Atlanta Public Schools
Teacher's Curriculum Supplement
Mathematics I: Unit 3

This document has been made possible by funding from the GE Foundation Developing Futures grant, in partnership with Atlanta Public Schools. It is derived from the Georgia Department of Education Math I Frameworks and includes contributions from Georgia teachers. It is intended to serve as a companion to the GA DOE Math I Frameworks Teacher Edition.

Preface

We are pleased to provide this supplement to the Georgia Department of Education's Mathematics I Framework. It has been written in the hope that it will assist teachers in the planning and delivery of the new curriculum, particularly in this first year of implementation. This document should be used with the following considerations.

- The importance of working the tasks used in these lessons cannot be overstated. In planning for the teaching of the Georgia Performance Standards in Mathematics teachers should work the task, read the teacher notes provided in the Georgia Department of Education's Mathematics I Framework Teacher Edition, and *then* examine the lessons provided here.

- This guide provides day-by-day lesson plans. While a detailed scope and sequence and established lessons may help in the implementation of a new and more rigorous curriculum, it is hoped that teachers will assess their students informally on an on-going basis and use the results of these assessments to determine (or modify) what happens in the classroom from one day to the next. Planning based on student need is much more effective than following a pre-determined timeline.

- It is important to remember that the Georgia Performance Standards provide a balance of concepts, skills, and problem solving. Although this document is primarily based on the tasks of the Framework, we have attempted to help teachers achieve this all important balance by embedding necessary skills in the lessons and including skills in specific or suggested homework assignments. The teachers and writers who developed these lessons, however, are not in your classrooms. It is incumbent upon the classroom teacher to assess the skill level of students on every topic addressed in the standards and provide the opportunities needed to master those skills.

- In most of the lesson templates, the sections labeled *Differentiated support/enrichment* have been left blank. This is a result of several factors, the most significant of which was time. It is also hoped that as teachers use these lessons, they will contribute their own ideas, not only in the areas of differentiation and enrichment, but in other areas as well. Materials and resources abound that can be used to contribute to the teaching of the standards.

On the topic of differentiation, it is critical to reiterate that many of the strategies used in a standards-based mathematics classroom promote differentiation. These strategies include

- the use of rich tasks with multiple points of entry and more than one path to a solution
- flexible grouping of students
- multiple representations of mathematical concepts
- writing in mathematics
- monitoring of progress through on-going informal and formative assessments
- and analysis of student work.

We hope that teachers will incorporate these strategies in each and every lesson.

It is hoped that you find this document useful as we strive to raise the mathematics achievement of all students in our district and state. Comments, questions, and suggestions for inclusions related to the document may be emailed to Dr. Dottie Whitlow, Executive Director, Mathematics and Science Department, Atlanta Public Schools, dwhitlow@atlantapublicschools.us

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Explanation of the Terms and Categories Used in the Lesson Template

Task: This section gives the suggested number of days needed to teach the concepts addressed in a task, the task name, and the problem numbers of the task as listed in the Georgia Department of Education's Mathematics I Framework Teacher Edition (GaDOE TE).

In some cases new tasks or activities have been developed. These activities have been named by the writers.

Standard(s): Although each task addresses many Math I standards and uses mathematics learned in earlier grades, in this section, only the key standards addressed in the lesson are listed.

New Vocabulary: Vocabulary is only listed here the first time it is used. It is strongly recommended that teachers, particularly those teaching Math Support, to use interactive word walls. Vocabulary listed in this section should be included on the word walls.

Mathematical concepts/topics: Here are listed the major concepts addressed in the lesson whether they are Math I concepts or were addressed in earlier grades.

Prior knowledge: Prior knowledge includes only those topics studied in previous grades. It does not include Math I content taught in previous lessons.

Essential Question(s): Essential questions may be daily and/or unit questions.

Suggested materials: In an attempt to list all materials that will be needed for the lesson, including consumable items, such as graph paper, and tools, such as graphing calculators and compasses. This list did not include those items that should always be present in a standards-based mathematics classroom such as markers, chart paper and rulers.

Warm-up: A suggested warm-up is included with every lesson. Warm-ups should be brief and should focus student thinking on the concepts that are to be addressed in the lesson.

Opening: Openings should set the stage for the mathematics to be done during the work time. The amount of class time used for an opening will vary from day-to-day but this should not be the longest part of the lesson.

Worktime: The problem numbers have been listed and the work that students are to do during the work time have been described. A student version of the day's activity follows the lesson template in every case. In order to address all of the standards in Math I, some of the problems in some of the tasks have been omitted and, in a few instances, substituted less time consuming activities for tasks. In many instances, in the student versions of the tasks, the writing of the original tasks has been simplified. In order to preserve all vocabulary, content, and meaning it is important that teachers work the original tasks as well as the student versions included here.

Teachers are expected to both facilitate and provide some direct instruction, when necessary, during the work time. Some suggestions, related to student misconceptions, difficult concepts, and deeper meaning in this section have been included. However, the teacher notes in the GaDOE Math I Framework are exceptional. In most cases, there is no need to repeat the

information provided there. Again, it is imperative that teachers work the tasks and read the teacher notes that are provided in GaDOE support materials.
Questioning is extremely important in every part of a standards-based lesson. We included suggestions for questions in some cases but did not focus on providing good questions as extensively as we would have liked. Developing good questions related to a specific lesson should be a focus of collaborative planning time.

Closing: The closing may be the most important part of the lesson. This is where the mathematics is formalized. Even when a lesson must be continued to the next day, teachers should stop, leaving enough time to "close", summarizing and formalizing what students have done to that point. As much as possible students should assist in presenting the mathematics learned in the lesson. The teacher notes are all important in determining what mathematics should be included in the closing.

Homework: In some cases, additional written homework suggestions are provided or used the homework provided in the GaDOE sample lessons. We hope that you will use your resources, including your textbook, to assign homework related to the lesson that addresses the needs of your students.

Homework should be focused on the skills and concepts presented in class, relatively short (30 to 45 minutes), and include a balance of skills and thought-provoking problems.

Differentiated support/enrichment: On the topic of differentiation, it is critical to reiterate that many of the strategies used in a standards-based mathematics classroom promote differentiation. These strategies include

- the use of rich tasks with multiple points of entry and more than one path to a solution
- flexible grouping of students
- multiple representations of mathematical concepts
- writing in mathematics
- monitoring of progress through on-going informal and formative assessments
- and analysis of student work.

Check for understanding: A check for understanding is a short, focused assessment-a ticket out the door, for example. The Coach Book may be a good resource for these items.

Resources/materials for Math Support: Again, in some cases, we have provided materials and/or suggestions for Math Support. This section should be personalized to your students, class, and/or school, based on your resources.

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Task 6: Centers of Triangles

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Unit 3 Timeline

Task 1: Robotic Gallery Guards	2 days
Task 2: Poor Captain Robot	1 day
Task 3: Constructing Pennants	3 days
Task 4: Constructing Diagonals	3 days
Task 5: Middles and Halves	2 days
Task 6: Centers of Triangles	2 day

Task Notes

Task 1: Robotic Gallery Guards
Problems #1 - #17 of the GaDOE TE Framework are included in these lessons. We believe that Problems 16 and 17 can be used as enrichment. The task has been edited in an effort to decrease the amount of time needed to develop understanding of both concepts and skills. In the student task included here, each student will measure the exterior and interior angles of **one** exhibit. They will compare the results for that exhibit with their group and then compare results for all four exhibits as a class.

Task 2: Poor Captain Robot
All parts of the original task from GaDOE TE Framework are included in the lesson plans. Edits were made for clarification.

Task 3: Constructing Pennants
It is suggested that teachers provide students an introduction to formal proof and understand its importance before beginning this task. Throughout the K-12 curriculum, students are expected to make conjectures and justify their work using various types of reasoning. In Math I, students formalize these previously informal mathematical arguments. Although being able to write a two-column proof is not nearly as important as being able to develop a logical sequence of steps leading to a conclusion, students do need to be exposed to a variety of formats for presenting formal mathematical proofs, including paragraph, flow, and two-column proofs. This introduction should include examples of and opportunities to develop both algebraic and geometric proofs.

Problems 2 - 16 of the GaDOE TE are addressed in these lesson plans. An introductory problem has been added to the task and existing problems have been revised and rearranged.

Task 4: Constructing With Diagonals
In order to capitalize on the work related to proof, this task has been moved to immediately follow *Constructing Pennants*.

Some discussion on the ideas of *necessary* versus *sufficient* conditions has been included in the closings for Days 3 and 4 of the lesson plans. The point of this discussion is not to place emphasis on a formal treatment of these ideas. However, it is important for students to understand the difference between questions asked relating to the two concepts.

All parts of the GaDOE task are included in these lesson plans.

Task 5: Middles and Halves
This task has been revised extensively. All content of the GaDOE task has been addressed.

Task 6: Centers of Triangles
All parts of the GaDOE task are included in these lesson plans.

Although there is no discussion here of the *Locations Tasks* included in the GaDOE TE, these tasks provide excellent opportunities for applying the mathematics addressed in this task.

Atlanta Public Schools
Teacher's Curriculum Supplement
Mathematics I: Unit 3
Task 1: Robot Gallery Guards

Mathematics I
Task 1: Robotic Gallery Guards **Day 1/2**
(GA DOE TE #1-10)

Standard(s): MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
a. Determine the sum of interior and exterior angles in a polygon.

New vocabulary: exterior angles, interior angles

Mathematical concepts/topics: convex polygons, measuring angles using a protractor, measurement error, exterior angles, supplementary angles, conjecture and proof, Exterior Angle Sum Theorem, sums of the measures of the interior angels of a convex polygon, measure of each interior angle of a regular polygon, measure of each exterior angle of a regular polygon

Prior knowledge: using protractors to measure angles, supplementary angles, extending patterns, convex polygons, measurement error, number of degrees in a triangle

Essential question(s): Are there special relationships that govern the measurements of the angles of a convex polygon?

