, and suggestions for inclusions related to the document may be emailed to Dr. Dottie Whitlow, Executive Director, Mathematics and Science Department, Atlanta Public Schools, dwhitlow@atlantapublicschools.us

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Explanation of the Terms and Categories Used in the Lesson Template

Task: This section gives the suggested number of days needed to teach the concepts addressed in a task, the task name, and the problem numbers of the task as listed in the Georgia Department of Education's Mathematics I Framework Teacher Edition (GaDOE TE).

In some cases new tasks or activities have been developed. These activities have been named by the writers.

Standard(s): Although each task addresses many Math I standards and uses mathematics learned in earlier grades, in this section, only the key standards addressed in the lesson are listed.

New Vocabulary: Vocabulary is only listed here the first time it is used. It is strongly recommended that teachers, particularly those teaching Math Support, use interactive word walls. Vocabulary listed in this section should be included on the word walls.

Mathematical concepts/topics: Major concepts addressed in the lesson are listed in this section whether they are Math I concepts or were addressed in earlier grades.

Prior knowledge: Prior knowledge includes only those topics studied in previous grades. It does not include Math I content taught in previous lessons.

Essential Question(s): Essential questions may be daily and/or unit questions.

Suggested materials: This is an attempt to list all materials that will be needed for the lesson, including consumable items, such as graph paper; and tools, such as graphing calculators and compasses. This list does not include those items that should always be present in a standards-based mathematics classroom such as markers, chart paper, and rulers.

Warm-up: A suggested warm-up is included with every lesson. Warm-ups should be brief and should focus student thinking on the concepts that are to be addressed in the lesson.

Opening: Openings should set the stage for the mathematics to be done during the work time. The amount of class time used for an opening will vary from day-to-day but this should not be the longest part of the lesson.

Worktime: The problem numbers have been listed and the work that students are to do during the worktime has been described. A student version of the day's activity follows the lesson template in every case. In order to address all of the standards in Math I, some of the problems in some of the tasks have been omitted and less time consuming activities have been substituted for those problems. In many instances, in the student versions of the tasks, the writing of the original tasks has been simplified. In order to preserve all vocabulary, content, and meaning it is important that teachers work the original tasks as well as the student versions included here.

Teachers are expected to both facilitate and provide some direct instruction, when necessary, during the work time. Suggestions related to student misconceptions, difficult concepts, and deeper meaning have been included in this section. However, the teacher notes in the GaDOE Math I Framework are exceptional. In most cases, there is no need to repeat the information

provided there. Again, it is imperative that teachers work the tasks and read the teacher notes that are provided in GaDOE support materials.
Questioning is extremely important in every part of a standards-based lesson. We included suggestions for questions in some cases but did not focus on providing good questions as extensively as we would have liked. Developing good questions related to a specific lesson should be a focus of collaborative planning time.

Closing: The closing may be the most important part of the lesson. This is where the mathematics is formalized. Even when a lesson must be continued to the next day, teachers should stop, leaving enough time to "close", summarizing and formalizing what students have done to that point. As much as possible students should assist in presenting the mathematics learned in the lesson. The teacher notes are all important in determining what mathematics should be included in the closing.

Homework: In some cases, homework suggestions are provided. Teachers should use their resources, including the textbook, to assign homework that addresses the needs of their students.

Homework should be focused on the skills and concepts presented in class, relatively short (30 to 45 minutes), and include a balance of skills and thought-provoking problems.

Differentiated support/enrichment: On the topic of differentiation, it is critical to reiterate that many of the strategies used in a standards-based mathematics classroom promote differentiation. These strategies include

- the use of rich tasks with multiple points of entry and more than one path to a solution
- flexible grouping of students
- multiple representations of mathematical concepts
- writing in mathematics
- monitoring of progress through on-going informal and formative assessments
- and analysis of student work.

Check for understanding: A check for understanding is a short, focused assessment-a ticket out the door, for example. The Coach Book may be a good resource for these items.

Resources/materials for Math Support: Again, in some cases, we have provided materials and/or suggestions for Math Support. This section should be personalized to your students, class, and/or school, based on your resources.

Table of Contents

Mathematics I
Unit 5

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Unit 5 Timeline

Task 1: Logo Symmetry	2 days
Task 2: Even, odd, or Neither?	2 days
Task 3: Just Jogging	1 day
Task 4: Resistance	2 days
Task 5: Shadows and Shapes	2 days

Task Notes

Task 1: Logos
The task entitled *Logo Symmetry Learning Task* as written in the GaDOE Framework has been separated into two major tasks for the purposes of this supplement. This first task deals primarily with transformations including; vertical shifts, vertical stretches and shrinks, and reflections across the x- and y- axes. Symmetry of graphs is discussed briefly here but the main emphasis on symmetry, including the concepts of even and odd functions, will be addressed in the next task.

Task 2: Even, Odd, or Neither
The main focus of this task is symmetry and the properties of even and odd functions. The task contains problems #1, #4, and #11 of the Logo Symmetry task in the GaDOE TE. It also contains a card sort activity that gives students opportunities to investigate and discover properties of even and odd functions and further develop their understanding of function notation.

Task 3: Just Jogging
This task was moved from Unit 2 of the GaDOE TE. The task introduces rational expressions and solving simple rational equations. All parts of the original task from the GaDOE TE are included in the lesson plans.

Task 4: Resistance
All parts of the GaDOE task are included in these lesson plans.

Computation with Rational Expressions

Task 5: Shadows and Shapes
All parts of the GaDOE task are included in these lesson plans.

Atlanta Public Schools
Teacher's Curriculum Supplement

Mathematics I: Unit 5

Task 1: Logos

Mathematics I
Task 1: Logos **Day 1/2**
(GaDOE TE # 2, #3, #5, #6)

Standard(s): MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.
c. Graph transformations of basic functions including vertical shifts, stretches, and shrinks, as well as reflections across the *x*- and *y*-axes.
d. Investigate and explain characteristics of a function: domain, range, zeros, intercepts, intervals of increase and decrease, maximum and minimum values, and end behavior
MM1A2. Students will simplify and operate with radical expressions, polynomials, and rational expressions.
a. Simplify algebraic and numeric expressions involving square root.
b. Perform operations with square roots.

New vocabulary: reflection of the graph of a function across the x- axis, reflection of the graph of a function across the y-axis

Mathematical concepts/topics: transformations of the graphs of $y = |x|$ and $y = \sqrt{x}$ including vertical shifts, vertical stretches, and reflections across the x- and y- axes; symmetry with respect to the y-axis; restricted domains and ranges; square roots of algebraic and numeric expressions; and function notation

Prior knowledge: square roots of positive numbers; transformations of objects in a plane, including translations, dilations, rotations, and reflections

Essential question(s): How can I use functions to solve problems involving art and design?

Suggested materials: graphing calculators, graph paper

Warm-up: Post the following:

Graph the functions below. For each function state the domain and range.

a. $y = |x|$
b. $y = \sqrt{x}$

Opening: Discuss the warm-up, reviewing absolute value, the square roots of positive numbers, and the domains and ranges of each function.

Explain to students that in their task today they will investigate the use of functions to create designs, logos, and other artistic creations. Have students read the scenario in problem 1 of the student task silently and then discuss the problem, making sure that students understand both the scenario and the tables used to describe the functions and the vertical lines used in the design.

Worktime: Students should work problems 1 – 4 of the student task. Given the work done in Unit 1 and the emphasis on symmetry throughout middle grades, problem 1 should be fairly easy for students.

Problems 2 and 3 address reflections across the x- and y- axes. Monitor student work carefully to be sure they understand the differences between these two processes. It is also important to help students realize that order matters when performing transformations on graphs of functions.

In problem 4, students use graphing calculators, or other technology, to investigate vertical stretches, reflections, shifts, and the effect of using the expression $\sqrt{|x|}$. Monitor student work for understanding of these concepts.

Closing: Concepts mentioned in the *Worktim*e section should be discussed carefully during the closing. The following chart shows the correspondence of problem numbers in the student task used in this supplement to the problem numbers of the GaDOE TE. See teacher notes.

Supplement Student Task	GaDOE TE
#1 a, b	#2 a, b
#1 c, d	#3 a, b
#2 a,b	#3 c, d
#3 a - g	#5 a - g
#4	#6

Homework: Students will need practice in graphing transformations of functions including vertical shifts, vertical stretches, and reflections across the x- and y- axes.

Differentiated support/enrichment:

Check for Understanding:

Resources/materials for Math Support: Concepts and vocabulary to be previewed: reflection of the graph of a function across the x- axis, reflection of the graph of a function across the y-axis, vertical shifts, and vertical stretches.

Students may need to review absolute value, finding square roots of positive numbers, graphs of basic functions, and restricted domains.

Mathematics I
Logos
Day 1 Student Task

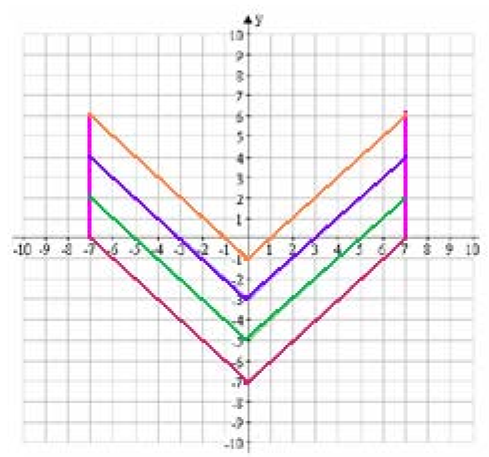
1. A textile company called "Uniform Universe" has been hired to manufacture some military uniforms. To complete the order, they need embroidered patches with the military insignia for a sergeant in the United States Army. To save on costs, Uniform Universe subcontracted a portion of their work to a foreign company. The machines that embroider the insignia design require a mathematical description. The foreign company ***incorrectly*** used the design at the right, which is the insignia for a British Sergeant. They sent the following description for the portion of the design to be embroidered in black.

