# Atlanta Public Schools

# Teacher's Curriculum Supplement

# Mathematics I: Unit 1

*This document has been made possible by funding from the GE Foundation Developing Futures grant, in partnership with Atlanta Public Schools. It is derived from the Georgia Department of Education Math I Frameworks and includes contributions from Georgia teachers. It is intended to serve as a companion to the GA DOE Math I Frameworks Teacher Edition.* 

**Preface**

We are pleased to provide this supplement to the Georgia Department of Education's Mathematics I Framework. It has been written in the hope that it will assist teachers in the planning and delivery of the new curriculum, particularly in this first year of implementation. This document should be used with the following considerations.

- The importance of working the tasks used in these lessons cannot be overstated. In planning for the teaching of the Georgia Performance Standards in Mathematics teachers should work the task, read the teacher notes provided in the Georgia Department of Education's Mathematics I Framework Teacher Edition, and *then* examine the lessons provided here.

- This guide provides day-by-day lesson plans. While a detailed scope and sequence and established lessons may help in the implementation of a new and more rigorous curriculum, it is hoped that teachers will assess their students informally on an on-going basis and use the results of these assessments to determine (or modify) what happens in the classroom from one day to the next. Planning based on student need is much more effective than following a pre-determined timeline.

- It is important to remember that the Georgia Performance Standards provide a balance of concepts, skills, and problem solving. Although this document is primarily based on the tasks of the Framework, we have attempted to help teachers achieve this all important balance by embedding necessary skills in the lessons and including skills in specific or suggested homework assignments. The teachers and writers who developed these lessons, however, are not in your classrooms. It is incumbent upon the classroom teacher to assess the skill level of students on every topic addressed in the standards and provide the opportunities needed to master those skills.

- In most of the lesson templates, the sections labeled *Differentiated support/enrichment* have been left blank. This is a result of several factors, the most significant of which was time. It is also hoped that as teachers use these lessons, they will contribute their own ideas, not only in the areas of differentiation and enrichment, but in other areas as well. Materials and resources abound that can be used to contribute to the teaching of the standards.

On the topic of differentiation, it is critical to reiterate that many of the strategies used in a standards-based mathematics classroom promote differentiation. These strategies include

- the use of rich tasks with multiple points of entry and more than one path to a solution
- flexible grouping of students
- multiple representations of mathematical concepts
- writing in mathematics
- monitoring of progress through on-going informal and formative assessments
- and analysis of student work.

We hope that teachers will incorporate these strategies in each and every lesson.
It is hoped that you find this document useful as we strive to raise the mathematics achievement of all students in our district and state. Comments, questions, and suggestions for inclusions

related to the document may be emailed to Dr. Dottie Whitlow, Executive Director, Mathematics and Science Department, Atlanta Public Schools, dwhitlow@atlantapublicschools.us

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## Explanation of the Terms and Categories Used in the Lesson Template

**Task:** This section gives the suggested number of days needed to teach the concepts addressed in a task, the task name, and the problem numbers of the task as listed in the Georgia Department of Education's Mathematics I Framework Teacher Edition (GaDOE TE).

In some cases new tasks or activities have been developed. These activities have been named by the writers.

**Standard(s):** Although each task addresses many Math I standards and uses mathematics learned in earlier grades, in this section, only the key standards addressed in the lesson are listed.

**New Vocabulary:** Vocabulary is only listed here the first time it is used. It is strongly recommended that teachers, particularly those teaching Math Support, to use interactive word walls. Vocabulary listed in this section should be included on the word walls.

**Mathematical concepts/topics:** Here are listed the major concepts addressed in the lesson whether they are Math I concepts or were addressed in earlier grades.

**Prior knowledge:** Prior knowledge includes only those topics studied in previous grades. It does not include Math I content taught in previous lessons.

**Essential Question(s):** Essential questions may be daily and/or unit questions.

**Suggested materials:** In an attempt to list all materials that will be needed for the lesson, including consumable items, such as graph paper, and tools, such as graphing calculators and compasses. This list did not include those items that should always be present in a standards-based mathematics classroom such as markers, chart paper and rulers.

**Warm-up:** A suggested warm-up is included with every lesson. Warm-ups should be brief and should focus student thinking on the concepts that are to be addressed in the lesson.

**Opening:** Openings should set the stage for the mathematics to be done during the worktime. The amount of class time used for an opening will vary from day-to-day but this should not be the longest part of the lesson.