Suggested materials: rulers, protractors

Warm-up: Provide students with a copy of an obtuse angle, a ruler, and a protractor. Ask them to do the following:

- *Measure the angle using your protractor.*
- *Write down 3 important things to keep in mind when measuring an angle with a protractor.*

Opening: Discuss the ideas in the warm-up. Using a protractor will be important in this task. Map out a quadrilateral on the floor. Have a student volunteer to walk along one side of the quadrilateral. When the student reaches a vertex, ask him/her to follow the outline of the quadrilateral. After just a few steps, stop the student and have a class discussion concerning the angle measurement of the student's turn around the vertex. Help the class understand the meaning of an exterior angle. Be sure all students understand before moving into the Worktime.

Worktime: Students should do problems 1-2 of the task. Each group of students will have a different "Exhibit". (The student task included here has been written so that there are four different copies of problems 1 and 2 related to four different exhibits. Problems 3 – 9 are the same for all students.) Every student should obtain their own results for problems 1 and 2 in order to compare with other members of the group. Discuss these two questions with the whole group before moving on to problems 3-9. A discussion of imprecision of measurement will be important here. As you monitor student progress, be sure that students are able to explain how they obtained their results and can justify their reasoning.

When the teacher notices that most everyone has finished problems 3-5 of the task, stop them for a short discussion of their results. Have students explain what they have learned about interior angles. Also, have them discuss their answers to #5 and why they believe this happens.

After the short discussion, allow students to continue working on problems 6-9 of the task.

Closing: It is important for students to share problems 6-9 before dismissing class. Make sure they are able to see the patterns for interior angels.

Homework:

1. Find the sum of the exterior angle measures of a nine sided polygon.
2. What is the sum of the interior angle measures of a hexagon?
3. If the sum of the interior angle measures of a polygon equals 900^o, how many sides does the polygon have?
4. How many sides does the polygon have if its interior angles measure 2160^o?

Differentiated support/enrichment: Should some students have difficulty understanding the angle to be measured in the opening of this lesson, it may be helpful for them to hold an arm out straight in front of them to "see" the angle (arc) being measured.

Check for Understanding: Using complete sentences, write a paragraph describing the relationships you discovered in today's lesson. Include what you learned about interior and exterior angles. Give any formulas that were found. You may want to use pictures, tables, or other representations to illustrate your work.

Resources/materials for Math Support: Students may need to review the number of degrees in a triangle as well as how to measure various angles with a protractor. It is important to use a hands-on approach with both of these concepts. Practice measuring small angles, angles greater than 180^o, angles with short sides that need to be extended, etc. Reviewing the names and shapes of various polygons would be advantageous for some students. For previewing, a fun kinesthetic activity could be used to assure students understand the relationship between interior and exterior angles of a polygon (i.e. walking a quadrilateral taped on the floor.). Other vocabulary to be previewed may include turn, linear pair, n-gon, concave, and convex.

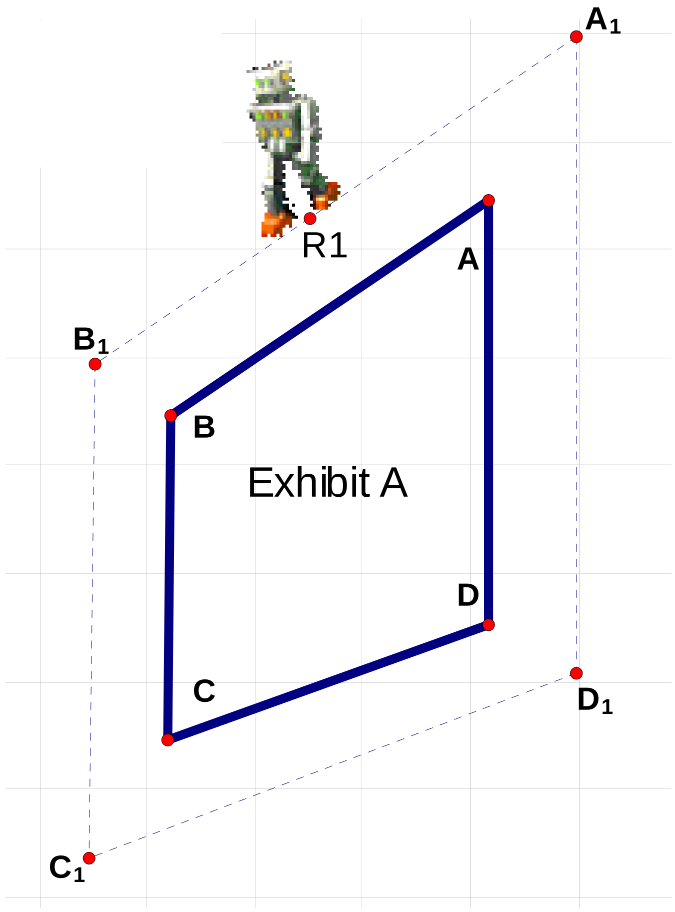
Mathematics I
Robot Gallery Guards
Day 1 Student Task: Group A

The Asimov Museum has contracted with a company that provides Robotic Security Squads to guard the exhibits during the hours the museum is closed. The robots are designed to patrol the hallways around the exhibits and are equipped with cameras and sensors that detect motion.

Each robot is assigned to patrol the area around a specific exhibit. They are designed to maintain a consistent distance from the wall of the exhibits. Since the shape of the exhibits change over time, the museum staff must program the robots to turn the corners of the exhibit.

When a robot reaches a corner, it will stop, turn through a programmed angle, and then continue its patrol. Your job is to determine the angles that R1 will need to turn as it patrols the assigned area. Keep in mind the direction in which the robot is traveling and make sure it always faces forward as it moves around the exhibits.

1. What angles will R1 need to turn? These are called ***exterior*** angles. What is the sum of the ***exterior*** angles of this exhibit?



2. Determine the measures of the ***interior*** angles of the exhibit. What is the sum of these measures?

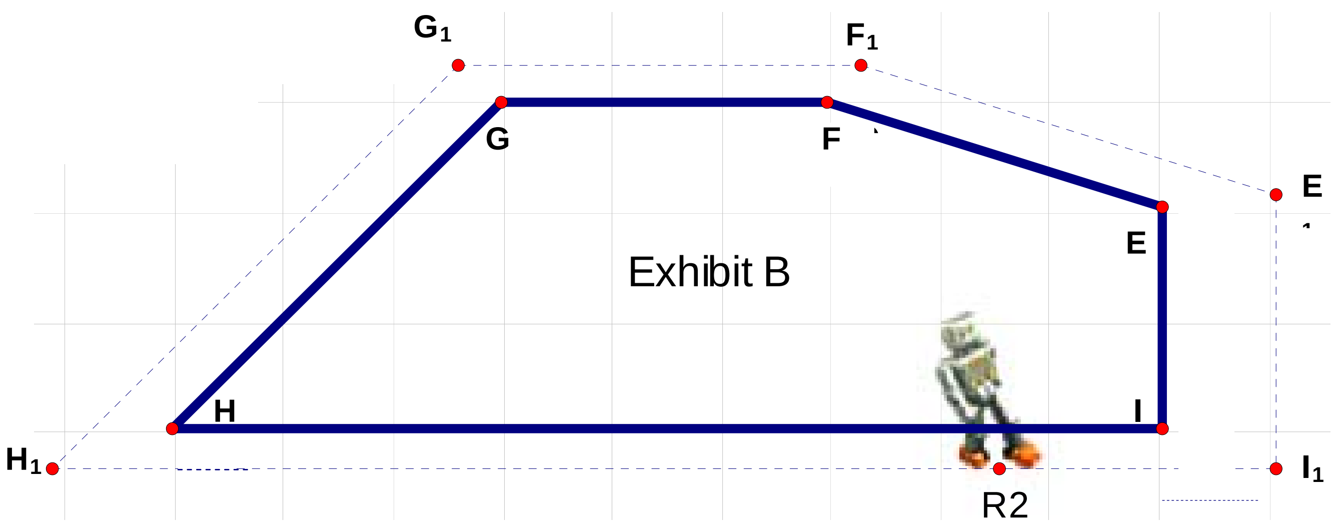
Mathematics I
Robot Gallery Guards
Day 1 Student Task: Group B

The Asimov Museum has contracted with a company that provides Robotic Security Squads to guard the exhibits during the hours the museum is closed. The robots are designed to patrol the hallways around the exhibits and are equipped with cameras and sensors that detect motion.

Each robot is assigned to patrol the area around a specific exhibit. They are designed to maintain a consistent distance from the wall of the exhibits. Since the shape of the exhibits change over time, the museum staff must program the robots to turn the corners of the exhibit.

When a robot reaches a corner, it will stop, turn through a programmed angle, and then continue its patrol. Your job is to determine the angles that R2 will need to turn as it patrols the assigned area. Keep in mind the direction in which the robot is traveling and make sure it always faces forward as it moves around the exhibits.

1. What angles will R2 need to turn? These are called ***exterior*** angles. What is the sum of the ***exterior*** angles of this exhibit?