Black embroidery instructions:

Vertical line	Restriction
$x = -7$	$0 \le y \le 6$
$x = 7$	$0 \le y \le 6$

Function	Domain
$y = \lvert x \rvert - 1$	$-7 \le x \le 7$
$y = \lvert x \rvert - 3$	$-7 \le x \le 7$
$y = \lvert x \rvert - 5$	$-7 \le x \le 7$
$y = \lvert x \rvert - 7$	$-7 \le x \le 7$

a. Match the lines in the design to the functions indicated in the table.



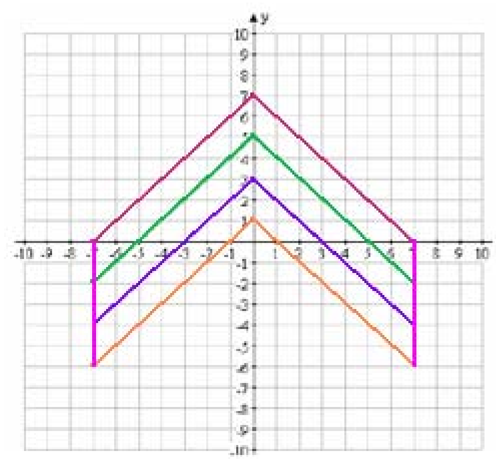
b. When the design is stitched on a machine, wide black stitching is **centered** along the lines given by the equations above. The British Sergeant's insignia has a light-colored embroidery between the lines of black. Write a description for the lines on which the light-colored stitching will be **centered**. Use a table format similar to that shown above.

c. What type of symmetry does this insignia have?

d. If it is symmetric about a point, line, or lines, write the associated coordinates of the point or equation(s) for the lines of symmetry.

2. Jessica, a manager at Uniform Universe, immediately noticed the design error when she saw some of the prototype uniforms. The sergeant's insignia was upside down from the correct insignia for a U.S. sergeant, which is shown at the right. Jessica checked the description that had been sent by the foreign contractor. She immediately realized how to fix the insignia. So, she emailed the foreign supplier to point out the mistake and to inform the company that the error could be corrected the by reflecting each of the functions in the *x*-axis.

Ankit, an employee at the foreign textile company, e-mailed Jessica back and included the graph at the right to verify that Uniform Universe would be satisfied with the new formulas.



a. Does the corrected insignia have the same symmetry as the original insignia?

b. Write the mathematical description of the design for the U.S. sergeant insignia, as shown in the graph at the right. Verify that your mathematical description yields the graphs shown.

3. When Jessica's supervisor, Malcom, saw the revised design, he told Jessica that he did not think that the U.S. Army would be satisfied. He pointed out that, while the revision did turn the design right-side up, it did not account for the slight curve in the lines in the real U.S. sergeant's insignia. He suggested that a square root function might be a better choice than an absolute value function and told Jessica to work with the foreign contractor to get a more accurate design. Jessica emailed Ankit to let him know that the design needed to be revised again to have lines with a curve similar to the picture from the U.S. Army website, as shown at right, and suggested that he try the square root function.

a. Ankit graphed the square root function, $y = \sqrt{x}$, and decided to limit the domain to $0 \le x \le 4$. Set up a grid using a scale of ½ -inch for each unit, and graph the square root function on this limited domain. For accuracy, plot points for the following domain values: 0, $\frac{1}{4}$, 1, $\frac{25}{16}$, $\frac{9}{4}$, $\frac{49}{16}$, 4.

b. Ankit saw that his graph of the square root function (on the domain $0 \le x \le 4$) looked like the curve that forms the lower right edge of the British sergeant's insignia. He knew that he could reflect the graph through the *x*-axis to turn the curve over for the U.S.

sergeant's insignia, but first he needed to reflect the graph through the *y*-axis in order to get the full curve to form the lower edge of the British sergeant's insignia. He knew that he needed to input a negative number and then get the square root of the corresponding positive number, so he tried graphing the function $y = \sqrt{-x}$ on the domain $-4 \le x \le 0$. Reproduce Ankit's graph using the scale of ½-inch for each unit and using the opposites of the domain values in part a as some accurate points on the graph: 0, $-\frac{1}{4}$, -1, $-\frac{25}{16}$, $-\frac{9}{4}$, $-\frac{49}{16}$, -4.

c. To see an additional example of reflecting a graph through the axes, graph each of the following and compare the graphs.

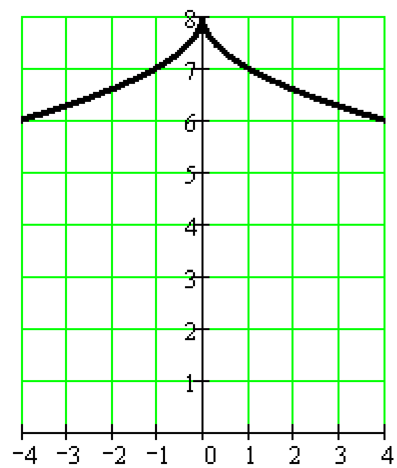
(i) $y = 2x + 3$ (ii) $y = -(2x+3) = -2x - 3$ (iii) $y = 2(-x) + 3 = -2x + 3$

d. Given an equation of the form $y = f(x)$, there is a specific process for creating a second equation whose graph is obtained by reflecting the graph of $y = f(x)$ **through the *x*-axis**, and there is different process for creating an equation whose graph is obtained by reflecting the graph of of $y = f(x)$ **through the *y*-axis**. Explain these processes in your own words.

e. Write a function that will reflect the graph of $y = \sqrt{x}$, $0 \le x \le 4$ (the graph in part a) across the *x*-axis and then shift it up 8 units. Call this function f_1, read "*f* sub one."

f. Write a function that will reflect the graph of $y = \sqrt{-x}$, $-4 \le x \le 0$ (the graph in part b) across the *x*-axis and then shift it up 8 units. Call this function f_2, read "*f* sub two."

When graphed on the same axes, the graphs of the functions f_1 and f_2 give the curve shown at the right.

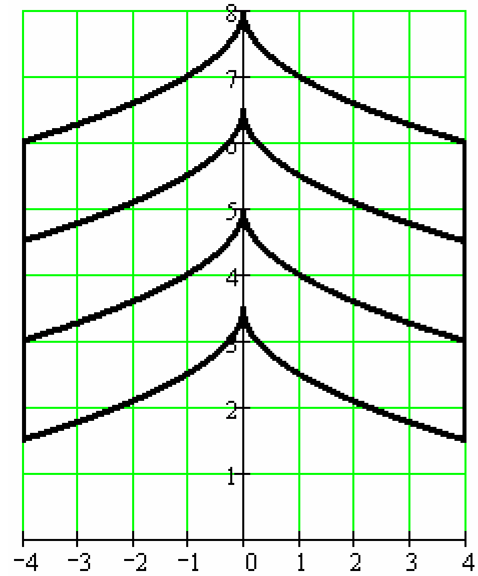


Graphs of f_1 and f_2

g. Ankit completed his mathematical definition for the U.S. Army sergeant's insignia and sent it to Jessica along with the graph shown below. Based on the graph and your work above, write Ankit's specifications for black embroidery of the insignia as shown in the graph.

Function	Domain
$y =$	
$y =$	
$y =$	
$y =$	
$y =$	
$y =$	
$y =$	
$y =$	

Vertical line	*Restriction*
$x =$	
$x =$	



Sergeant insignia, U.S. Army

4. A few days after Ankit sent the completed specifications to Uniform Universe, he looked back at the picture of the insignia and his graphs and thought that he could improve upon the proportions of the design and the efficiency of his mathematical formula. He sent the revised instructions below. Use graphing technology to examine Ankit's final mathematical definition for the insignia. Explain the effect of the changes on the functions you wrote in answer to item 3i.

Vertical line	Restriction
$x = -4$	$0.4 \le y \le 3.5$
$x = 4$	$0.4 \le y \le 3.5$

Function	Domain
$y = 7.5 - 2.0\sqrt{\lvert x \rvert}$	$-4 \le x \le 4$
$y = 6.0 - 1.8\sqrt{\lvert x \rvert}$	$-4 \le x \le 4$
$y = 4.7 - 1.65\sqrt{\lvert x \rvert}$	$-4 \le x \le 4$
$y = 3.5 - 1.55\sqrt{\lvert x \rvert}$	$-4 \le x \le 4$

Standard(s): MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.

c. Graph transformations of basic functions including vertical shifts, stretches, and shrinks, as well as reflections across the *x*- and *y*-axes.

d. Investigate and explain characteristics of a function: domain, range, zeros, intercepts, intervals of increase and decrease, maximum and minimum values, and end behavior

i. Understand that any equation in *x* can be interpreted as the equation f(*x*) = g(*x*), and interpret the solutions of the equation as the *x*-value(s) of the intersection point(s) of the graphs of *y* = f(*x*) and *y* = g(*x*).

MM1A3. Students will solve simple equations.

b. Solve equations involving radicals such as $\sqrt{x}+b=c$, using algebraic techniques.

New vocabulary: reflection of a point through the origin

Mathematical concepts/topics: transformations of the graphs including vertical shifts, vertical stretches, and reflections across the x- and y- axes; restricted domains and ranges; solving simple radical equations both algebraically and geometrically; reflections of points through the x- and y-axes; and the rotation of a point through the origin

Prior knowledge: square roots of positive numbers; transformations of objects in a plane including translations, dilations, rotations, and reflections

Essential question(s): How can I use functions to solve problems involving art and design?

Suggested materials: graphing calculators**,** graph paper

Warm-up: Post the following:

Graph the function below. Give its domain and range and discuss any other important characteristics.

$$y=\frac{1}{x}$$

Opening: Discuss the graph of $y=\frac{1}{x}$. Discussion should include asymptotes (and why they occur),domain, and range.

Worktime: Students should work problems 5 - 9 of the student task.

Closing: The following chart shows the correspondence of problems numbers in the student task used in this supplement to the problem numbers of the GaDOE TE. See teacher notes.