**Worktime:** The problem numbers have been listed and the work that students are to do during the worktime have been described. A student version of the day's activity follows the lesson template in every case. In order to address all of the standards in Math I, some of the problems in some of the tasks have been omitted and, in a few instances, substituted less time consuming activities for tasks. In many instances, in the student versions of the tasks, the writing of the original tasks has been simplified. In order to preserve all vocabulary, content, and meaning it is important that teachers work the original tasks as well as the student versions included here.

Teachers are expected to both facilitate and provide some direct instruction, when necessary, during the worktime. Some suggestions, related to student misconceptions, difficult concepts, and deeper meaning in this section have been included. However, the teacher notes in the GaDOE Math I Framework are exceptional. In most cases, there is no need to repeat the information provided there. Again, it is imperative that teachers work the tasks and read the teacher notes that are provided in GaDOE support materials.
Questioning is extremely important in every part of a standards-based lesson. We included suggestions for questions in some cases but did not focus on providing good questions as

extensively as we would have liked. Developing good questions related to a specific lesson should be a focus of collaborative planning time.

**Closing:** The closing may be the most important part of the lesson. This is where the mathematics is formalized. Even when a lesson must be continued to the next day, teachers should stop, leaving enough time to "close", summarizing and formalizing what students have done to that point. As much as possible students should assist in presenting the mathematics learned in the lesson. The teacher notes are all important in determining what mathematics should be included in the closing.

**Homework:** In some cases, additional written homework suggestions are provided or used the homework provided in the GaDOE sample lessons. We hope that you will use your resources, including your textbook, to assign homework related to the lesson that addresses the needs of your students.

Homework should be focused on the skills and concepts presented in class, relatively short (30 to 45 minutes), and include a balance of skills and thought-provoking problems.

**Differentiated support/enrichment:** On the topic of differentiation, it is critical to reiterate that many of the strategies used in a standards-based mathematics classroom promote differentiation. These strategies include

- the use of rich tasks with multiple points of entry and more than one path to a solution
- flexible grouping of students
- multiple representations of mathematical concepts
- writing in mathematics
- monitoring of progress through on-going informal and formative assessments
- and analysis of student work.

**Check for understanding:** A check for understanding is a short, focused assessment-a ticket out the door, for example. The Coach Book may be a good resource for these items.

**Resources/materials for Math Support:** Again, in some cases, we have provided materials and/or suggestions for Math Support. This section should be personalized to your students, class, and/or school, based on your resources.

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## Mathematics I
## Unit 1

## Unit 1 Timeline

| | |
|---|---|
| Task 1: Functions with Fiona | 2 days |
| Task 2: Fences and Functions | 2 days |
| Task 3: Wonderland to Functionland | 3 days |
| Task 4: Sequences as Functions | 2 days |
| Task 5: Walking, Falling, and Making Money | 2 days |
| Task 6: Southern Yard and Garden | 2 days |
| Task 7: The Six Basic Functions of Mathematics I | 1 day |

## Task Notes

**Task 1: Functions with Fiona**
Parts of the original GaDOE framework task have been re-written. Problems 1, 3, 4, 5 and 6 of the GaDOE TE are included. It is suggested that #7 be used in Math Support.

**Task 2: Fences and Functions**
All parts of the original task from GaDOE framework are included in the lesson plans. It is suggested that problem #6 may be assigned for homework but not skipped.

**Task 3: Wonderland to Functionland**
All problems in this task from GaDOE framework are included in the lesson plans. However, several of the problems have been rewritten or the wording has been changed in order to simplify or eliminate some of the reading in the task.

The extra problems included in the table of contents at the end of the task can be used anytime after Day 1. Number 3 of these problems foreshadows the biconditional statement and can be used as enrichment.

**Task 4: Sequences as Functions**
All parts of the original task from GADOE framework are included in the lesson plans. Problem 1 is suggested as a warm-up for the first day of the task.

**Task 5: Walking, Falling, and Making Money**
All parts of the original task from GaDOE framework are included in the lesson plans. Graphs for the warm-up on Day 1 are included immediately following the Day 1 Lesson Plan. Use of CBRs is highly recommended for this activity.

**Task 6: Southern Yard and Garden**
Problems 1, 2, 3, 6, and 7 of the original task from GaDOE framework are included in the lesson plans. We strongly suggest that problems 4 and 5 be used as enrichment if time permits.