2. Determine the measures of the ***interior*** angles of the exhibit. What is the sum of their measures?

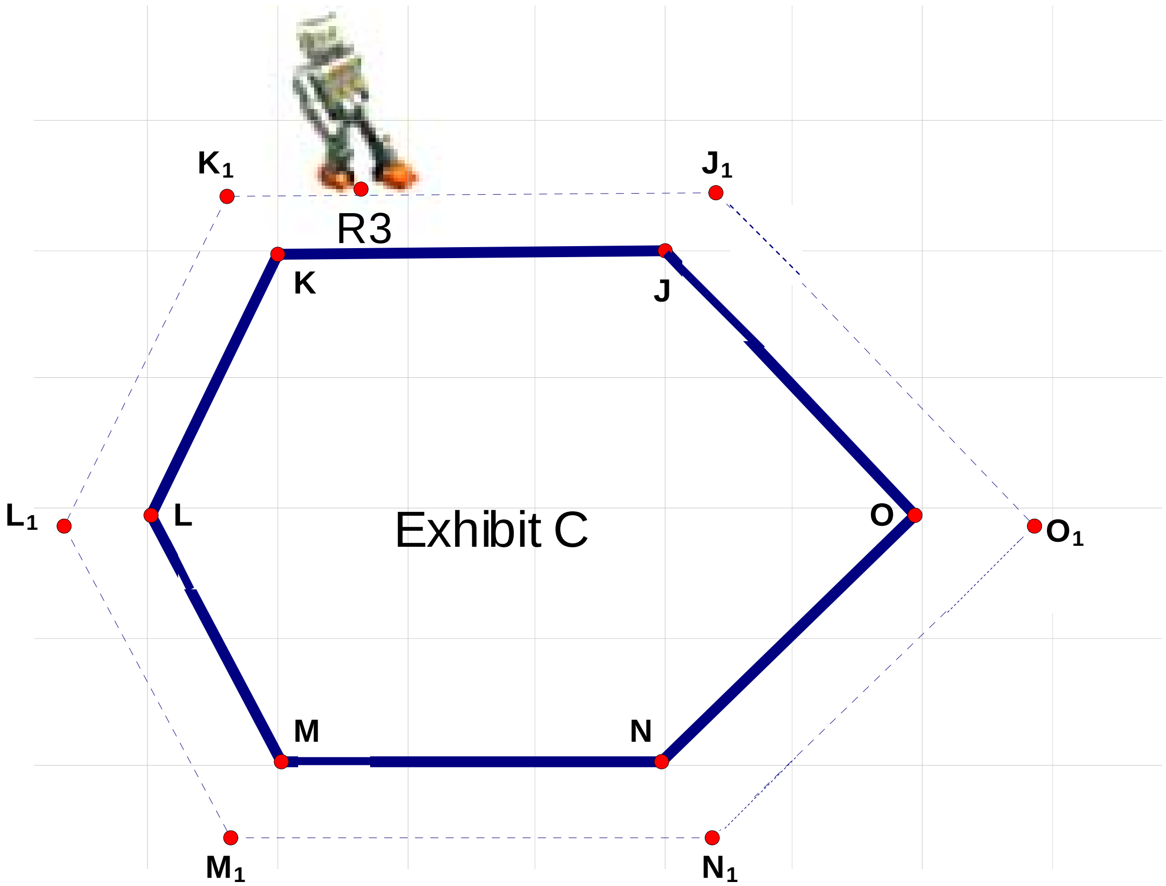
Mathematics I
Robot Gallery Guards
Day 1 Student Task: Group C

The Asimov Museum has contracted with a company that provides Robotic Security Squads to guard the exhibits during the hours the museum is closed. The robots are designed to patrol the hallways around the exhibits and are equipped with cameras and sensors that detect motion.

Each robot is assigned to patrol the area around a specific exhibit. They are designed to maintain a consistent distance from the wall of the exhibits. Since the shape of the exhibits change over time, the museum staff must program the robots to turn the corners of the exhibit.

When a robot reaches a corner, it will stop, turn through a programmed angle, and then continue its patrol. Your job is to determine the angles that R3 will need to turn as it patrols the assigned area. Keep in mind the direction in which the robot is traveling and make sure it always faces forward as it moves around the exhibits.

1. What angles will R3 need to turn? These are called ***exterior*** angles. What is the sum of the ***exterior*** angles of this exhibit?



2. Determine the measures of the ***interior*** angles of the exhibit. What is the sum of their measures?

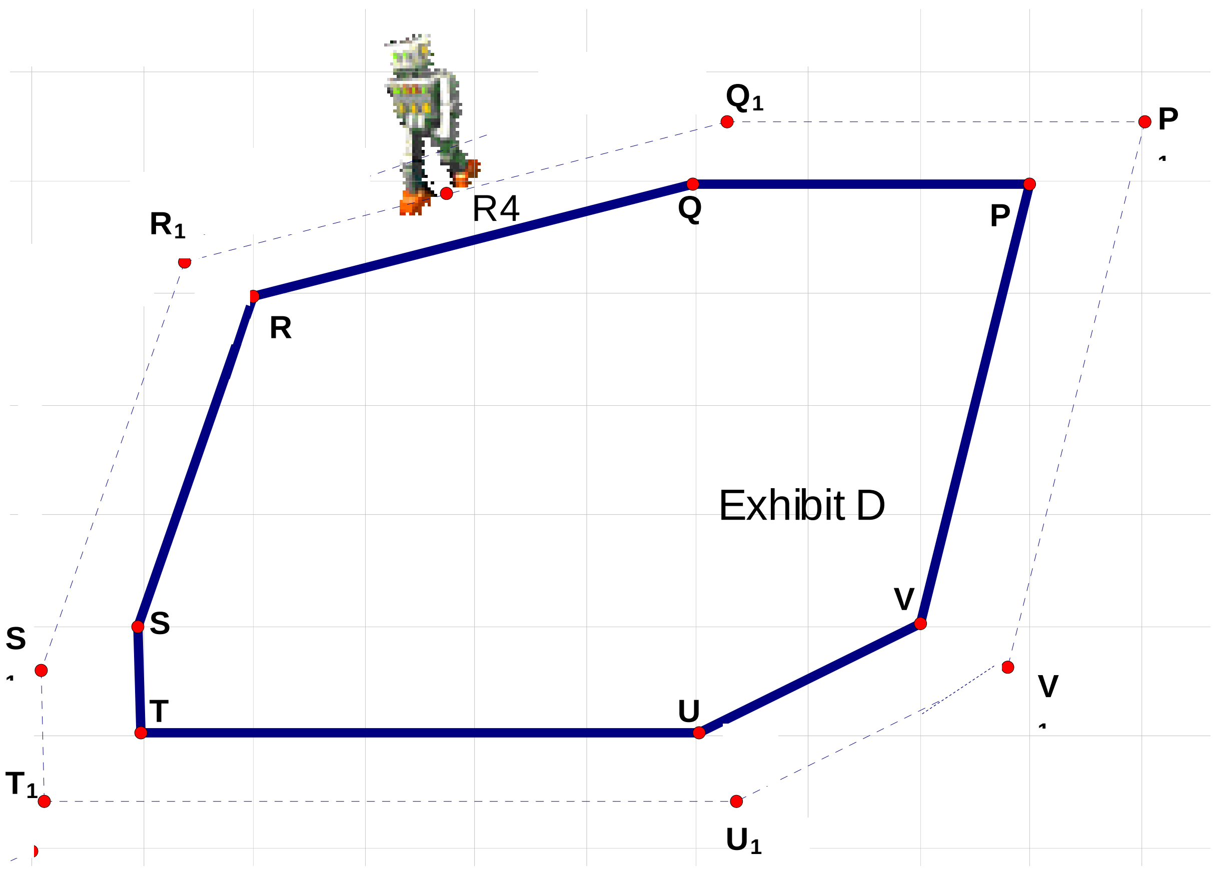
Mathematics I
Robot Gallery Guards
Day 1 Student Task: Group D

The Asimov Museum has contracted with a company that provides Robotic Security Squads to guard the exhibits during the hours the museum is closed. The robots are designed to patrol the hallways around the exhibits and are equipped with cameras and sensors that detect motion.

Each robot is assigned to patrol the area around a specific exhibit. They are designed to maintain a consistent distance from the wall of the exhibits. Since the shape of the exhibits change over time, the museum staff must program the robots to turn the corners of the exhibit.

When a robot reaches a corner, it will stop, turn through a programmed angle, and then continue its patrol. Your job is to determine the angles that R4 will need to turn as it patrols the assigned area. Keep in mind the direction in which the robot is traveling and make sure it always faces forward as it moves around the exhibits.

1. What angles will R4 need to turn? These are called ***exterior*** angles. What is the sum of the ***exterior*** angles of this exhibit?



2. Determine the measures of the ***interior*** angles of the exhibit. What is the sum of their measures?

Mathematics I
Robot Gallery Guards
Day 1 Student Task Continued: All Groups

3. Compare your results with the results of other groups.

4. What do you notice about the sum of the ***exterior*** angles?

5. How could you determine the sum of the ***exterior*** angles in the exhibits without using a protractor?

6. What do you notice about the sum of the ***interior*** angles?

7. How could you determine the sum of the ***interior*** angles in the exhibits without using a protractor?

8. Find the sum of the ***interior*** angles of a decagon. A dodecagon. An *n*-gon.

9. Choose one of the exhibits from the map and look at the interior and exterior angle found at a vertex. What do you notice about the sum of these two angles? Find this sum at each vertex. Explain your observations of each pair of interior and exterior angles found at every vertex.