Supplement Student Task	GaDOE TE
#5	#7 table and graph only
#6	#8

#7	#9
#8	#10
#9	#12

Homework: Students will need practice in solving simple radical equations of the form $\sqrt{x}+b=c$ algebraically and graphically. They will also need practice in graphing transformations of the basic functions.

Differentiated support/enrichment:

Check for Understanding:

Resources/materials for Math Support: Preview should include graphing and investigating the function $y=\frac{1}{x}$. Students should discuss all characteristics of the function including: domain, range, asymptotes, and whether the function is increasing or decreasing. They should understand *why* division by zero is undefined.

Students should also preview rotation of a point through the origin, solving simple radical equations of the form $\sqrt{x}+b=c$ both algebraically and graphically, and graphing transformations of the six basic functions.

Mathematics I

Logos

Day 2 Student Task

5. Now, we will use both square root and linear functions to create a different logo. The functions are listed in the table at the bottom of this page. The logo is the shape **completely enclosed** by the graphs of the functions. Thus, in order to draw the logo, you will need to find the points of intersections among the graphs. Once you have the points of intersections, you can determine how to limit the domain of each function to specify the boundary of the logo.

Finding points of intersection:
To find the *x*-coordinates of the points of intersection algebraically, solve the following equations. (In solving the equations remember the definition of square root: $\sqrt{a} = b$ if and only if *a* and *b* are nonnegative real numbers with $a = b^2$.)

$2\sqrt{x} = 4$ $\qquad\qquad$ $2\sqrt{-x} = 4$

$-2\sqrt{x} = -4$ $\qquad\qquad$ $-2\sqrt{-x} = -4$

Verify your solutions geometrically by checking that each solution is the *x*-coordinate of the point where the graph of the function given by the expression on the left side of the equation intersects either the line $y = 4$ or the line $y = -4$

Graphing the logo;
In the table below, you are also asked to specify the relationship of the other graphs to the graph of $y = 2\sqrt{x}$ and to find the range for each function after you have restricted the domain.

Fill in the table **and** graph the logo with the limited domain. Be creative and color the logo after you are finished graphing it.

Function	Relation of the graph to graph of (i)	Domain	What is the range of the function with limited domain?
(i) $y = 2\sqrt{x}$			
(ii) $y = 2\sqrt{-x}$	Reflection through		
(iii) $y = -2\sqrt{x}$	Reflection through		
(iv) $y = -2\sqrt{-x}$	Rotation of		
(v) $y = 4$	Intersects at ( )		
(vi) $y - -4$	No intersection		

The last part of this task explores another logo, but first we explore the coordinate geometry of reflections and rotations a little further.

6. Start with a point (a, b).
   a. Assuming that (a, b) is in Quadrant I, reflect the point across the *x*-axis. Which coordinate stays the same? Which coordinate changes? What are the coordinates of the new point obtained by reflecting in the *x*-axis?

   b. Repeat part a assuming that (a, b) is in Quadrant II.

   c. Repeat part a assuming that (a, b) is in Quadrant III.

   d. Repeat part a assuming that (a, b) is in Quadrant IV.

   e. Repeat part a assuming that you have a point on the *y*-axis.

   f. Summarize: For any point (a, b), reflecting the point through the *x*-axis results in a point whose coordinates are (_ , _). Explain why this rule applies to points on the *x*-axis even though reflecting such points in the *x*-axis results in the same point as the original one.

7. Start with a point (a, b).
   a. Assuming that (a, b) is in Quadrant I, reflect the point in the *y*-axis. Which coordinate stays the same? Which coordinate changes? What are the coordinates of the new point obtained by reflecting in the *y*-axis?

   b. Repeat part a assuming that (a, b) is in Quadrant II.

   c. Repeat part a assuming that (a, b) is in Quadrant III.

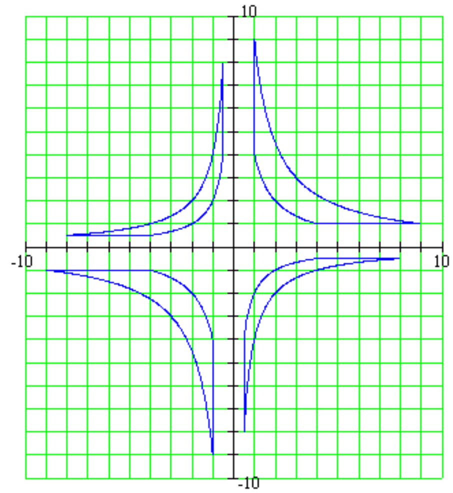
   d. Repeat part a assuming that you have a point on the *x*-axis.

   e. Summarize: For any point (a, b), reflecting the point through the *y*-axis results in a point whose coordinates are (?, ?). Explain why this rule applies to points on the *y*-axis even though reflecting such points in the *y*-axis results in the same point as the original one.

8. Start with a point (a, b).
   a. Reflect the point through the *x*-axis and then reflect that point through the *y*-axis. What are the coordinates of the twice-reflected point in terms of *a* and *b*? Does the location of the point (Quadrant I, II, III, IV, or on one of the axes) affect your answer?
   b. Start with point (a, b) again. This time reflect through the *y*-axis first and then the *x*-axis. What are the coordinates of this twice-reflected point in terms of *a* and *b*? Does the location of the point (Quadrant I, II, III, IV or on one of the axes) affect your answer?
   c. Compare the results from parts a and b. Does the order in which you perform the two different reflections have an effect on the final answer?
   d. Using the concept of slope, explain why the points (a, b) and $(-a, -b)$ are on the same line through the origin. What happens if $a = 0$ and slope is not defined for the line through the origin and the point (a, b)?

e. What are the coordinates of the point obtained by rotating the point (a, b) through 180° about the origin? Explain how to use reflection through the axes to rotate a point 180° about the origin. Note: the two points are symmetric with respect to the point (0, 0), so find this second point is also called a ***reflection of the point (a, b) through the origin***.

9. Next we explore a logo based on transformations of the basic function $y = \frac{1}{x}$. The graph of the logo is shown at the right. The curved parts of the logo lie on the graphs of the functions listed in the table below.

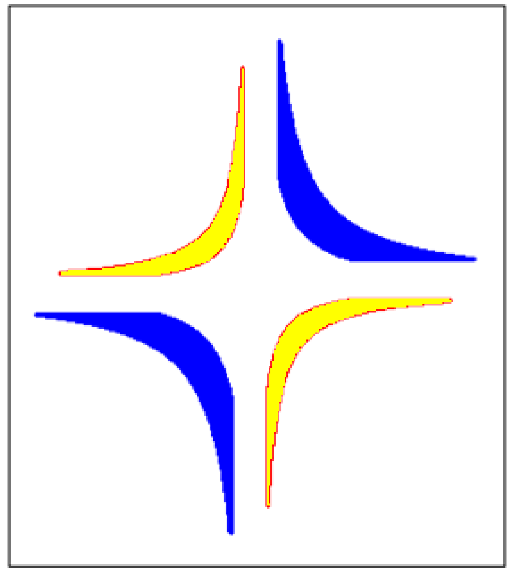


a. Complete the mathematical definition for this logo by completing the table. Determine the remaining information by applying what you know about the graphs of the functions listed to obtain equations in x and y for each curved or straight line shown at the right.

Function	Domain
$y = \frac{4}{x}$	$1 \le x \le 4$ or
$y = \frac{9}{x}$	$1 \le x \le 9$ or
$y = \frac{-2}{x}$	$\frac{1}{2} \le x \le 4$ or
$y = \frac{-4}{x}$	$\frac{1}{2} \le x \le 8$ or
$y =$	
$y =$	
$y =$	
$y =$	

Vertical line	Restriction
$x = 1$	$4 \le y \le 9$
$x = \frac{1}{2}$	$-8 \le y \le -\frac{1}{2}$

b. The completed logo, without grid lines, is shown at the right. What symmetry does this logo have?



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Mathematics I: Unit 5

Task 2: Even, Odd, or Neither?

Mathematics I

Task 2: Even, Odd, or Neither? **Day 1/2**

(For purposes of this supplement, the Logo Symmetry task of the GaDOE TE has been separated into two different tasks. The first task in this unit dealt primarily with transformations, particularly reflections. The main focus of this second task is symmetry and the properties of even and odd functions. It contains problems #1, #4, and #11 of the GaDOE TE Logo Symmetry task as well as a card sort activity that gives students opportunities to investigate and discover properties of even and odd functions and further develop their understanding of function notation.

Standard(s): MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.

a. Represent functions using function notation.

b. Graph the basic functions $f(x) = x^n$ where $n = 1$ to 3, $f(x) = |x|$, $f(x) = \sqrt{x}$, and $f(x) = 1/x$.

c. Graph transformations of basic functions including vertical shifts, stretches, and shrinks, as well as reflections across the *x*- and *y*-axes.

h. Determine graphically and algebraically whether a function has symmetry and whether it is even, odd or neither.

New vocabulary: even function, odd function

Mathematical concepts/topics: line, point, and rotational symmetry; properties of even and odd functions; graphs of the six basic functions of Mathematics I; domain and range; function notation; simplifying algebraic expressions; opposites of algebraic expressions

Prior knowledge: point, line, and rotational symmetry

Essential question(s): How can I use symmetry to understand, explain, and graph functions?

Suggested materials:

- Copy of company logos (included below), preferably projected on a white board or Promethean Board so that students can draw lines and points of symmetry directly on the logos.
- One set of symmetry cards and a symmetry table for each pair of students. (Be sure to include 3 blank cards. All are included below). It is best if the cards and table are copied on card stock.
- Copy of student task, problems 1 - 9, for each student.
- Scissors
- Tape or glue
- Graph paper
- Graphing calculators

Warm-up: Project the picture of the 17 company logos or give students copies of the logos. It is preferable that students view them in color. Allow students plenty of time, working in pairs or groups, to discuss the symmetry they see in each of the pictures. Point, line, and rotational symmetry were studied extensively in middle school so this is an opportunity to connect to prior knowledge.