**Task 7: The Six Basic Functions of Mathematics I** (substitution for Painted Cubes)
This activity was substituted, in the interest of time, for the task *Painted Cubes* GaDOE framework. Several of the transformations in *Painted Cubes* involve horizontal shifts, which are not tested in Math I.

# Atlanta Public Schools
# Teacher's Curriculum Supplement
# Mathematics I: Unit 1
# Task 1: Functions with Fiona

# Mathematics I
## Task 1: Exploring Functions with Fiona Day 1/2
(GaDOE TE #1, #3, and #4)

**Standard(s): MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.**
a. Represent functions using function notation.
e. Relate to a given context the characteristics of a function, and use graphs and tables to investigate its behavior.

**New Vocabulary:** function notation

**Mathematical concepts/topics:** independent and dependent variables; intuitive introduction to the characteristics of a function including continuous versus discrete domains, intervals of increase and decease, and rates of change; representing functions using tables and graphs; and function notation

**Prior knowledge:** independent and dependent variables, continuous versus discrete data, slope, representing functions using tables and graphs

**Essential Question(s):** What are the characteristics of a function? What is function notation? Why is it useful?

**Suggested materials:** graph paper

**Warm-up:** How big were you when you were born? Do you know how much you weighed? How long you were? If not, estimate your height and length at birth. What about at age 2? 5? 10? Now? Sketch a graph showing how you have grown since birth.

**Opening:** Discuss students' answers to the warm-up questions. This should be a fun activity and could be a good 'getting to know you' time with your students. Not many students will know their length at birth while many will know their weight. Discuss students' estimates about their heights at different ages. Have a yardstick handy to show the measurements students might be using. The goal of this opening is to hook students into the idea of recording growth over time.

**Worktime:** Students should do parts 1, 2, and 3 (GaDOE TE 1, 3, and 4) of the restructured task. Students will need to pay close attention to the scale they use for Julia's height. Counting by 3's or 4's will work with most graph paper. Encourage them to create a large graph so they can see the smaller differences in her height. Make sure that students can determine which variable is dependent and which is independent; and describe what function notation means in their own words. The following question has been added to question 1 in lieu of GaDOE #2:

*g. In earlier grades, you discussed relationships between two quantities that were functions and relationships that were not functions. Does the relationship between Julia's age and her height represent a function? Explain how you know.*

Students should have spent extensive time in the eighth grade discussing the difference between relationships that are functions and those that are not. You may need to spend time discussing the definition of a function and how you can tell quickly whether a graph does or does not represent a function. Have students explain their answers to *g* and give examples of relationships that are not functions.

**Closing:** Have students present their work, making sure that ALL questions are explored and answered. (See teacher notes.)

**Homework:** The homework is adapted from GaDOE and follows the student task.

**Differentiated support/enrichment:**

**Check for understanding:**

**Resources/materials for Math Support:** Make sure students can graph data and can label all the important parts of a graph. Have them pay close attention to scale. They may need to spend time discussing the definition of a function, and giving examples of relationships that are functions and relationships that are not functions.

# Mathematics I
# Exploring Functions with Fiona
### Day 1 Student Task

1. While visiting her grandmother, Fiona Evans found markings on the inside of a closet door showing the heights of her mother, Julia, and her mother's brothers and sisters on their birthdays growing up. From the markings in the closet, Fiona wrote down her mother's height each year from ages 2 to 16. Her grandmother found the measurements at birth and one year by looking in her mother's baby book. The data is provided in the table below, with heights rounded to the nearest inch.

| Age (yrs.) | $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Height (in.) | $y$ | 21 | 30 | 35 | 39 | 43 | 46 | 48 | 51 | 53 | 55 | 59 | 62 | 64 | 65 | 65 | 66 | 66 |

   a. Which variable is the independent variable, and which is the dependent variable? Explain your choice.
   b. Make a graph of the data.
   c. Should you connect the dots on your graph? Explain.
   d. Describe how Julia's height changed as she grew up.
   e. How tall was Julia on her 11th birthday? Explain how you can see this in both the graph and the table.
   f. What do you think happened to Julia's height after age 16? Explain. How could you show this on your graph?
   g. In earlier grades, you discussed relationships between two quantities that were functions and relationships that were not functions. Does the relationship between Julia's age and her height represent a function? Explain how you know.