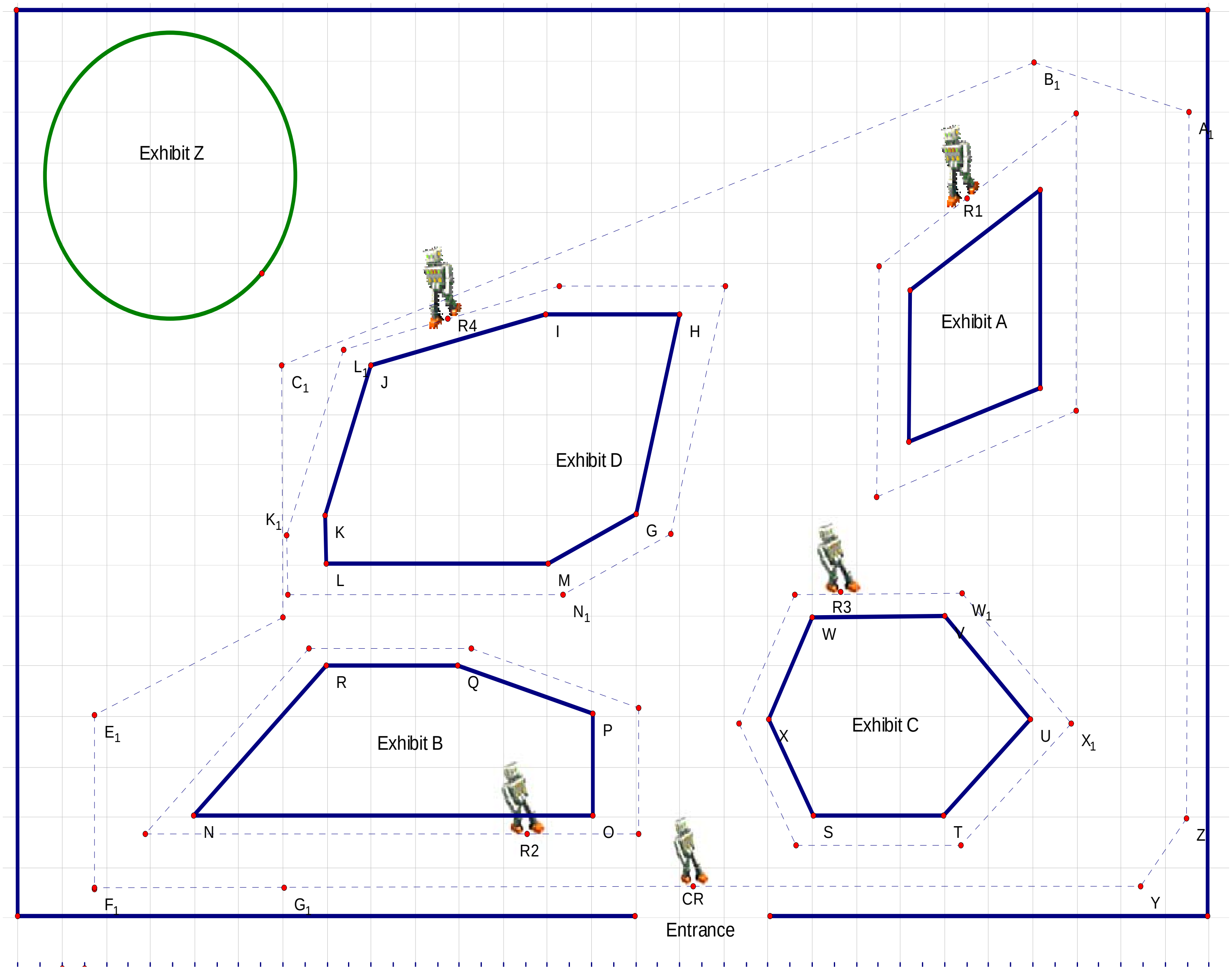


Exhibit Z
Exhibit A
Exhibit D
Exhibit C
Exhibit B
Entrance
R1
R2
R3
R4
CR
A1
B1
C1
E1
F1
G1
K1
L1
N1
W1
X1
H
I
J
K
L
M
G
N
O
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T
U
V
W
X
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Z

Mathematics I
Task 1: Robotic Gallery Guards
Day 1 Homework

1. Find the sum of the exterior angle measures of a nine sided polygon.

2. What is the sum of the interior angle measures of a hexagon?

3. If the sum of the interior angle measures of a polygon equals 900^o, how many sides does the polygon have?

4. How many sides does the polygon have if its interior angles measure 2160^o?

Mathematics I
Task 1: Robotic Gallery Guards **Day 2/2**
(GA DOE TE #11-17)

Standard(s): MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
a. Determine the sum of interior and exterior angles in a polygon.

New vocabulary: exterior angles, interior angles

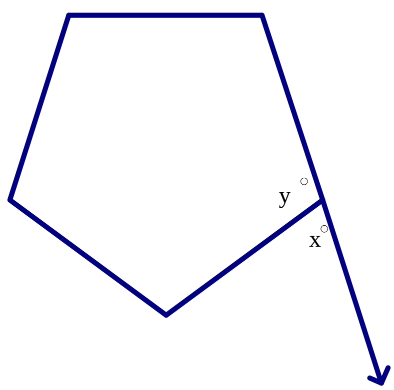
Mathematical concepts/topics: supplementary angles, regular polygons, exterior angle sum theorem, sum of the measures of the interior angles of a convex polygon, measure of each interior angle of a regular n-gon, measure of each exterior angle of a regular n-gon, algebraic proofs of geometric properties

Prior knowledge: using protractors to measure angles, supplementary angles, number of degrees in a triangle, extending patterns

Essential question(s): Are there special relationships that govern the measurements of the interior and exterior angles of a convex polygon?

Suggested materials: rulers, protractors

Warm-up:



1. Using the figure drawn here, find x in terms of y.
2. Find the sum of the exterior angles in a heptagon.
3. Find the sum of the interior angles of a heptagon.

Opening: Discuss the warm-up. Be sure students are able to clarify their thinking and that any misconceptions are corrected.

Worktime: Students should do problems 10-16 of the task. As you monitor progress, be sure students are using the proper formulas for problems 11-14. On problem #15, encourage students to draw at least one concave polygon and measure the interior and exterior angles to see if the generalizations derived hold for all polygons.

Closing: There should be a thorough discussion of problems 10 – 15. Students should share their work and thinking with the whole group. Problem 16 may be addressed by the whole class or used as enrichment depending upon time and student understanding of the previous problems.

Homework:

Differentiated support/enrichment:

Check for Understanding: The following additional questions might be used to check for understanding of the concepts and formulas developed in the task.

1. The sum of the interior angle measures of a convex polygon is 1260^o. Name the polygon.
2. Every exterior angle of a convex polygon measures 24^o. How many sides does the polygon have?
3. A regular polygon has one interior angle that measures 160^o. How many sides does it have?
4. Explain why the sum of the exterior angle measures of a convex polygon is always 360^o.

Resources/materials for Math Support: Students may need review in simplifying algebraic expressions of the type needed to complete the proof in problem 10 of the student task.

Mathematics I
Robotic Gallery Guards
Day 2 Student Task

10. In the previous lesson, you discovered a formula for the sum of the interior angles of a polygon. You also found that the sum of the interior and exterior angels at every vertex of a polygon is 180^o. Can you use these two pieces of information to help you prove your conjecture, from problem #5, that the sum of the exterior angles of a polygon is always 360^o?

11. The museum intends to create regular polygons for its next exhibition; how can the directions for the robots be determined for a regular pentagon? Hexagon? Nonagon? N-gon?

12. A sixth exhibit was added to the museum. The robot patrolling this exhibit only makes 15º turns. What shape is the exhibit? What makes it possible for the robot to make the same turn each time?

13. Robot 7 makes a total of 360 º during his circuit. What type of polygon does this exhibit create?

14. The sum of the interior angles of Robot 7's exhibit is 3,420 º. What type of polygon does the exhibit create?

15. All of the current exhibits are designed in the shape of convex polygons. Would the generalizations you have made for these shapes hold for a concave polygon?

16. The museum is now using Exhibit Z. Complete a set of instructions that will help the robot monitor this circular exhibit. Compare your instructions with a partner. Which instructions are better and why?

Atlanta Public Schools
Teacher's Curriculum Supplement
Mathematics I: Unit 3
Task 2: Poor Captain Robot

Mathematics I
Task 2: Poor Captain Robot **Day 1/1**
(GA DOE TE #1-7)

Standard(s): MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
b. Understand and use the triangle inequality, the side-angle inequality, and the exterior-angle inequality.

New vocabulary: triangle inequality, side-angle inequality, remote interior angles, exterior angle inequality

Mathematical concepts/topics: triangle inequality, measures of interior angles of a triangle, measures of exterior angles of a triangle, remote interior angles, the relationship between the measures of the angles of a triangle and the lengths of the sides (side-angle inequality), the relationship between an exterior angle of a triangle and its two remote interior angles (exterior angle inequality)

Prior knowledge: using protractors to measure angles, supplementary angles, number of degrees in a triangle

Essential question(s): Are there special relationships among the lengths of the sides of any triangle? Are there special relationships between the lengths of the sides of a triangle and the measures of the angles of a triangle? What special relationships exist between the exterior and interior angles of a triangle?

Suggested materials: string, straws, spaghetti, pipe cleaners, or coffee stirrers; protractors; rulers marked in centimeters; Geometer's Sketchpad, if possible

Warm-up: Have students read the scenario for *Poor Captain Robot*. Ask them to make notes in the margin and be ready to explain the situation.

Opening: Discuss the warm-up by reading the scenario and asking a student to model a possible path that Captain Robot might take. Explain the materials available for use during the task.

Worktime: Students should do problems 1-7 of the task. Working with straws, spaghetti, or other manipulatives can give students a conceptual understanding of the inequalities addressed here fairly quickly. Be sure to allow enough time for students to *discover* the concepts.

Closing: Discuss all 7 problems thoroughly. Make sure that multiple ways of visualizing the concepts, using both manipulatives and software, are shared by students. (See teacher notes.)

Homework: Jovani is designing a triangular frame for his latest art project. Two sides of the frame measure 12 inches and 23 inches.

1. What possible lengths could Jovani choose for the third side of his frame?
2. Choose an appropriate length for the third side of Jovani's frame and sketch the frame using a scale of 1 cm to 3 inches.
3. Label your sketch with the lengths of all 3 sides and the measures of all 3 angles.
4. Use your measures to write a numerical inequality and a verbal statement that illustrate the side-angle inequality.
5. Use your measures to write a numerical inequality and a verbal statement that illustrate the exterior angle inequality.

Differentiated support/enrichment: Students may need more of an introduction to proof. This is an excellent time to introduce formal proof. One suggestion for this introduction is to have students prove the measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles.

Check for Understanding:

Resources/materials for Math Support: Students may need to practice measuring length using centimeters and review the fact that the sum of the measures of the interior angles of a triangle is 180°. Students will also benefit from practice using Geometer's Sketchpad.

Mathematics I
Poor Captain Robot
Day 1 Student Task

Captain Robot's positronic brain is misfiring and he will only take instructions to move three distances. He will no longer acknowledge angles given in directions and chooses all of them himself. He will not travel the same path twice and refuses to move at all if he cannot end up back at his starting point.