Opening: Allow students to discuss the symmetry they see in the 17 logos. (Solutions can be found in problem #1 of the Logo Symmetry Learning Task in the GaDOE TE.) During this discussion, guide students to develop definitions of *line symmetry*, *point symmetry*, and *rotational symmetry*. Definitions are given here but it is hoped that students can develop comparable definitions in their own words.

- A figure has **line symmetry** if there is a line that divides the figure into two parts that are mirror images of each other.
- A figure has **rotational symmetry** if, when rotated by an angle of 180 degrees or less about its center, the figure aligns with itself.
- A figure has **point symmetry** if when rotated about the point 180 degrees it aligns with itself.

Worktime: Students should work in pairs to complete problems 1 - 8 of the student task before beginning the card sort. Students should be monitored for understanding as they work; however, they should be quite capable of completing problems 1 – 8 given the work they have done in middle school, in the first task of this unit, and in the warm-up.

Once students have completed problems 1 – 8, have a brief discussion of the problems, making sure that all students understand graphical concepts of even and odd functions.

Discuss the directions for the card sort with students. This type of activity may be new to many. The directions are shown below.

> As a way of further investigating the properties of even and odd functions, you will be asked to complete a card sort. You have been given a set of cards. Each card contains a statement, an equation, or a graph. You are to decide whether the information on the card is representative of an even function, an odd function, or a function that is neither even nor odd. Once you have made your decision, place the card in the appropriate column in the table labeled *Even, Odd, or Neither*.
>
> You have three *blank* cards. Use these cards to create one of your own graphs, equations, or statements to match **each** category.
>
> After you have placed each of your cards, choose two cards from each column and **write a justification** of your placement of these cards. Among the six cards you choose for justification, you should have at least one card that contains a graph, one that contains an equation, and one card that contains a statement. You should use multiple representations in your justifications, when possible. Equations should be justified algebraically as well as geometrically. Label each justification with the card number.

It is important that students are given time to investigate the ideas inherent in this activity. While the simple fact that a function can be even, odd, or neither is not terribly important, other ideas and skills contained here are very important; including, a deeper understanding of function notation and of the coordinate plane, algebraic representations of a variable and its opposite, and simplification of algebraic expressions.

Closing: Clear up any questions that students may still have related to whether simple graphs represent functions that are even, odd, or neither. It is important at this point to include graphs that represent functions that are neither even nor odd.

Tell students they will finish placing their cards and justifying their decisions during the next class period.

Homework:

1. For each basic function you classified as even (problem 1 of today's lesson), let *g* be the function obtained by shifting the graph down five units. Determine whether *g* is even, odd, or neither. Represent the translation algebraically and justify your answer.

2. For each basic function you classified as odd (problem 5 of today's lesson), let *h* be the function obtained by shifting the graph up three units. Determine whether *h* is even, odd, or neither. Represent the translation algebraically and justify your answer.

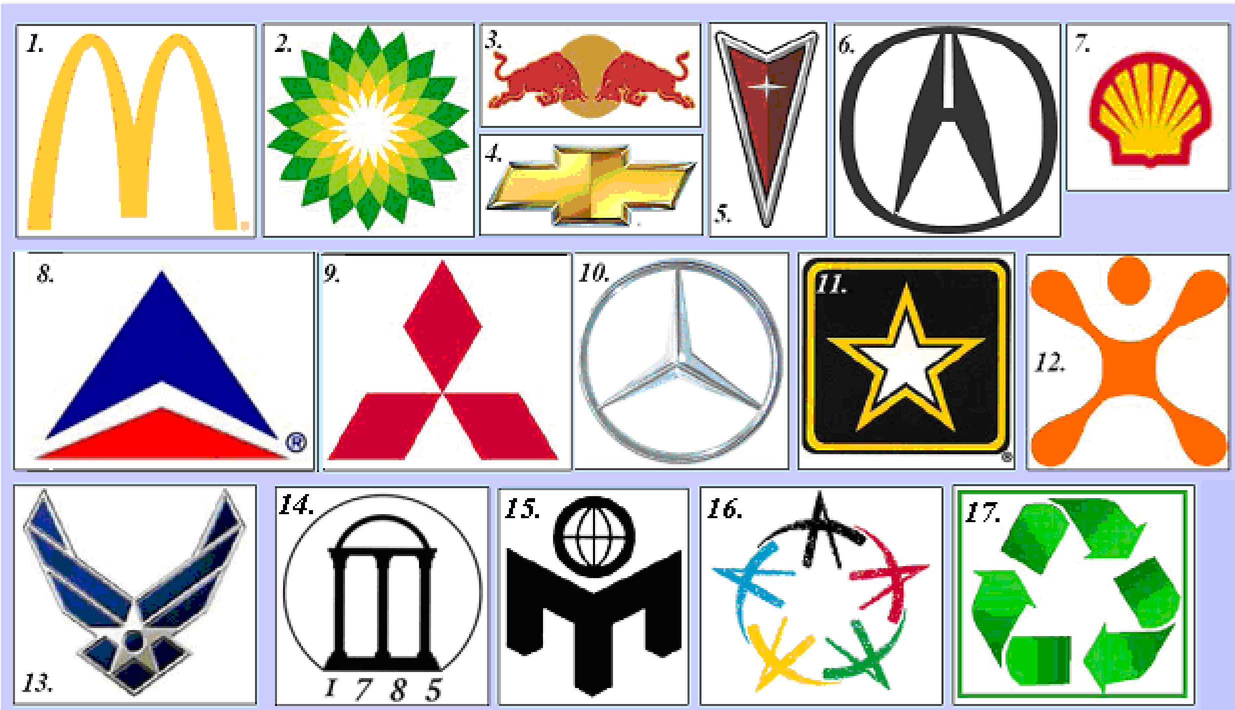
Differentiated support/enrichment:

Check for Understanding:

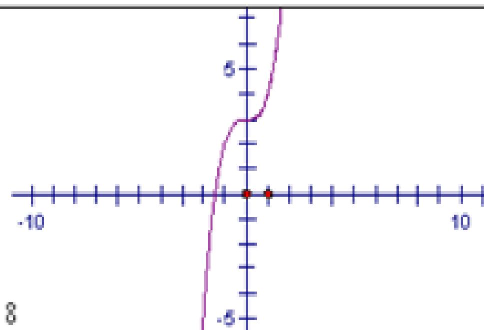
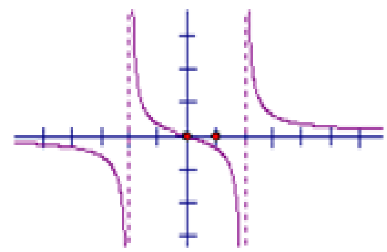
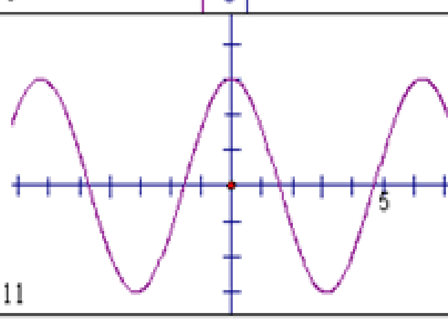
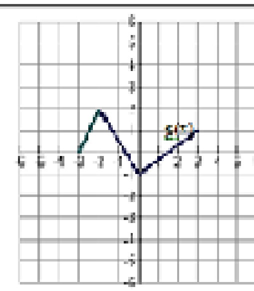
Resources/materials for Math Support:

- Preview line, point, and rotational symmetry.
- Review the relationship between a variable and its opposite. Many students still may not understand that x can represent a negative number and $-x$ can represent a positive number.
- Provide skills work in substituting $-x$ for x in algebraic expressions and then simplifying those expressions. Students will need help in determining when $f(-x) = -f(x)$, particularly when the expression representing the function is a rational expression.

We all see many company logos every day. These logos often have symmetry. For each logo shown below, identify and explain any symmetry you see.



Symmetry Cards

$f(x) = \frac{4}{x - 2}$ 1	If the point (a,b) is on the graph of f(x), then the point (-a,b) is on the graph of f(x). 2	$f(-x) = -f(x)$ 3
$f(-x) = f(x)$ 4	$f(x) = \frac{x^2}{x^2 - 4}$ 5	If (a,b) is on the graph of f(x) then (b,a) is on the graph of f(x). 6
If the point (a,b) is on the graph of f(x), then the point (-a,-b) is on the graph of f(x). 7	 8	 9
$f(x) = 2x^5$ 10	 11	 12
13	14	15

Even, Odd or Neither		
Even Functions	Odd Functions	Functions that are Neither Even nor Odd

Mathematics I
Even, Odd or Neither
Day 1 Student Task

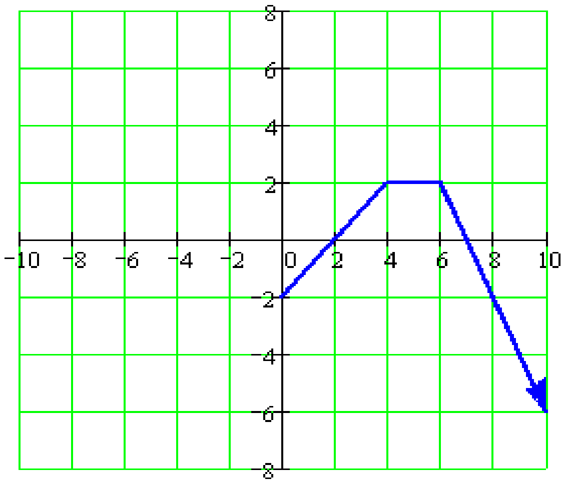
If the graph of a function f is symmetric with respect to the y-axis, then f is referred to as an **<u>even function.</u>**

1. Are any of the six basic function you have studied in Math I even functions? Explain your thinking using pictures and words.

2. Suppose f is an even function and the point (3, 5) is on the graph of f. What other point do you know must be on the graph of f?

3. Suppose f is an even function and the point (– 2, 4) is on the graph of f. What other point do you know must be on the graph of f?

4. Consider the function k, which is an even function. Part of the graph of k is shown below. Using the information that k is an even function, complete the graph for the rest of the domain.



The graph of the function k
for the nonnegative part of the domain.