2. In Mathematics I and all advanced mathematics, *function notation* is used as an efficient way to describe relationships between quantities that vary in a functional relationship. In order to discuss function notation, we first need to give our function a mathematical name. Since the outputs of the function above are Julia's heights, we will name the function $J$ (for Julia's height). In Mathematics I and following courses, we'll use one letter names for functions as a way to refer to the whole relationship between inputs and outputs. So, for this example, we mean that $J$ consists of all the input-output pairs of Julia's age and corresponding height in inches. We can say that the graph you drew in part b above is *the graph of the function J* because the graph shows all of these input-output pairs.

Consider the notation $J(2)$. We note that function notation gives us another way to write about ideas that you began learning in middle school, as shown in the table below.

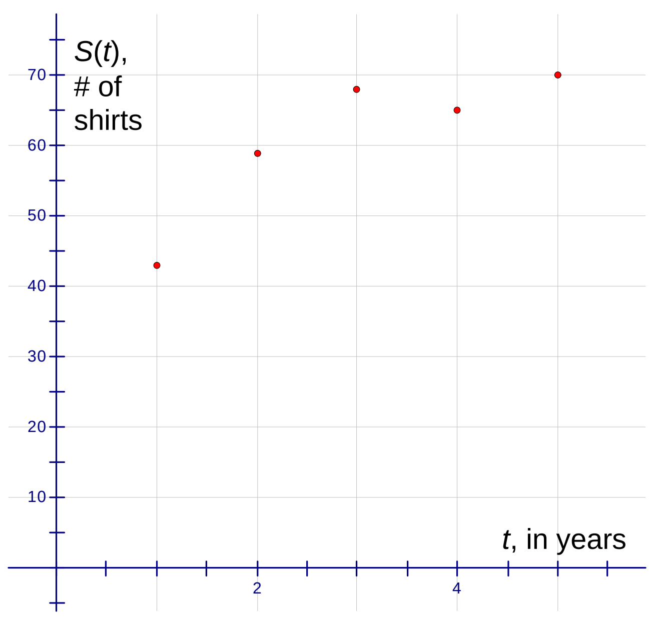
| *Statement* | *Type* |
|---|---|
| At age 2, Julia was 35 inches tall. | Natural language |
| When $x$ is 2, $y$ is 35. | Statement about variables |
| When the input is 2, the output is 35. | Input-output statement |
| $J(2) = 35$. | Function notation |

The notation $J(x)$ is typically read "$J$ of $x$," but thinking "$J$ at $x$" is also useful since $J(2)$ can be interpreted as "height at age 2," for example.

Note: Function notation looks like a multiplication calculation, but the meaning is very different. To avoid misinterpretation, be sure you know which letters represent functions. For example, if $g$ represents a function, then $g(4)$ is ***not*** multiplication of g and 4 but is rather the value of "$g$ at 4," that is, the output value of the function $g$ when the input is value is 4.

a. What is $J(11)$? What does this mean?
b. When $x$ is 3, what is $y$? Express this fact using function notation.
c. Find an $x$ so that $J(x) = 53$. Explain your method. What does your answer mean?
d. From your graph or your table, estimate $J(6.5)$. Explain your method. What does your answer mean?
e. Estimate a value for $x$ so that $J(x)$ is approximately 60. Explain your method. What does your answer mean?
f. Describe what happens to $J(x)$ as $x$ increases from 0 to 16.
g. What can you say about $J(x)$ for $x$ greater than 16?
h. Describe the similarities and differences you see between these questions and the questions in #1.