1. Captain Robot was given the following sets of instructions. Determine which instructions Captain Robot will use and which sets he will ignore. Be ready to defend your choices.

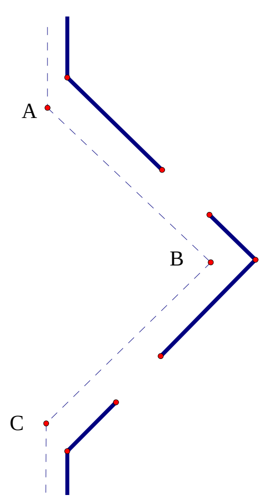
    Instruction Set #1: 10 m, 5 m, 8 m
    Instruction Set #2: 7 m, 4.4 m, 8 m
    Instruction Set #3: 10 m, 2 m, 8 m
    Instruction Set #4: 5 m, 5 m, 2.8 m
    Instruction Set #5: 7 m, 5.1 m, 1 m

2. Determine a method that will always predict whether or not Captain Robot will move.

3. Captain Robot traveled from point A to point B to point C. His largest turn occurred at point C and his smallest turn occurred at point B. Order the sides of the triangle that Captain Robot traversed by length from longest to shortest.

4. Is there a relationship between the lengths of the sides and the measures of the angles of a triangle? Explain why or why not.

    After repair, the museum has decided Captain Robot needs to patrol two access doors off the side of the museum. Captain Robot's addition to his route is shown below.



5. Determine the measures of the angles Captain Robot will need to turn.

6. Consider the triangle formed by points A, B, and C. Look carefully at the angles you measured for the robot's route. How would you describe these angles in relation to triangle ABC? Do you see a relationship between the interior angles at A and C and the exterior angle at B?

7. Draw several triangles. Measure the interior angles and one exterior angle. What pattern do you notice? Make a generalization relating the interior and exterior angles.

ATLANTA PUBLIC SCHOOLS

Making A Difference

Atlanta Public Schools

Teacher's Curriculum Supplement

Mathematics I: Unit 3

Task 3: Constructing Pennants

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Mathematics & Science Initiative

Mathematics I
Task 3: Constructing Pennants **Day 1/3**
(GaDOE TE #2, #5, #6, and 8)

Standard(s): MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
c. Understand and use congruence postulates and theorems for triangles (SSS, SAS, ASA, AAS, HL).

New vocabulary:

Mathematical concepts/topics: similarity; congruence; corresponding parts of congruent and similar figures; counterexample; investigating whether congruence of triangles can be determined given 1 or 2 pieces of information; SSS, ASA, and AAS postulates and theorem

Prior knowledge: similarity, congruence, corresponding parts of congruent and similar figures

Essential question(s): What information is sufficient to guarantee that two triangles are congruent?

Suggested materials: string, straws, spaghetti, pipe cleaners, or coffee stirrers; protractors; rulers; compass and straight edge; patty paper; Geometer's Sketchpad, if possible

Warm-up: Have students read the introductory paragraphs of the task. Ask them to note important points in the margins of the introduction and be prepared to discuss the scenario.

Opening: Have a student read the introduction aloud. Ask students to explain the situation. Thinking in terms of the minimum requirements needed to assure congruent triangles, ask the students to discuss whether one given side measurement or one given angle measurement would be enough information to guarantee congruent triangles. Have them share counterexamples. Students should easily realize that one piece of information is not enough to guarantee congruence.

Worktime: Students should complete problems 1 – 4 of the student task. The chart in problem 5 should be used as a graphic organizer and completed as students finish each part of the task. Allow time for students to experiment and discover for themselves what information is sufficient to guarantee congruent triangles.

Closing: Allow students to share their work for each part of the task. Students may have different conjectures about problem 3. The key is that two angles and a side will always produce congruent triangles as long as the order in which the triangles are constructed is the same (i.e. AAS or ASA). See the teacher notes for problems 5 and 6 in the GaDOE TE.

Homework: If not finished in class, have students complete the first three rows of the chart in problem 5.

Differentiated support/enrichment:

Check for Understanding:

Resources/materials for Math Support: Students should review/preview similarity, congruence, corresponding parts of congruent and similar figures, and counterexample.

Mathematics I
Constructing Pennants
Day 1 Student Task

The students at Hometown High School decided to make large pennants for all 8 high schools in their district. The picture to the right shows typical team pennants. The Hometown High students wanted their pennants to be unique and possibly shaped differently than the typical isosceles triangle used for pennants. They plan to hang the final products in the gym as a welcome to all the schools who visit Hometown High.

Jamie had learned in middle school that two figures are congruent if all of their corresponding sides are congruent and all of their corresponding angles are congruent. As chairperson of the pennant committee, she wanted to be sure that all eight of the pennants were congruent to each other. While she knew that all corresponding parts of congruent figures are themselves congruent, she suspected that it is not always necessary to have all of that information. The entire committee decided to find the minimum requirements needed before they started making the triangular pennants.

Your group has been asked to help the committee experiment in order to find this information.

1. You have discovered that one piece of information is not enough to prove congruent triangles. Now you will experiment with only 2 pieces of information. Before you begin, predict whether the two given pieces of information will be enough to guarantee congruent triangles.
   a) Two given sides
   b) Two given angles
   c) One given side and one given angle

   Was your prediction correct? Why or why not? State a true generalization for each of these three experiments.

2. Conduct an experiment using three sides of a triangle. Construct a triangle with sides of 3 inches, 4 inches, and 6 inches. Compare your triangle to other students' triangles. Are any of the triangles congruent? Are all of the triangles congruent? Will this always happen? Conduct at least one additional experiment with 3 different side lengths. Do you think 3 sides are enough to guarantee congruent triangles? Explain.

3. Next, experiment with one side and two angles. Start with a side of 7 in. and angles of 35° and 57°. Construct and compare triangles. Do you think that 2 angles and 1 side are enough information to guarantee congruent triangles? Explain.

4. Jim noticed that Sasha drew her conclusion given two angles and the included side. He wondered if the results would be the same if you were given any two angles and a side. What do you think?

5. To help the pennant committee understand your findings, you will summarize results of this task using the chart below. You should be ready now to summarize your results for parts 2 through 4 in the first 3 rows of the chart. This chart will be revisited concerning the additional rows throughout the rest of the task.

Information	Does it always, produce a unique triangle?	Make a complete statement regarding your findings and illustrate with a diagram.
SSS		
ASA		
AAS		
SAS		
SSA		
AAA		
HL		
LL		

Mathematics I
Task 3: Constructing Pennants **Day 2/3**
(GaDOE TE #3, 4, 7, 8, 9, and 10)

Standard(s): MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
c. Understand and use congruence postulates and theorems for triangles (SSS, SAS, ASA, AAS, HL).

New vocabulary: postulate, theorem

Mathematical concepts/topics: similarity, congruence, corresponding parts of congruent and similar figures, counterexample, investigating whether congruence of triangles can be determined given three angles, SAS postulate, definition of postulate and theorem, proof using the congruence postulates

Prior knowledge: similarity, congruence, corresponding parts of congruent and similar figures

Essential question(s): What information is sufficient to guarantee that two triangles are congruent?

Suggested materials: string, straws, spaghetti, pipe cleaners, or coffee stirrers; protractors; rulers; compass and straight edge; patty paper; Geometer's Sketchpad, if possible

Warm-up: If the first 3 rows of the chart in problem 5 have been completed, allow pairs of students to compare their responses. If not, give the chart to students and have them complete the first 3 rows as a way of reviewing the postulates and theorem addressed in the previous lesson. Have students read problem 6 of the student task silently, noting important information in the margins.

Opening: Allow students to share their responses in the chart. Then ask them to explain the situations that they will be investigating today.

Worktime: Students should complete problems 6 - 10 of the student task. Problems 6 and 7 may be challenging for students. See the teacher notes for problems #3 and #4 in the GaDOE TE.

Closing: Allow students to share their work for each part of the task.

Homework: Complete appropriate parts of the chart in problem 5.

Differentiated support/enrichment:

Check for Understanding:

Resources/materials for Math Support: Students should review/preview similarity, congruence, corresponding parts of congruent and similar figures, and counterexample.

Mathematics I
Constructing Pennants
Day 2 Student Task

6. Conduct an experiment using only two sides and one angle. Start with sides of 5 inches and 7 inches and an angle of 38°. Using these measures, construct a triangle and compare it to the triangles of other students. Are any of the triangles congruent? Are all of the triangles congruent? Will this always happen? Explain.

7. Joel and Cory ended up with different triangles. Joel argued that Cory put her angle in the wrong place. Joel constructed his triangle with the angle between the two sides. Cory constructed her sides first then constructed her angle at the end of the 7 in. side not touching the 5 in. side. Everybody quickly agreed that these two triangles were different. They all tried Cory's method, what happened? Which method, Joel's or Cory's will always produce the same triangle?

8. As a last experiment to help the pennant committee, your group will work with three angles. Make a prediction before beginning the experiment concerning the results using angles of 20°, 40°, and 120°. Construct a triangle using these three angles and compare with others. Was your prediction correct? Why or why not? Conduct at least one additional experiment with 3 different angle measures. Do you think that three angle measures are enough information to guarantee congruent triangles? Explain why or why not.

9. SSS, SAS, and ASA are generally accepted as postulates. Look up the definition of a postulate. Discuss the need for postulates in this case. Why can't we just use SSS as a single postulate? Could we use less than the three given postulates or could we choose a different set as postulates?

10. Use the postulates discussed in this lesson to PROVE that AAS is always true. What do we call a statement that can be proved using a postulate?

Mathematics I
Task 3: Constructing Pennants **Day 3/3**
(GaDOE TE #11 - 16)

Standard(s): MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
c. Understand and use congruence postulates and theorems for triangles (SSS, SAS, ASA, AAS, HL).