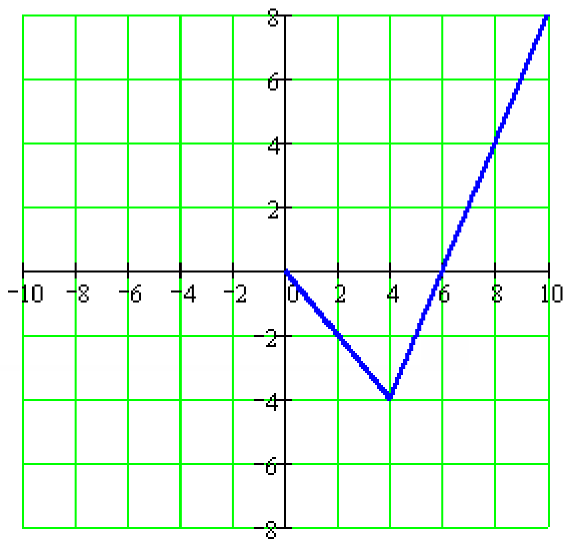
If the graph of a function f is symmetric with respect to the origin, then f is referred to as an **<u>odd function.</u>**

5. Are any of the six basic functions you have studied in Math I odd functions? Explain your thinking using pictures and words.

6. Suppose f is an odd function and the point (3, 5) is on the graph of f. What other point do you know must be on the graph of f?

7. Suppose f is an odd function and the point (– 2, 4) is on the graph of f. What other point do you know must be on the graph of f?

8. Consider the function *k*, which is an odd function. The part of the graph of *k* which has nonnegative numbers for the domain is shown at the right below. Using the information that *k* is an odd function, complete the graph for the rest of the domain.



The graph of the function *k*
for the nonnegative part of the domain.

9. As a way of further investigating the properties of even and odd functions, you will be asked to complete a card sort. You have been given a set of cards. Each card contains a statement, an equation, or a graph. You are to decide whether the information on the card is representative of an even function, an odd function, or a function that is neither even nor odd. Once you have made your decision, place the card in the appropriate column in the table labeled *Even, Odd, or Neither*.

   You have three *blank* cards. Use these cards to create one of your own graphs, equations, or statements to match **each** category.

   After you have placed each of your cards, choose two cards from each column and **write a justification** of your placement of these cards. Among the six cards you choose for justification, you should have at least one card that contains a graph, one that contains an equation, and one card that contains a statement. You should use multiple representations in your justifications, when possible. Equations should be justified algebraically as well as geometrically. Label each justification with the card number.

Mathematics I
Task 2: Even, Odd, or Neither? **Day 2/2**

Standard(s): MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.
a. Represent functions using function notation.
b. Graph the basic functions $f(x) = x^n$ where $n = 1$ to 3, $f(x) = |x|$, $f(x) = \sqrt{x}$, and $f(x) = 1/x$.
c. Graph transformations of basic functions including vertical shifts, stretches, and shrinks, as well as reflections across the *x*- and *y*-axes.
h. Determine graphically and algebraically whether a function has symmetry and whether it is even, odd or neither.

New vocabulary: even function, odd function

Mathematical concepts/topics: line, point, and rotational symmetry; properties of even and odd functions; graphs of the six basic functions of Mathematics I; domain and range; function notation; simplifying algebraic expressions; opposites of algebraic expressions

Prior knowledge: point, line, and rotational symmetry

Essential question(s): How can I use symmetry to understand, explain, and graph functions?

Suggested materials: Materials from previous lesson:

- Copy of company logos, preferably projected on a white board or Promethean Board so that students can draw lines and points of symmetry directly on the objects. (Included below.)
- One set of symmetry cards and a symmetry table for each pair of students. (Be sure to include 3 blank cards. All are included below). It is best if the cards and table are copied on card stock.
- Copy of student task, problems 1 - 9, for each student.
- Scissors
- Tape or glue
- Graph paper
- Graphing calculators

Warm-up: Allow students to compare homework.

Opening: Review ideas presented in the previous lesson by discussing the homework.

Worktime: Students should complete the placement and justification of the cards in their card sorts. Monitor student work for placements and justifications to be shared during the closing.

Closing: Use a Promethean Board, document camera, or overhead projector to project the card sort. Hopefully, you can choose students to *place and justify* each card. Allow several students to share the cards they created.
(Solutions are shown immediately following this lesson plan.)

Homework:

Differentiated support/enrichment:

Check for Understanding: This is an excellent mini-project that can be used to check for understanding of concepts in Tasks 1 and 2. It is problem #14 of the Logo Symmetry Task in the GaDOE TE.

Create a logo using any combination of vertical shifts, vertical stretches or shrinks, reflection through the x-axis, or reflection through the y-axis of the basic functions listed below as well as horizontal and vertical lines.

$$f(x) = x,\ f(x) = x^2,\ f(x) = x^3,\ f(x) = \sqrt{x},\ f(x) = |x|, \text{ and } f(x) = \frac{1}{x}.$$

Your logo should be aesthetically appealing and must include the following:

- *at least one of the functions from the list of basic functions*
- *at least four different equations*
- *at least two examples of vertical shifts, vertical stretches and/or shrinks*
- *at least one reflection*
- *at least one type of symmetry*

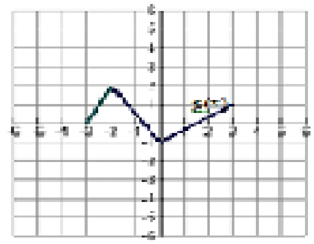
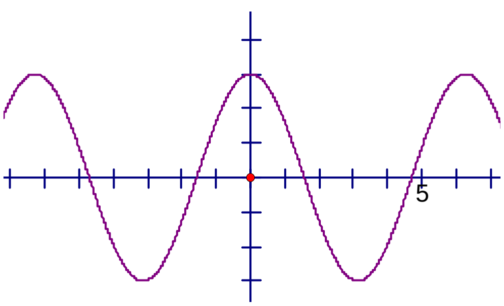
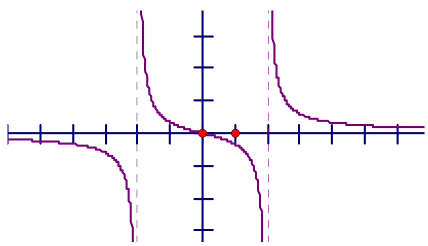
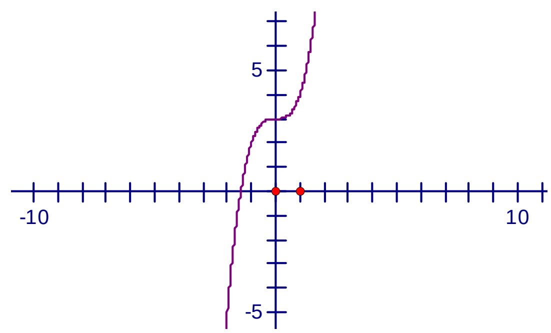
- o *Explain how your logo meets each of the requirements listed above.*
- o *Identify any important points, lines, and or angles associated with your logo's symmetry.*

Resources/materials for Math Support:

- Preview line, point, and rotational symmetry.
- Review the relationship between a variable and its opposite. Many students still may not understand that x can represent a negative number and $-x$ can represent a positive number.
- Provide skills work in substituting $-x$ for x in algebraic expressions and then simplifying those expressions. Students will need help in determining when $f(-x) = -f(x)$, particularly when the expression representing the function is a rational expression.

Even Functions	Odd Functions	Functions that are Neither Even nor Odd

Solutions for Card Sort

<table>
<tr><td>If the point (a,b) is on the graph of f(x), then the point (-a,b) is on the graph of f(x).</td><td>If the point (a,b) is on the graph of f(x), then the point (-a,-b) is on the graph of f(x).</td><td>If (a,b) is on the graph of f(x) then (b,a) is on the graph of f(x).</td></tr>
<tr><td>f(-x) = f(x)</td><td>f(-x) = -f(x)</td><td></td></tr>
<tr><td>f(x) = $\frac{x^2}{x^2 - 4}$</td><td>$f(x) = 2x^5$</td><td>f(x) = $\frac{4}{x - 2}$</td></tr>
<tr><td></td><td></td><td></td></tr>
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Atlanta Public Schools
Teacher's Curriculum Supplement

Mathematics I: Unit 5

Task 3: Just Jogging

Mathematics I
Task 3: Just Jogging **Day 1/1**
(GaDOE TE #1 - #10)

Standard(s):
MM1A2. Students will simplify and operate with radical expressions, polynomials, and rational expressions.
d. Add, subtract, multiply, and divide rational expressions.
MM1A3. Students will solve simple equations.
a. Solve quadratic equations in the form $ax^2 + bx + c = 0$, where $a = 1$, by using factorization and finding square roots where applicable.
d. Solve simple rational equations that result in linear equations or quadratic equations with leading coefficient of 1.

New vocabulary: rational expression

Mathematical concepts/topics: computation with simple fractions, proportional reasoning, calculation of time given distance and rate, simple complex fractions, algebraic fractions, rational expressions, operations with rational expressions, average speed, solving simple quadratic equations

Prior knowledge: computation with fractions; proportional reasoning; relationship between distance, rate, and time ($d = r \times t$)

Essential question(s): How can I use algebra to solve problems involving distance, rate, and time?

Suggested materials: graphing calculators

Warm-up: Post the following:

Ishan's grandmother lives 4 miles from his house. On Saturday morning, Ishan rode his bike the 4 miles to her house at a rate of 12 miles per hour. In the afternoon, he returned home at a rate of 15 miles per hour. How many hours did Ishan spend riding his bike to and from his grandmother's? What was his average speed over the total 8 miles he traveled?

Opening: Discuss the warm-up. (Ishan spent a total of 9/15 hours on his bike. His average speed for the entire trip was 120/9 or approximately 13.3 miles per hour.) Discussion should include addition of fractions with unlike denominators, the relationship between distance, rate, and time ($d = r \times t$), and the fact that average speed is $\frac{\text{total distance}}{\text{total time}}$.