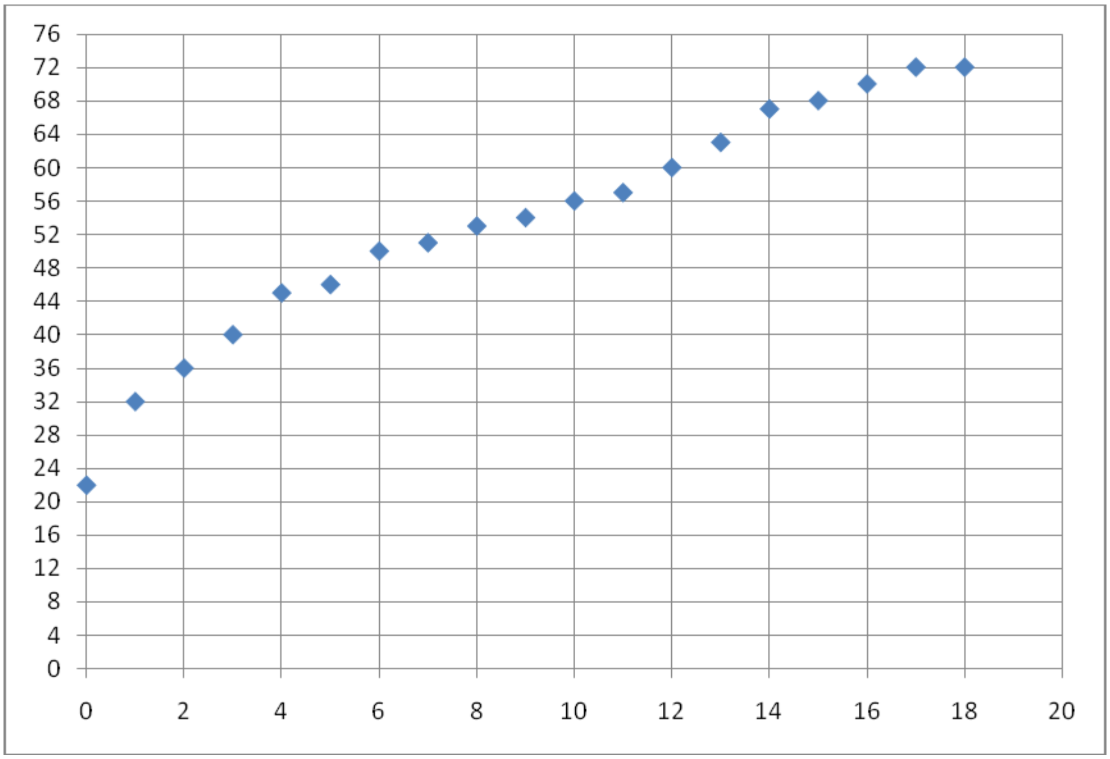
3. Fiona attends Peachtree Plains High School. When the school opened five years ago, a few teachers and students put on Fall Fest, featuring contests, games, prizes, and performances by student bands. To raise money for the event, they sold Fall Fest T-shirts. The event was very well received, and so Fall Fest has become a tradition. This year Fiona is one of the students helping with Fall Fest and is in charge of T-shirt sales. She gathered information about the growth of T-shirt sales for the Fall Fests so far and created the graph below that shows the function $S$.



a. What are the independent and dependent variables shown in the graph?
b. For which years does the graph provide data?
c. Does it make sense to connect the dots in the graph? Explain.
d. What were the T-shirt sales in the first year? Use function notation to express your result.
e. Find $S(3)$, if possible, and explain what it means or would mean.
f. Find $S(6)$, if possible, and explain what it means or would mean.
g. Find $S(2.4)$, if possible, and explain what it means or would mean.
h. If possible, find a $t$ such that $S(t) = 65$. Explain.
i. If possible, find a $t$ such that $S(t) = 62$. Explain.
j. Describe what happens to $S(t)$ as $t$ increases, beginning at $t = 1$.
k. What can you say about $S(t)$ for values of $t$ greater than 6?

# Mathematics I
# Functions with Fiona
### Day 1 Homework

Fiona has an uncle named Joe. His heights were also marked on the door and are shown in the graph below.