New vocabulary:

Mathematical concepts/topics: similarity, congruence, determining corresponding parts of congruent triangles, determining congruence of right triangles given the hypotenuse and one leg or two legs, indirect measurement, using congruent triangles to solve authentic problems

Prior knowledge: similarity, congruence, corresponding parts of congruent and similar figures, right triangles, the Pythagorean Theorem

Essential question(s): How can I use congruent triangles to help me solve real-world problems?

Suggested materials: string, straws, spaghetti, pipe cleaners, or coffee stirrers; protractors; rulers; compass and straight edge; patty paper; Geometer's Sketchpad, if possible

Warm-up: Sketch a right triangle and label with appropriate side lengths. Explain your thinking.

Opening: Discuss the warm-up. Students have studied right triangles extensively and should be able to use the Pythagorean Theorem to determine appropriate lengths for the sides of a right triangle. Review findings from previous lessons by allowing students to share their comments in the charts completed for homework.

Worktime: Students should complete problems 11 - 16 of the student task.

Closing: Allow students to share their work for each part of the task.

Homework: All parts of the chart in problem 5 should be completed. Students will need more practice on real-world problems and indirect measurement and in writing proofs using the theorems and postulates learned in this task. A sample problem is given below.

In middle school, and perhaps in this task, you used a compass and a straight edge to construct the perpendicular bisector of a segment. Construct the perpendicular bisector of a segment and use the triangle congruence postulates you learned in this task to prove that your construction does create a perpendicular bisector.

Differentiated support/enrichment:

Check for Understanding: Problem 11 of the student task

> *Write a letter to the pennant committee giving your recommendation for the triangular pennant shape and the minimum required information to assure that all eight pennants are congruent? Be sure to include reasons for your suggestions.*

Resources/materials for Math Support: Students should review/preview similarity, congruence, corresponding parts of congruent and similar figures, counterexample, right triangles, the Pythagorean Theorem.

Mathematics I
Constructing Pennants
Day 3 Student Task

11. The postulates and theorem listed in the table in problem 5, which can be used for proving two triangles congruent, require three parts of one triangle to be congruent to three corresponding parts of another triangle. Nakita wondered if the methods might be a bit shorter for right triangles. She chose to create two right triangles with congruent hypoteni and one congruent leg. Would this guarantee congruent triangles? Why or why not?

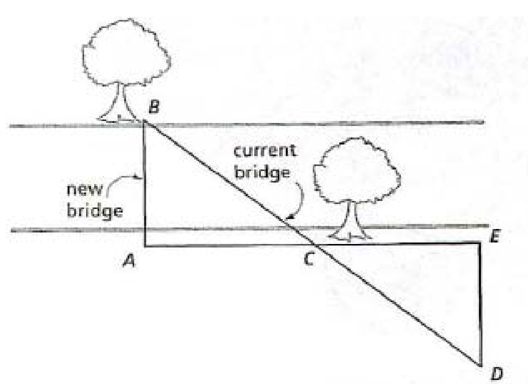
12. What if Nakita had chosen right triangles with the two corresponding legs congruent to each other? Would this guarantee congruent triangles? Why or why not?

13. What are the minimum parts needed to justify that two right triangles are congruent? Add this information to the last two rows of your chart.

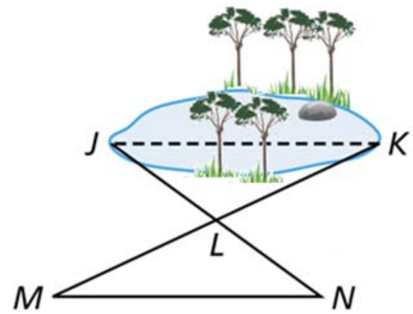
14. Once it is known that two triangles are congruent, what can be said about the parts of the triangles? Write a statement relating the parts of congruent triangles.

Congruent triangles can be used to solve problems encountered in everyday life. The next two situations are examples of these types of problems.

15. In order to construct a new bridge, to replace the current bridge, an engineer needed to determine the distance across a river, without swimming to the other side. The engineer noticed a tree on the other side of the river and suddenly had an idea. She drew a quick sketch and was able to use this to determine the distance. Her sketch is to the right. How was she able to use this to determine the length of the new bridge? You do not have to find the distance; just explain what she had to do to find the distance.



16. A landscape architect needed to determine the distance across a pond. Why can't he measure this directly? He drew the following sketch as an indirect method of measuring the distance. He stretched a string from point J to point N and found the midpoint of this string, point L. He then stretched a string from M to K making sure it had same center. He found the length of segment MN was 43 feet and the length of segment LK is 19 feet. Find the distance across the pond. Justify your answer.



Atlanta Public Schools

Teacher's Curriculum Supplement

Mathematics I: Unit 3

Task 4: Constructing with Diagonals

Task 4: Constructing with Diagonals **Day 1/3**
(GaDOE TE #1 - #3)

Standard(s): MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.

d. Understand, use, and prove properties of and relationships among special quadrilaterals: parallelogram, rectangle, rhombus, square, trapezoid, and kite.

New vocabulary:

Mathematical concepts/topics: attributes of special quadrilaterals; use of Euclidean tools; constructions; properties of special quadrilaterals; proof, including paragraph proofs, flow proofs, and two column proofs

Prior knowledge: attributes of special quadrilaterals, measuring with protractors, use of Euclidean tools, constructions

Essential question(s): How can I determine which quadrilateral(s) can be constructed based on specific information about the diagonals?

Suggested materials: Depending upon the method of construction, students may need manipulatives such as spaghetti noodles, straws, pipe cleaners, d-stix, etc.; protractors; compasses; straightedges; and/or Geometer's Sketchpad. (See teacher notes for further information.)

Warm-up: Post the following:

Construct two segments of different length that are perpendicular bisectors of each other.

Opening: Discuss the warm-up, making sure that all students are comfortable with the construction. Ask students what they know about quadrilaterals. Spend a few minutes charting the information they already know without going deeply into the properties of quadrilaterals. The goal is for students to give you a definition of a quadrilateral and name some special quadrilaterals in order to set the stage for the investigations in the task.

Worktime: Students should complete problems 1 - 3 of the student task with a partner. Each of these problems requires students to develop a proof. Teachers should have exposed students to all 3 types of proofs: two-column, flow chart, and paragraph. When students have had ample time to prove their conjectures in problem 1 using one type of proof, stop and discuss the problem as a whole class. Allow students to share their work. Students should have ownership and do the work while the teacher leads them, if necessary, with guiding questions through the proof. Share all 3 kinds of proof, if possible.

The proof of problem 3 can be assigned for homework, if class time is an issue.

Closing: Allow students to share finished work. Students should understand not only that they may use various types of proofs, but also that proofs can be logically organized with differing orders of steps. (See teacher notes.)

Homework:

Differentiated support/enrichment:

Resources/materials for Math Support: Students may need to review basic constructions (particularly perpendicular bisectors), and the names and attributes of special quadrilaterals. Give students lots of opportunities to develop proofs, exposing them to all 3 types of proof. Including both simple algebraic and geometric proofs will help students gain proficiency in understanding and developing logical sequences of steps.

Mathematics I
Constructing with Diagonals
Day 1 Student Task

1. Construct two segments of different lengths that are perpendicular bisectors of each other. Connect the four end points to form a quadrilateral. What names can be used to describe the quadrilaterals formed using these constraints? Write a proof to justify your answer.

2. Repeat #1 with two congruent segments. Connect the four end points to form a quadrilateral. What names can be used to describe the quadrilaterals formed using these constraints? Write a proof to justify your answer.

3. Construct two segments that bisect each other but are not perpendicular. Connect the four end points to form a quadrilateral. What names can be used to describe the quadrilaterals formed using these constraints? Write a proof to justify your answer.

Mathematics I
Task 4: Constructing with Diagonals **Day 2/3**
(GaDOE TE #4 - 8)

Standard(s): MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.

e. Understand, use, and prove properties of and relationships among special quadrilaterals: parallelogram, rectangle, rhombus, square, trapezoid, and kite.

New vocabulary:

Mathematical concepts/topics: attributes of special quadrilaterals; use of Euclidean tools; constructions; properties of special quadrilaterals; proof, including paragraph proofs, flow proofs, and two column proofs; necessary conditions

Prior knowledge: attributes of special quadrilaterals, use of Euclidean tools

Essential question(s): How can I determine which quadrilateral(s) can be constructed based on specific information about the diagonals?

Suggested materials: Depending upon the method of construction, students may need manipulatives such as spaghetti noodles, straws, pipe cleaners, d-stix, etc.; protractors; compasses; straightedges; and/or Geometer's Sketchpad. (See teacher notes for further information.)

Warm-up: Have pairs of students compare the proofs they wrote for problem 3.

Opening: As a means of review and setting the stage for today's lesson, have students share and discuss different types of proofs for problem 3.

Worktime: Students should work in pairs to complete problems 4 – 8 of the student task.

Closing: Allow students to share their work. Problem 7 may be difficult for students but it is a very important question. Notice students are asked to name the quadrilaterals for which the condition is *necessary, not sufficient.* In other words, the quadrilaterals for which this condition must exist. There may be other conditions that must exist as well in order to determine a specific quadrilateral but this is not the question. A condition or set of conditions that will guarantee a specific figure are referred to as *sufficient* for determining the figure. This is the question that is asked in problem 11 of the student task. As an example, row 5 of the table in problem 7 reads as follows:

Diagonals are perpendicular and bisect each other.

These two conditions are *necessary* for both rhombi and squares, which should be the answers given in the table. They are not *sufficient* for proving that the figure is a square. In order to prove a square, we must know that the diagonals are congruent.
Answers in the sixth row of the table should be rectangle and square.

Homework:

Differentiated support/enrichment:

Resources/materials for Math Support: Continue to review the names and attributes of special quadrilaterals and to provide practice in developing proofs.

Mathematics I
Constructing with Diagonals
Day 2 Student Task

4. What if the two segments in #3 above are congruent in length? What type of quadrilateral is formed? What names can be used to describe the quadrilaterals formed using these constraints? Write a proof to justify your answer.