Worktime: Students should complete problems 1 – 10 of the task. Students progress from computing with simple fractions to writing and simplifying algebraic fractions and rational expressions. Special attention should be paid to the concept addressed in problem 9 - *average rate for the total trip is not found by averaging the rates of each leg of the trip*.

Closing: Allow students to share their work for each part of the task. See teacher notes.

Homework: Practice should include operating with simple algebraic fractions and rational expressions similar to those found in the task.

Check for Understanding:

Resources/materials for Math Support: Students should review/preview proportional reasoning, including writing and solving proportions and using double number lines; computation with simple fractions; writing simple complex fractions as single fractions using division of fractions; operating with algebraic fractions and rational expressions, including finding common denominators; writing and solving equations involving distance, rate, and time.

Mathematics I
Just Jogging
Student Task

For distances of 12 miles or less, a certain jogger can maintain an average speed of 6 miles per hour while running on level ground.

1. If this jogger runs around a level track at an average speed of 6 mph, how long in hours will the jogger take to run each of the following distances? [Express your answers as fractions of an hour in simplest form.]

   (a) 3 miles (b) 9 miles (c) 1 mile (d) $\frac{1}{2}$ mile (e) $\frac{1}{10}$ mile

2. Analyze your work in Question 1. Each answer can be found by using the number of miles, a single operation, and the number 6. What operation should be used? Write an algebraic expression for the time it takes in hours for this jogger to run *x* miles on level ground at an average speed of 6 miles per hour.

3. Each day this jogger warms up with stretching exercises for 15 minutes, jogs for a while, and then cools down for 15 minutes. How long would this exercise routine take, in hours, if the jogger ran for 5 miles? [Express your answer as a fraction in simplest form.]

4. Let *T* represent the total time in hours it takes for this workout routine when the jogger runs for *x* miles. Write a formula for calculating *T* given *x*, where, as in Question 2, *x* is number of miles the jogger runs. Express the formula for *T* as a single algebraic fraction.

5. If the jogger skipped the warm-up and cool-down period and used this additional time to jog, how many more miles would be covered? Does this answer have any connection to the answer to question 4 above?

   Suppose this same jogger decides to go to a local park and use one of the paths there for the workout routine one day each week. This path is a gently sloping one that winds its way to the top of a hill.

6. If the jogger can run at an average speed of 5.5 miles per hour up the slope and 6.5 miles per hour going down the slope, how long, in hours, will it take for the jogger to cover 2 miles by going uphill for 1 mile and then returning 1 mile back down the hill? Give an exact answer expressed as a fraction in simplest terms and then give a decimal approximation correct to three decimal places.

7. If the jogger can run at an average speed of 5.3 miles per hour up the slope and 6.7 miles per hour going down the slope, how long, in hours, will it take for the jogger to cover 2 miles by going uphill for 1 mile and then returning 1 mile back down the hill? Give an exact answer expressed as a fraction in simplest terms and then give a decimal approximation correct to three decimal places.

8. Write an algebraic expression for the total time, in hours, that it takes the jogger to cover 2 miles by going uphill for 1 mile and then returning 1 mile back down the hill if the jogger runs uphill at an average speed that is c miles per hour slower than the level-ground speed of 6 miles per hour and runs downhill at an average speed that is c miles per hour faster than the level-ground speed of 6 miles per hour. Simplify your answer to a single algebraic fraction. Verify that your expression gives the correct answers for Questions 6 and 7.

9. The average speed in miles per hour is defined to be the distance in miles divided by the time in hours spent covering the distance.

   (a) What is the jogger's average speed for a two mile trip on level ground?

   (b) What is the jogger's average speed for the two mile trip in question 6?

   (c) What is the jogger's average speed for the two mile trip in question 7?

   (d) Write an expression for the jogger's average speed over the two-mile trip (one mile up and one mile down) when the average speed uphill is c miles per hour slower than the level-ground speed of 6 miles per hour and the average speed downhill at an average speed that is c miles per hour faster than the level-ground speed of 6 miles per hour. Express your answer as a simplified algebraic fraction.

   (e) Use the expression in part (d) to recalculate your answers for parts (b) and (c)? What value of c should you use in each part?

10. For what value of c would the jogger's average speed for the two-mile trip (one mile up and one mile down) be 4.5 miles per hour?

Atlanta Public Schools

Teacher's Curriculum Supplement

Mathematics I: Unit 5

Task 4: Resistance

Task 4: Resistance **Day 1/2**
(GaDOE TE #1 - #6)

Standard(s): MM1A3. Students will solve simple equations.
a. Solve quadratic equations in the form $ax^2 + bx + c = 0$ where $a = 1$, by using factorization and finding square roots where applicable.
d. Solve simple rational equations that result in linear equations or quadratic equations with leading coefficient of 1.

New vocabulary: resistance, resistors, resistors in parallel, resistors in series, rational expression, complex fraction

Mathematical concepts/topics: translating from verbal to numeric and algebraic expressions, division by zero, addition of simple numeric fractions with unlike denominators, writing simple complex fractions as single fractions, finding least common denominators for simple rational expressions, solving simple rational equations

Prior knowledge: translating from verbal to numeric and algebraic expressions, addition of simple numeric fractions with unlike denominators, solving simple linear equations

Essential question(s): How can I use algebra to understand and determine resistance in electrical circuits?

Suggested materials:

Warm-up: Post the following:

Perform the following computations. Give you final answer in simplest form.

a. $\frac{3}{4} + \frac{4}{7} - \frac{5}{6}$ b. $\frac{2}{\frac{3}{4}} + \frac{3}{8}$

Opening: Allow students to share their solutions for the problems in the warm-up. Discussion should include methods for finding the least common denominator and writing a simple complex fraction as a single fraction using division of fractions.

Discuss the information related to electrical circuits and resistance included at the beginning of the task. (Video on these topics can be found at Discovery Education using the link provided here. There are seven video segments totaling about 17 minutes. Showing a short clip directly related to resistors in parallel and in series may peak student interest in the task.
http://player.discoveryeducation.com/index.cfm?guidAssetId=2A8797BB-21A1-4E43-9800-811B0A61EF84&blnFromSearch=1&productcode=US

Worktime: Students should work in pairs to complete problems 1 - 6 of the student task. Once students have had time to complete problems 1 – 4, allow them to share their work on problems 1-3, discussing any misconceptions or difficult concepts identified as you monitored student work on these problems.

In problem 4, students are asked to find a common denominator for simple rational expressions and to determine why x cannot be zero in the expression $1/x$. After they have had an opportunity to struggle with these ideas on their own, a short mini-lesson to discuss both concepts may be beneficial.

In the context of problem 4, $x \neq 0$ because x represents resistance and resistance is always a positive number. It is important for students to note that if we multiply both sides of an equation by an expression that is equal to 0, we will not obtain an equivalent equation. Ask students why this is true.

This is also a good time to discuss *why* the expression $1/x$ is undefined when $x = 0$.
You might begin by asking the question: *What is division?* Most often students will respond by saying that division is the opposite or inverse of multiplication. Provide simple examples that will allow students to confirm this fact. For example:

> 15/3 is ? ____because? ____ Students will respond that 15/3 is 5 because 3 x 5 is 15.
> 32/4 is ? ____because? _____
> 54/6 is? ____because ?______
>
> Finally ask 7/0 is ? ____ because? _____ Students will quickly see that there is no number that can be multiplied by 0 to obtain 7. Therefore division by 0 is *literally undefined.*

Rational expressions in problems 1 – 6 of the student task are very simple. Begin the discussion of finding least common denominators using expressions similar to those contained in these problems. Denominators for the expressions in problems 7 and 8 become increasingly complex and more complex common denominators should be discussed when those problems are encountered.

Closing: Allow students to share finished work. Emphasis should be placed on checking solutions in both the equations used to model the situations and in the original situations themselves. (See teacher notes.)

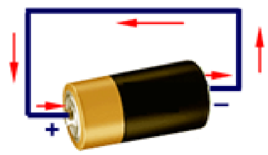
Homework: Practice should be provided in finding common denominators of simple rational expressions, and in writing and solving simple rational equations that model real world situations.

Differentiated support/enrichment:

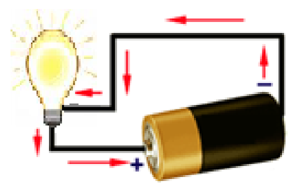
Resources/materials for Math Support: Students may need to review/preview computation with basic fractions, solving simple linear and quadratic equations, and writing rational equations to model real-world situations. Terminology identified above as *new vocabulary* should also be previewed.

Mathematics I
Resistance
Day 1 Student Task

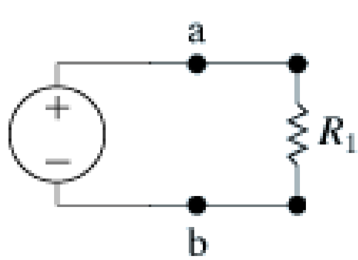
We use electricity every day to do everything from cooking food to charging ipods and cell phones. Electricity results from the presence and flow of electric charges. Electrons with a negative charge are attracted to those with a positive charge. Electrons cannot travel through the air. They need a path to move from one charge to the other. This path is called a circuit. A simple circuit can be seen in the connection of the negative and positive ends of a battery.



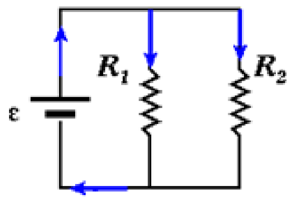
When a circuit is created, electrons begin moving from the one charge to the other. In the circuit below, a bulb is added to the circuit. The electrons pass through the filament in the bulb heating it and causing it to glow and give off light.