1. Use the graph to complete the following chart.

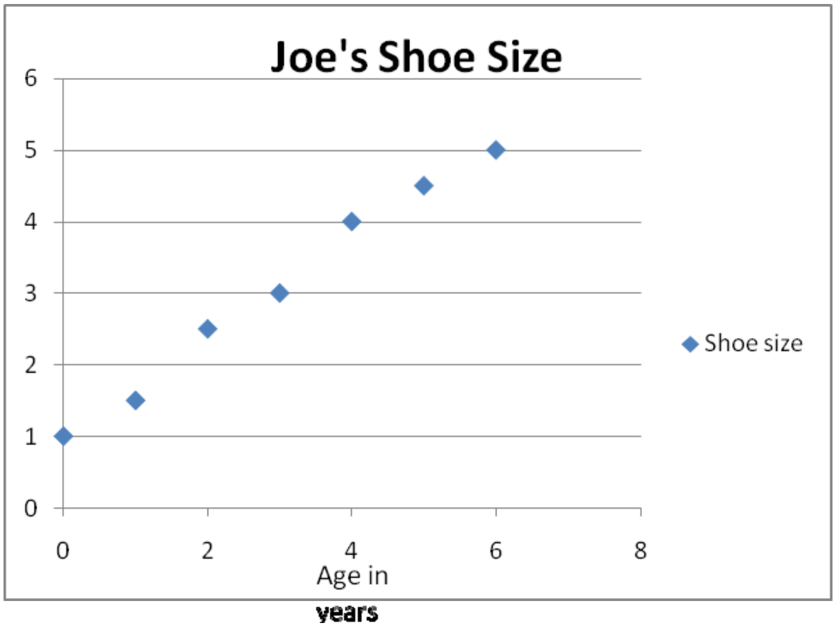
| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Age (yrs.) | *x* | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| Height (in.) | *y* | 22.5 | 32 | | 40 | | | 50 | | | 54 | 56 | | 60 | 63 | | 68 | 70 | | |

2. Should the dots in the graph be connected? Explain your thinking.

Let h(x) represent Joe's height at age x.

3. What does h(5) = y mean?
4. What is h(4)?
5. Estimate the value of h(0.5). Explain in words what h(0.5) means.
6. Can you use the chart or graph to find h(7)? Explain your answer.

Use the graph below to answer the following questions.




7. What information does this graph give you?
8. What is the dependent and independent variable?
9. At what age did Joe wear a size 2 ½ ?
10. If j(x) is Joe's shoe size at *x* years of age, what is j(6)?
11. What is *x* if j(x) = 4.5?

# Mathematics I

## Task 1: Exploring Functions with Fiona Day 2/2

(GaDOE TE #5 and #6)

**Standard(s): MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.**
a. Represent functions using function notation.
e. Relate to a given context the characteristics of a function, and use graphs and tables to investigate its behavior.

**New Vocabulary:** function notation

**Mathematical concepts/topics:** independent and dependent variables; intuitive introduction to the characteristics of a function including continuous versus discrete domains, intervals of increase and decease, and rates of change; translating between verbal, tabular, graphic, and algebraic representations of functions; and function notation

**Prior knowledge:** independent and dependent variables; continuous versus discrete data; slope; and translating between verbal, tabular, graphic, and algebraic representations of functions

**Essential Question(s):** How can we describe a function with an equation?

**Suggested materials:** graph paper

**Warm-up:** Have students compare responses to their homework with a partner. Ask them to be prepared to discuss any differences or questions they have.

**Opening:** Use the homework from day 1 as an opening to this continuation of the task. Talk to students about the fact that the functions they worked with yesterday were represented by tables and graphs but that functions can also be represented using formulas. The functions they will work with today will be represented by formulas that they will develop or that they will be given. Have a student read the first problem to be investigated during worktime (GaDOE TE #5). Ask guiding questions to be sure that they understand the situation.

**Worktime:** Students should do parts 4 and 5 (GaDOE TE 5 and 6) of the restructured task. In problem 5, make sure students understand they are measuring the distance the ball has dropped, not the distance the ball is from the ground. Check students' sketches to make sure they understand the question. The sketch should show more distance between the points as time goes by.

**Closing:** Have students present their work, making sure that ALL questions are explored and answered. (See teacher notes.) Connecting the points in the graph of the T-shirt problem is a bit tricky. Be sure to discuss this thoroughly with students. As you discuss the solutions for problem 5, be sure that students recognize the relationship between the picture and the graph of the ball drop. The idea that the actual path of the ball and the graph of time vs. distance are different is important for preparation for future mathematics. Students need to begin to understand that the graph does not always represent the actual path traveled.

**Homework:** The homework, which follows the student task, is adapted from GaDOE TE.

**Differentiated support/enrichment:**

**Check for understanding:**

**Resources/materials for Math Support:** GaDOE TE #7. This part of the task was not included in the lessons for the Math I class and would be an excellent problem for Math Support. The problem involves a simple linear function but the questions asked give students an opportunity to work with different kinds of rational numbers, which students in Support are likely to need.

# Mathematics I
# Exploring Functions with Fiona
### Day 2 Student Task

As you saw in the last lesson, functions can be described by tables and by graphs. In high school mathematics, functions are often given by formulas. In the remaining items for this task, we develop, or are given, a formula for each function under consideration, but it is important to remember that not all functions can be described by formulas.