5. Draw a segment and mark the midpoint. Now construct a segment that is perpendicular to the first segment at the midpoint but is not bisected by the original segment. Connect the four end points to form a quadrilateral. What names can be used to describe the quadrilaterals formed using these constraints? Write a proof to justify your answer.

6. In the above constructions you have been discovering the properties of the diagonals of each member of the quadrilateral family. Stop and look at each construction. Summarize any observations you can make about the special quadrilaterals you constructed. If there are any quadrilaterals that have not yet been constructed, investigate any special properties of their diagonals.

7. Complete the chart below by identifying the quadrilateral(s) for which the given condition is necessary.

Conditions	Quadrilateral(s)	Explain your reasoning
Diagonals are perpendicular.		
Diagonals are perpendicular and only one diagonal is bisected.		
Diagonals are congruent and intersect but are not perpendicular.		•
Diagonals bisect each other.		
Diagonals are perpendicular and bisect each other.		
Diagonals are congruent and bisect each other.		
Diagonals are congruent, perpendicular and bisect each other.		

8. As you add more conditions to describe the diagonals, how does it change the types of quadrilaterals possible? Why does this make sense?

Mathematics I
Task 4: Constructing with Diagonals **Day 3/3**
(GaDOE TE #9 - 11)

Standard(s): MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.

e. Understand, use, and prove properties of and relationships among special quadrilaterals: parallelogram, rectangle, rhombus, square, trapezoid, and kite.

New vocabulary:

Mathematical concepts/topics: attributes of special quadrilaterals, properties of special quadrilaterals, necessary conditions, sufficient conditions

Prior knowledge: attributes of special quadrilaterals

Essential question(s): How can I identify and verify properties of special quadrilaterals?

Suggested materials: rulers, protractors, Geometer's Sketchpad, if possible

Warm-up: Post one of the quadrilaterals A – F from problem 9 and the following directions.

> *Name the figure above using as many names as possible and state as many properties as you can about the figure.*

Opening: To this point in the task, students have focused on properties related to the diagonals of special quadrilaterals. In this lesson they will extend their investigations to include properties of sides and angles as well as diagonals. Discuss the warm-up allowing students to fill in the table in problem 9 for this figure during the discussion. Be sure to list as many properties as possible, particularly those related to angles and sides.

Worktime: Students should complete problems 9 – 11.

Closing: Allow students to share their work. Problems 10 and 11 should help students better understand the ideas of necessary and sufficient conditions. In problem 10 students are asked to identify the properties that are *always* true for the given quadrilateral. These properties would be necessary for the given figure. In problem 11 of the student task, revised for this lesson, students are asked to *list at least one **minimum** set of conditions **sufficient** to prove that a quadrilateral is* one of the figures listed. There are multiple solutions. Encourage students to list more than one set of conditions. (See teacher notes.)

Homework:

Differentiated support/enrichment:

Resources/materials for Math Support:

Mathematics I
Constructing with Diagonals
Day 3 Student Task

9. Name each of the figures below using as many names as possible and state as many properties as you can about each figure.

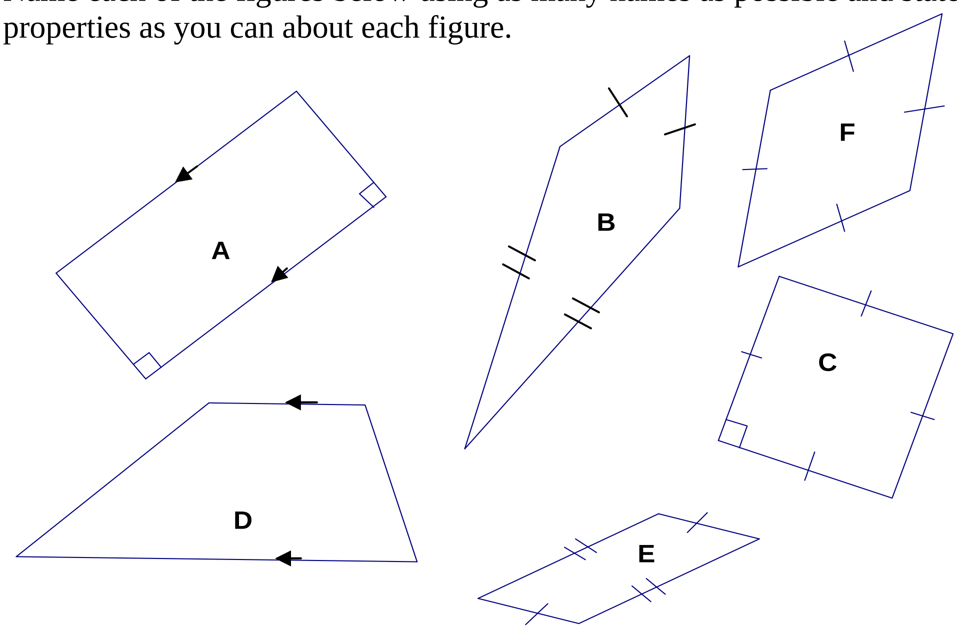


Figure	Names	Properties
A		
B		
C		
D		•
E		
F		

10. Identify the properties that are always true for the given quadrilateral by placing an X in the appropriate box.

Property	Parallelogram	Rectangle	Rhombus	Square	Isosceles Trapezoid	Kite
Opposite sides are parallel						
Only one pair of opposite sides is parallel						
Opposite sides are congruent						
Only one pair of opposite sides is congruent						
Opposite angles are congruent						
Only one pair of opposite angles is congruent						
Each diagonal forms 2 ≅ Triangles						
Diagonals bisect each other						
Diagonals are perpendicular						
Diagonals are congruent						
Diagonals bisect vertex angles						
All ∠s are right ∠s						
All sides are congruent						
Two pairs of consecutive sides are congruent						

11. Using the properties in the table above, list at least one **minimum** set of conditions sufficient to prove that a quadrilateral is:

a. a parallelogram
b. a rectangle
c. a rhombus
d. a square
e. a kite
f. an isosceles trapezoid
g. a trapezoid

Atlanta Public Schools

Teacher's Curriculum Supplement

Mathematics I: Unit 3

Task 5: Middles and Halves

Mathematics I
Task 5: Middles and Halves **Day 1/2**
(GaDOE TE 1- 3)

Standard(s): MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.

e. Find and use points of concurrency in triangles: incenter, orthocenter, circumcenter, and centroid.

New vocabulary: median (of a triangle)

Mathematical concepts/topics: classification of triangles by sides and angles; definitions of angle bisectors, perpendicular bisectors, altitudes, and medians; basic constructions, including angle bisectors, perpendicular bisector, and medians; special properties of isosceles and equilateral triangles; using geometric tools

Prior knowledge: classification of triangles by sides and angles; basic constructions, including angle bisectors, perpendicular lines, and perpendicular bisector; definitions of angle bisectors, perpendicular bisectors, and altitudes; special properties of isosceles and equilateral triangles

Essential question(s): How are angle bisectors, perpendicular bisectors, altitudes, and medians of triangles related?

Suggested materials: miras (or a similar reflective tool), patty paper, compasses, straightedges, and Geometer's Sketchpad or similar geometry software, if possible

Warm-up: Post the following:

You have learned how to classify triangles by the lengths of their sides and by the measures of their angles.

- *How many different ways might you classify triangles using **both** the lengths of their sides and the measures of their angles?*
- *Explain why it is possible (or not possible) to make each of these types of triangles.*

Opening: Discuss the warm-up as a means of reviewing classification and properties of triangles. This is review for students and the discussion should be brief.

Following the discussion, give each group of students a card on which you have written one of the four geometric terms investigated in today's lesson: angle bisector, perpendicular bisector, altitude (of a triangle), and median (of a triangle). Terms should be written in the colors used throughout the student task.

Explain to students that their group has been given a geometric term. They are to develop a definition for their term and illustrate the definition with a diagram. Throughout this activity, students should be encouraged to use the corresponding colors represented in the actual task. Students should be given 5 – 7 minutes to accomplish this. At the end of this time, allow groups to share. As each group shares, the class should agree on a final definition for each term. Students have worked extensively with angle bisectors, perpendicular bisectors, and altitudes in middle school. The only new term in this lesson is *median*.

Worktime: Students should complete problems 1 and 2 of the task. If your students are competent in the use of a variety of geometric tools, including miras, patty paper, compass and straight edge, and software, this lesson should move quickly. If not, it is worth the time to allow students to explore these tools.

Closing: Allow students to share their work. Make sure that all misconceptions are addressed. After the closing, students should be aware of the following:

- altitudes may fall "outside" the triangle from the vertex to the line that contains the opposite side
- perpendicular bisectors do not necessarily intersect a vertex
- a point on the bisector of an angle, is equidistant from the sides of the angle
- a point equidistant from the sides of an angle, lies on the bisector of the angle

Homework:

1. Tell whether each of the following statements is always true, sometimes true, or never true. Explain each of your choices.
   a. A median to a side of a triangle bisects that side of the triangle.
   b. A bisector of an angle of a triangle is perpendicular to the side opposite the angle.
   c. An altitude drawn to a side of a triangle passes through the midpoint of that side.

2. Draw and label a right isosceles triangle and an obtuse scalene triangle on your paper. Discuss how the altitudes of these triangles will be alike and how they will be different. Use words and pictures to describe your work.

Differentiated support/enrichment:

Resources/materials for Math Support: Students should review/preview classification of triangles by sides and angles; definitions of angle bisectors, perpendicular bisectors, and altitudes; and special properties of isosceles and equilateral triangles. Students should perform basic constructions, including angle bisectors, perpendicular bisectors, altitudes, and medians. Allowing students to perform constructions using a variety of tools, including Geometer's Sketchpad or other software, is particularly effective in helping them discover properties and retain concepts.

Mathematics I
Middles and Halves
Day 1 Student Task

1. Draw a scalene triangle on a blank piece of paper.
    - Classify your triangle based on its angles.
    - Make three copies of your triangle. Each triangle should be labeled in exactly the same way.