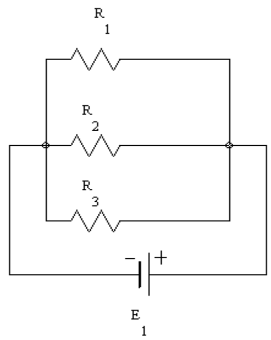


Electrons try to move as quickly as possible. If a circuit is not set up carefully, too many electrons can move across at one time causing the circuit to break. We can limit the number of electrons crossing over a circuit to protect it. Adding objects that use electricity, such as the bulb in the above circuit, is one way to limit the flow of electrons. This limiting of the flow of electrons is called resistance. It is often necessary to add objects called **resistors** to protect the circuit and the objects using the electricity passing through the circuit. In the circuit below, R_1 represents a resistor.

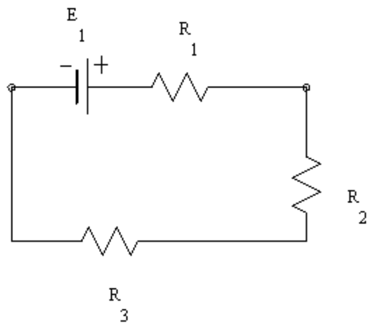


More than one resistor can be placed on a circuit. The placement of the resistors determines the total effect on the circuit. The resistors in the diagram below are placed in parallel (this refers to the fact that there are no resistors directly between two resistors, not to the geometric definition of parallel). Parallel resistors allow multiple paths for the electricity to flow. Two examples of parallel resistors are shown below.





The resistors in the next circuit below are not parallel. These resistors are placed in **series** because the electricity must travel through all three resistors as it travels through the circuit.



Resistance is measured in units called ohms and must always be a positive number. The omega symbol, Ω, is used to represent ohms.

For *n* **resistors in parallel**, R_1, R_2, R_3, etc. the total resistance, R_T, across a circuit can be found using the equation:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

For *n* **resistors is series**, R_1, R_2, R_3, etc. the total resistance, R_T, across a circuit can be found using the equation:

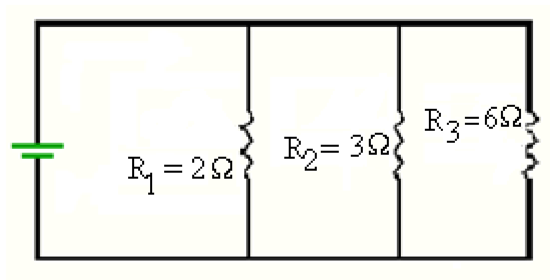
$$R_T = R_1 + R_2 + R_3 + \cdots + R_n$$

1. What is the total resistance for a circuit with three resistors in series if the resistances are 2 ohms, 5 ohms, and 4 ohms, respectively?

2. What is the total resistance for a circuit with two parallel resistors, one with a resistance of 3 ohms and the other with a resistance of 7 ohms?

3. What is the total resistance for a circuit with four resistors in parallel if the resistances are 1 ohm, 3 ohms, $\frac{5}{2}$ ohms, and $\frac{3}{5}$ ohms, respectively?

4. What is the total resistance for the circuit to the right?



5. A circuit with a total resistance of $\frac{28}{11}$ has two parallel resistors. One of the resistors has a resistance of 4 ohms.
   a. Let *x* represent the resistance of the other of the other resistor, and write an equation for the total resistance of the circuit.

   b. The equation in part *a* contains rational expressions. If you have any complex fractions, simplify them. In your equation containing no complex fractions, what is the least common denominator of the rational expressions?

   c. Use the Multiplication Principle of Equality to obtain a new equation that has the same solutions as the equation in part a but does not contain any rational expressions. Why do you know that $x \neq 0$? How does knowing that $x \neq 0$ allow you to conclude that this new equation has the same solutions as, or is equivalent to, the equation from part a.

   d. Solve the new equation to find the resistance in the second resistor. Check your answer.

6. A circuit has been built using two parallel resistors.
   a. One resistor has twice the resistance of the other. If the total resistance of the circuit is $\frac{3}{4}$ ohms, what is the resistance of each of the two resistors?

   b. One resistor has a resistance of 4 ohms. If the total resistance is one-third of that of the other parallel resistor, what is the total resistance?

Task 4: Resistance **Day 2/2**

(GaDOE TE #7 - #8)

Standard(s): MM1A3. Students will solve simple equations.

a. Solve quadratic equations in the form $ax^2 + bx + c = 0$ where $a = 1$, by using factorization and finding square roots where applicable.

d. Solve simple rational equations that result in linear equations or quadratic equations with leading coefficient of 1.

New vocabulary:

Mathematical concepts/topics: writing rational equations to model real situations, finding a common denominator for a set of rational expressions, solving simple rational equations, extraneous solutions, solving simple quadratic equations by factoring

Prior knowledge: translating from verbal to numeric and algebraic expressions, addition of simple numeric fractions with unlike denominators, solving simple linear equations

Essential question(s): How can I use algebra to understand and determine resistance in electrical circuits?

Suggested materials:

Warm-up: Post the following and have students work in pairs to find the least common denominators. Do not allow more than 5 – 10 minutes for this activity. Students probably have not had much instruction to this point in finding common denominators of this type. The idea is to determine what they know and build on that knowledge.

Find the least common denominator for each of the following sets of rational expressions:

a. $\frac{1}{3x} + \frac{1}{x-5}$ *b.* $\frac{1}{2x+4} + \frac{1}{x+2}$ *c.* $\frac{1}{4} + \frac{1}{4x+8} + \frac{1}{x+3}$

Opening: Conduct a mini-lesson on finding common denominators using the problems in the warm-up.

Worktime: Students should work in pairs to complete problems 7 and 8 of the student task.

Closing: Allow students to share finished work. Emphasis should be placed on checking solutions in both the equations used to model the situations and in the original situations themselves. Re-visit the idea, introduced in Unit 2, that solutions to an equation may need to be eliminated based on physical constraints of the situation modeled by the equation. Extraneous solutions of rational equations should also be discussed. (See teacher notes.)

Homework: Practice in writing and solving rational equations to model real world situations should be provided.

Differentiated support/enrichment:

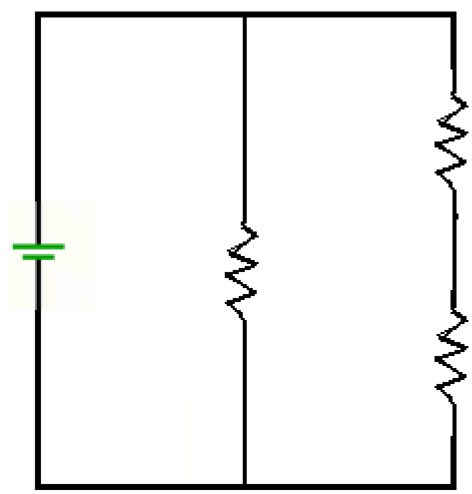
Resources/materials for Math Support: Students may need to review/preview computation with basic fractions, solving simple linear and quadratic equations, and writing rational equations to model real-world situations. Many of these students will need significant practice in finding the least common denominator for sets of rational expressions.

Mathematics I
Resistance
Day 2 Student Task

7. A circuit has been built using two paths for the flow of the current; one of the paths has a single resistor and the other has two resistors in series as shown in the diagram below.

   a. Assume that, for the two resistors in series, the second has a resistance that is three times the resistance of the first one in the series. The single resistor has a resistance that is 6 ohms more than the resistance of the first resistor in series, and the total resistance of the circuit is 4 Ω. Write an equation to model this situation, and solve this equation. What is the solution set of the equation? What is the resistance of the each of the resistors?

   b. Assume that, for the two resistors in series, the second has a resistance that is 3 ohms more than twice the resistance of the first one in the series. The single resistor has a resistance that is 1 ohm more than the resistance of the first resistor in series, and the total resistance of the circuit is 3 Ω. Write an equation to model this situation, and solve this equation. What is the solution set of the equation? What is the resistance of the each of the resistors?



   c. Assume that, for the two resistors in series, the second has a resistance that is 2 ohms more than the first one in the series. The single resistor has a resistance that is 3 ohms more than the resistance of the first resistor in series, and the total resistance of the circuit is 2 Ω. Write an equation to model this situation, and solve this equation. What is the solution set of the equation? What is the resistance of the each of the resistors?

d. Assume that, for the two resistors in series, the second has a resistance that is 4 ohms more than the first one in the series. The single resistor has a resistance that is 3 ohms less than the resistance of the first resistor in series, and the total resistance of the circuit is 4 Ω. . Write an equation to model this situation, and solve this equation. What is the solution set of the equation? What is the resistance of the each of the resistors?

8. A circuit has three resistors in parallel. The second resistor has a resistance that is 4 ohms more than the first. The third resistor has a resistance of 8 ohms. The total resistance is one-half the resistance of the first resistor. Find each of the unknown resistances.

Computation with Rational Expressions

The following problems are representative of the computation with rational expressions required of students in Mathematics 1.

Perform the indicated division in problems 1-3.

1. The total volume of $2x$ boxes is $4x^3 + 8x^2 + 10x$. Find the average volume of the $2x$ boxes by using the expression shown below.

$$\frac{4x^3 + 8x^2 + 10x}{2x}$$

2. $$\frac{x^2 + 2x - 3}{x - 1}$$

3. $$\frac{5y + y^2 + 4}{2 + y}$$

Simplify the algebraic fractions in problems 4-6.

4. If $-48a^2$ represents the amount of debt owed by 16a corporations, use the expression below to find the average amount of debt per corporation.

$$\frac{-48a^2}{16a}$$

5. $$\frac{g + 4}{g^2 - 16}$$

6. $$\frac{6z^2 - 24z}{2z^2 - 8z}$$

Multiply and/or divide the rational expressions in problems 7-9.

7. The length of a corn field is represented by the expression $\dfrac{5x}{x^2+5x+6}$.

The width of the same field is represented by the expression $x + 3$.
Us the expression below to determine the area of land available to plant corn.

$$\frac{5x}{x^2+5x+6}(x+3)$$

8. $$\frac{x^2+x-2}{x^2+2x} \bullet \frac{2x^2+2x}{5x^2-15x+10}$$

9. $$\frac{m^2-4}{2m^2+4m} \div \frac{6m-3m^2}{4m+44}$$

Simplify the complex fractions in problems 10-11.