2. Use a different method (mira, patty paper, compass and straight edge, or Geometer's Sketchpad) to complete each of the four constructions below.
    - Using the first triangle, construct an **angle bisector** and mark it in the color blue. Using blue, label the paper "**angle bisector**". Include on your paper the definition of **angle bisector** agreed upon by your class.
    - Using the second triangle, construct a **perpendicular bisector** and mark it in the color purple. Using purple, label the paper "**perpendicular bisector**". Include on your paper the definition of **perpendicular bisector** agreed upon by your class.
    - Using the third triangle, construct an **altitude** (of a triangle) and mark it in the color red. Using red, label the paper "**altitude**". Include on your paper the definition of **altitude** agreed upon by your class.
    - Using the final triangle, construct a **median** (of a triangle) and mark it in the color green. Using green, label the paper "**median**". Include on your paper the definition of **median** agreed upon by your class.

Mathematics I
Task 5: Middles and Halves **Day 2/2**
(GaDOE TE 4 and 6)

Standard(s): MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
f. Find and use points of concurrency in triangles: incenter, orthocenter, circumcenter, and centroid.

New vocabulary:

Mathematical concepts/topics: classification of triangles by sides and angles; basic constructions, including angle bisectors, perpendicular bisector, and medians; definitions of angle bisectors, perpendicular bisectors, altitudes, and medians; special properties of isosceles and equilateral triangles; proof

Prior knowledge: classification of triangles by sides and angles; basic constructions, including angle bisectors, perpendicular lines, and perpendicular bisector; definitions of angle bisectors, perpendicular bisectors, and altitudes; special properties of isosceles and equilateral triangles

Essential question(s): How are angle bisectors, perpendicular bisectors, altitudes, and medians of triangles related?

Suggested materials: miras (or a similar reflective tool), patty paper, compasses, straightedges, and Geometer's Sketchpad or similar geometry software, if possible

Warm-up: Allow students to compare homework with a partner

Opening: Discuss the homework as a means of setting the stage for the two problems to be completed in this lesson.

Worktime: Students should complete problems 3 and 4 of the task.

Closing: Allow students to share their work. After the closing, students should be aware of the following:

- When drawn from the vertex angle of an isosceles triangle, the perpendicular bisector, angle bisector, median, and altitude are all the same line segment. The segment divides the triangle into halves.
- As a special case of the statement above, when drawn from any one vertex of an equilateral triangle; the perpendicular bisector, angle bisector, median, and altitude are all the same line segment.

Homework:

Differentiated support/enrichment:

Resources/materials for Math Support: Students will continue to need opportunities to develop simple proofs.

Mathematics I
Middles and Halves
Day 2 Student Task

3. Suppose your triangle had been an isosceles triangle. Make and verify at least three predictions about how your four constructions would have been different.

4. Construct an equilateral triangle. Find the point that when connected to the vertices will divide it into 3 congruent isosceles triangles. Write a proof to justify your results.

Atlanta Public Schools

Teacher's Curriculum Supplement

Mathematics I: Unit 3

Task 6: Centers of Triangles

Mathematics I
Task 6: Centers of Triangles **Day 1/2**
(GaDOE TE #1- 3)

Standard(s): MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
g. Find and use points of concurrency in triangles: incenter, orthocenter, circumcenter, and centroid.

New vocabulary:

Mathematical concepts/topics: basic constructions, including angle bisectors, perpendicular bisector, medians, and altitudes; discovering points of concurrency

Prior knowledge: basic constructions, including angle bisectors, perpendicular lines, and perpendicular bisectors; definitions of angle bisectors, perpendicular bisectors, and altitudes

Essential question(s): How can I use geometric ideas to solve real-world problems?

Suggested materials: miras (or a similar reflective tool), patty paper, compasses, straightedges, and Geometer's Sketchpad or similar geometry software, if possible

Warm-up: Give students the task. Working alone, have them read the scenario and answer question 1.

Opening: Choose a student to read the scenario aloud. Discuss the situation and allow students to share their answers and thinking for problem 1.

Worktime: Students should complete problems 2 and 3 of the task. The goal is for students to discover that there is a point of intersection for all 4 sets of constructions and to begin to discover the significance of the points . Monitor students carefully to be sure that errors in measurement do not prevent them from determining points of intersection. Error in measurement will probably need to be discussed with the whole class as students get into the task. Geometer's Sketchpad is extremely powerful in helping students see that points of intersection will occur despite the type of triangle.

(Note: It is helpful for students to trace 4 copies of the triangle used in the task rather than trying to find all four points of concurrency on one triangle.)

Closing: Have students share their work. A demonstration using Sketchpad or other dynamic geometry software is recommended.

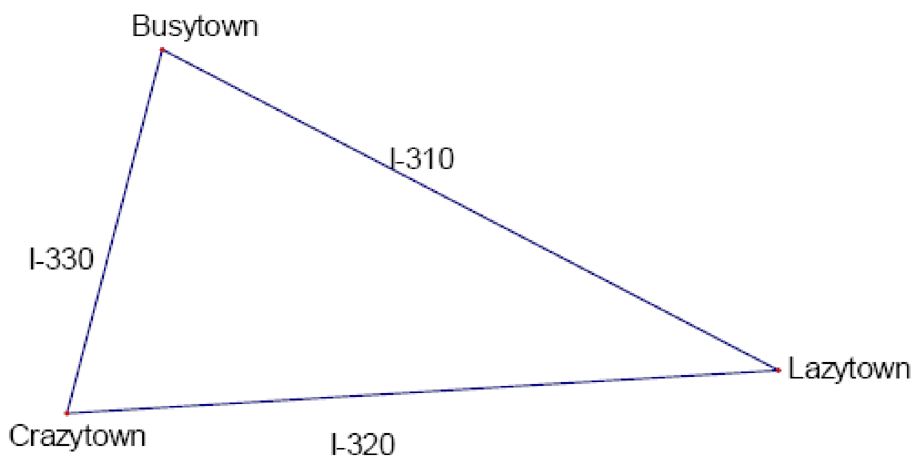
Homework:

Differentiated support/enrichment:

Resources/materials for Math Support: Allowing students to perform constructions using a variety of tools, including Geometer's Sketchpad or other software, is particularly effective in helping them discover properties and retain concepts. Remaining consistent with the same color assignments with the use of software will help many students to "see" and remember these ideas.

Mathematics I
Centers of Triangles
Day 1 Student Task

A developer plans to build an amusement park but wants to locate it within easy access of the three largest towns in the area as shown on the map below. The developer has to decide on the best location and is working with the ABC Construction Company to minimize costs wherever possible. No matter where the amusement park is located, roads will have to be built for direct access to the towns or to the existing highways.



1. Mark a point, P, on the map indicating what you think will be the best location for the amusement park. Explain why you chose this location.

2. Investigate other possible locations for the amusement park by constructing the following:
   a) all 3 **angle bisectors** of the triangle
   b) all 3 **perpendicular bisectors** of the triangle
   c) all 3 **altitudes** of the triangle
   d) all 3 **medians** of the triangle
   You have four different kinds of tools at your disposal – patty paper, MIRA, compass and straight edge, and dynamic geometry software. Use a different tool for each of your constructions.

3. What did you notice about each of the four sets of constructions? Would this occur regardless of the type of triangle? Why or why not?

4. Choose a location for the amusement park based on the work you did in part 2. Explain why you chose this point. How close is this point to your point P?

Mathematics I
Task 6: Centers of Triangles **Day 2/2**
(GaDOE TE #4 - 7)

Standard(s): MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.

g. Find and use points of concurrency in triangles: incenter, orthocenter, circumcenter, and centroid.

New vocabulary: incenter, orthocenter, circumcenter, and centroid

Mathematical concepts/topics: names, significance, and applications of points of concurrency

Prior knowledge: basic constructions, including angle bisectors, perpendicular lines, and perpendicular bisectors; definitions of angle bisectors, perpendicular bisectors, and altitudes

Essential question(s): How can I use geometric ideas to solve real-world problems?

Suggested materials: miras (or a similar reflective tool), patty paper, compasses, straightedges, and Geometer's Sketchpad or similar geometry software, if possible

Warm-up: Allow students to work with a partner to answer the follow question.

> *Consider the four points of intersection that you discovered in the previous lesson. Are there any special relationships that exist between each of the points and the triangle? If so, describe them.*

Opening: Discuss the warm-up. Both the names and significance of the points of concurrency may be discovered during this discussion. Require students to justify their conjectures by using geometric tools or by using properties they have learned related to the constructions.

Worktime: Much of this content may be discovered during the opening as students share the thinking that took place during the warm-up. Ideas should be formalized and put to paper during the worktime. Encourage students to be creative in their letters to the president of ABC Construction Company. Remind them that justifying their answer, the most important part of this assignment, will require that they be specific and detailed.

Closing: Have students share their letters.

Homework:

Differentiated support/enrichment:
Resources/materials for Math Support: Allowing students to perform constructions using a variety of tools, including Geometer's Sketchpad or other software, is particularly effective in helping them discover properties and retain concepts.

Mathematics I
Centers of Triangles
Day 2 Student Task

5. You have discovered that each set of segments resulting from the constructions above has a point of intersection. These four points of intersection are called the ***points of concurrency*** of a triangle.
Match the names of the ***points of concurrency*** with what you think is the appropriate point of intersection. Give reasons why you believe each match is correct.

A) Intersection of three altitudes
B) Intersection of three angle bisectors
C) Intersection of three medians
D) Intersection of three perpendicular bisectors

____ 1. Centroid
____ 2. Circumcenter
____ 3. Incenter
____ 4. Orthocenter

Use the table below to help you summarize what you have learned about points of concurrency.

Triangle Center:	***Point of Concurrency of:***	***Significance of:***

Which triangle center did you recommend for the location of the amusement park?

6. The president of ABC Construction Company is concerned about the cost of building roads from the three towns to the new amusement park. What recommendation would you give her? Write a formal letter to the president explaining why she should consider your recommendation. Be specific and include details.