10. Let $x^2 - 1$ represent the amount of money allocated for a company's monthly payroll. If each of the $\dfrac{x+1}{x-1}$ employees receives the same amount of money each month, use the expression below to determine the amount of money each employee will receive.

$$\frac{x^2-1}{\left(\dfrac{x+1}{x-1}\right)}$$

11. $$\frac{\left(\dfrac{-9x^5}{7}\right)}{-12x^2}$$

Add or subtract the rational expressions in problems 12-15.

12. If Julie ate $\frac{2}{y}$ amount of pizza and Jane ate $\frac{y+1}{y}$ amount of pizza use $\frac{2}{y}+\frac{y+1}{y}$ to determine the total amount of pizza the two girls ate.

13. $$\frac{4x+1}{2x-1}-\frac{2x-3}{2x-1}$$

14. $$\frac{5x}{4}+\frac{2}{5x}$$

15. $$\frac{k-7}{k^2+6k+9}+\frac{k-5}{k^2-5k-24}$$

Atlanta Public Schools

Teacher's Curriculum Supplement

Mathematics I: Unit 5

Task 5: Shadows and Shapes

Mathematics I
Task 5: Shadows and Shapes **Day 1/1**
(GaDOE TE #1 - #7)

Standard(s):
MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.
c. Graph transformations of basic functions including vertical shifts, stretches, and shrinks, as well as reflections across the *x*- and *y*-axes.
d. Investigate and explain characteristics of a function: domain, range, zeros, intercepts, intervals of increase and decrease, maximum and minimum values, and end behavior
MM1A2. Students will simplify and operate with radical expressions, polynomials, and rational expressions.
a. Simplify algebraic and numeric expressions involving square root.
e. Factor expressions by greatest common factor, grouping, trial and error, and special products limited to the formulas below.

$$(x+y)^2 = x^2 + 2xy + y^2$$
$$(x-y)^2 = x^2 - 2xy + y^2$$
$$(x+y)(x-y) = x^2 - y^2$$
$$(x+a)(x+b) = x^2 + (a+b)x + ab$$
$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$
$$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

MM1A3. Students will solve simple equations.
a. Solve quadratic equations in the form $ax^2 + bx + c = 0$ where $a = 1$, by using factorization and finding square roots where applicable.
c. Use a variety of techniques, including technology, tables, and graphs to solve equations resulting from the investigation of $x^2 + bx + c = 0$.
d. Solve simple rational equations that result in linear equations or quadratic equations with leading coefficient of 1.

New vocabulary:

Mathematical concepts/topics: similar triangles, proportions, Pythagorean Theorem, solving simple rational equations, solving quadratic equations by factoring and by understanding that x-intercepts of the graph of *f(x)* are solutions of the equation *f(x) = 0*

Prior knowledge: similar triangles, proportions, Pythagorean Theorem, solving simple one-step equations

Essential question(s): How can I use algebra to solve problems?

Suggested materials: graphing calculators

Warm-up: Have students work problem 1 of the task. Students have worked extensively with writing and solving proportions so this problem should be review.

Opening: Discuss problem 1. When solving the proportion that arises here, emphasis should be placed on multiplying both sides of the equation by the common denominator of the two existing fractions rather than "cross-multiplying". Students often do not understand *why* they "cross-multiply", leading to significant confusion.

Worktime: Students should complete problems 2 – 7 of the task. Much of the content in this task is review. It is important that students can factor expressions in the form $x^2 - y^2$ and see x-intercepts of the graph of $f(x)$ as solutions of the equation $f(x) = 0$.

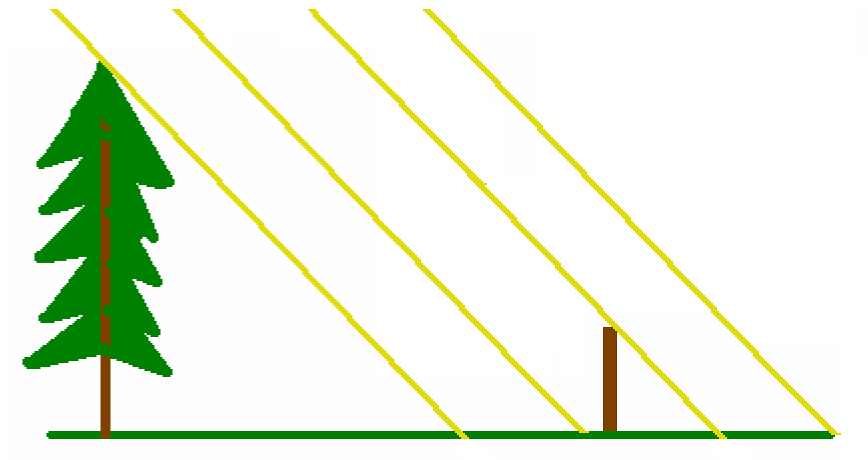
Closing: Allow students to share their work. See teacher notes.

Homework: **Check for Understanding:**

Resources/materials for Math Support: Students should review/preview proportional reasoning, vertical shifts of $y = x^2$, and the special products studied in Unit 2.

Mathematics I
Shadows and Shapes
Student Task

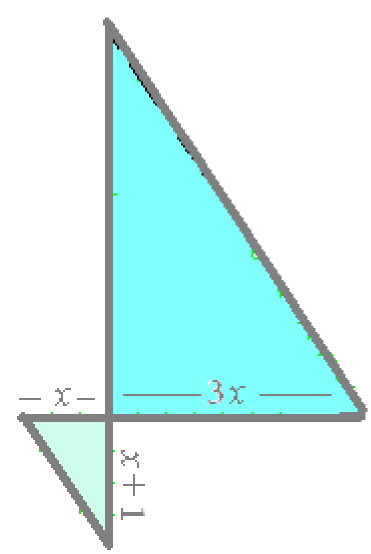
1. At a particular time one spring day in a park area with level ground, a pine tree casts a 60-foot shadow while a nearby post that is 5 feet high casts an 8-foot shadow.
   a. Using the figure below, draw appropriate lines to indicate the rays of sunlight, which are always parallel.



   b. Let *h* represent the height of the pine tree. Use relationships about similar triangles to write an equation involving *h*.

   c. Find the height of the pine tree, to the nearest foot.

2. On bright sunny day, two men are standing in an open plaza. The shorter man is wearing a hat that makes him appear to be the same height as the other man, who is not wearing a hat. The hat is designed so that the top of the hat is 4 inches above the top of the wearer's head. When the shorter man takes off his hat, he casts a 51-inch shadow while the taller man casts a 54-inch shadow. How tall is each man?



3. A plaza floor has a geometric design. The design includes similar right triangles with the relationships shown in the figure at the right.
   a. If the hypotenuse of the larger right triangle is 15 inches, what is the length of the other hypotenuse? Explain why.

   b. Assuming the hypotenuse of the larger right triangle is 15 inches, for each triangle, write an equation expressing a relationship among the lengths of the sides of the triangle.

c. Solve at least one of the equations from part b, and find the lengths of the legs of both triangles.

4. Another shape in the plaza floor design is a right triangle with hypotenuse 26-inches long and one leg 24 inches long.

   a. Let x denote the length of the other leg of this triangle. Write a quadratic equation expressing a relationship among the lengths of the sides of the triangle.

   b. Put this equation in the standard form $x^2 + bx + c = 0$. What are the values of b and c?

   c. What is the length of the other leg of the triangle?

5. When you put the equation you solved in item 4 in standard form, you were working with an equation in the form $x^2 - c = 0$. In item 4, the value of c is a perfect square, so you could use the Difference of Square identity to solve the equation by factoring. Use this identity and factoring to solve each of the equations below, each of which has the form $x^2 - c = 0$.

   a. $x^2 - 25 = 0$

   b. $x^2 - 49 = 0$

   c. $x^2 - 4 = 0$

   d. $x^2 - 81 = 0$

   e. $x^2 - 121 = 0$

   f. $x^2 - 64 = 0$

5. Make a quick sketch of the graphs of each of the functions listed below, and discuss how the solutions from item 5 can be seen in the graphs.

   a. $f(x) = x^2 - 25$

   b. $f(x) = x^2 - 49$

   c. $f(x) = x^2 - 4$

   d. $f(x) = x^2 - 81$

   e. $f(x) = x^2 - 121$

   f. $f(x) = x^2 - 64$

6. Use the graphs of appropriate functions to solve each of the following equations.

   a. $x^2 - 5 = 0$

   b. $x^2 - 11 = 0$

   c. $x^2 - 5 = 0$

   d. $x^2 - 8 = 0$

   e. $x^2 - 2 = 0$

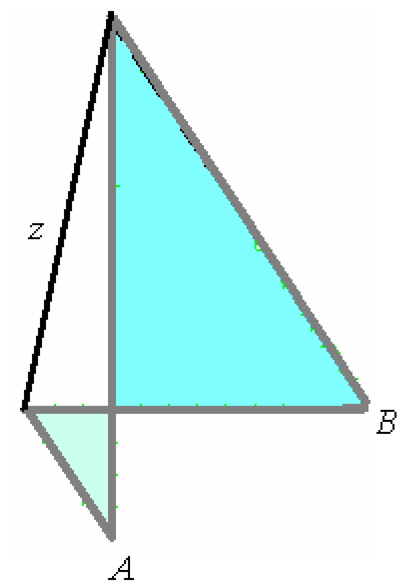
   f. $x^2 - 12 = 0$

g. Explain why each equation has two solutions.

h. For a quadratic equation of the form $x^2 - c = 0$ with $c > 0$, how many solutions does the equation have? In terms of *c*, what are the solutions?

7. In the figure at the right, a line has been added to the part of the design from item 3. As shown, the shorter leg of the smaller right triangle and the longer leg of the larger right triangle form the legs of this new right triangle.

a. Let *z* represent the length of the hypotenuse of this new right triangle, as indicated. Use the Pythagorean Theorem, and the lengths you found in item 3 to find z.



b. If a line segment were drawn from point *A* to point *B* in the figure, what would be the length of the segment?