4. Fiona's T-shirt committee decided on a long sleeve T-shirt in royal blue, one of the school colors, with a FallFest logo designed by the art teacher. The committee needs to decide how many T-shirts to order. Fiona was given the job of collecting price information so she checked with several suppliers, both local companies and some on the Web. She found the best price at Peachtree Plains Promotions, a local company owned by parents of a Peachtree Plains High School senior.

The salesperson for Peachtree Plains Promotions told Fiona that there would be a $50 fee for setting up the imprint design and different charges per shirt depending on the total number of shirts ordered. For an order of 50 to 250 T-shirts, the cost is $9 per shirt. Based on sales from the previous five years, Fiona is sure that they will order at least 50 T-shirts and will not order more than 250. If $x$ is the number of T-shirts to be ordered for this year's FallFest, and $y$ is the total cost in dollars of these shirts, then $y$ is a function of $x$. Let's name this function C, for cost function. Fiona started the table below.

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 50 | 100 | 150 | 200 | 250 |
| $9x$ | 450 | 900 | 1350 | | |
| $y = C(x)$ | 500 | 950 | | | |

a. Fill in the missing values in the table above.
b. Make a graph to show how the cost depends upon the number of T-shirts ordered. You can start by plotting the points corresponding to values in the table. What points are these? Should you connect these points? Explain. Should you extend the graph beyond the first or last point? Explain.
c. Write a formula showing how the cost depends upon the number of T-shirts ordered. For what number of T-shirts does your formula apply? Explain.
d. What does C(70) mean? What is the value of C(70)? Did you use the table, the graph, or the formula?
e. If the T-shirt committee decides to order only the 67 T-shirts that are pre-paid, how much will it cost? Show how you know. Express the result using function notation.
f. If the T-shirt committee decides to order the 67 T-shirts that are pre-paid plus 15 more, how much will it cost? Show how you know. Express the result using function notation.

5. Fiona is taking physics. Her sister, Hannah, is taking physical science. Fiona decided to use functions to help Hannah understand one basic idea related to gravity and falling objects. Fiona explained that, if a ball is dropped from a high place, such as the Tower of Pisa in Italy, then there is a formula for calculating the distance the ball has fallen. If $y$, measured in

meters, is the distance the ball has fallen and $x$, measured in seconds, is the time since the ball was dropped, then $y$ is a function of $x$, and the relationship can be approximated by the formula $y = d(x) = 5x^2$. Here we name the function $d$ because the outputs are distances.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | … |
| $x^2$ | 0 | 1 | 4 | 9 | | | | … |
| $y = d(x) = 5x^2$ | 0 | 5 | 20 | | | | | … |

a. Fill in the missing values in the table above.
b. Suppose the ball is dropped from a building at least 100 meters high. Measuring from the top of the building, draw a picture indicating the position of the ball at times indicated in your table of values.
c. Draw a graph of $x$ versus $y$ for this situation. Should you connect the dots? Explain.
d. What is the relationship between the picture (part b) and the graph (part c)?
e. You know from experience that the speed of the ball increases as it falls. How can you "see" the increasing speed in your table? How can you "see" the increasing speed in your picture?
f. What is $d(4)$? What does this mean?
g. Estimate $x$ such that $d(x) = 50$. Explain your method. What does it mean?
h. In this context, $y$ is proportional to $x^2$. Explain what that means. How can you see this in the table?

# Mathematics I
# Functions with Fiona
### Day 2 Homework

1. If $f(x) = 3x^2 - 6x - 5$, find:
   a. f(2)
   b. f(-3)
   c. f(0)
2. If $g(x) = 2x + 5(3x – 2)$, find:
   a. g(1)
   b. g(0)
   c. g(-3)
   d. g(x) = 6, x =
   **e.** g(x) = -2, x =

# Atlanta Public Schools

# Teacher's Curriculum Supplement

# Mathematics I: Unit 1

# Task 2: Fences and Functions

# Mathematics I

## Task 2: Fences and Functions Day 1/2

(GaDOE TE #1, #2, and #3)

**Standard(s): MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.**
d. Investigate and explain the characteristics of a function: domain, range, zeros, intercepts, intervals of increase and decrease, maximum and minimum values, and end behavior.
e. Relate to a given context the characteristics of a function, and use graphs and tables to investigate its behavior.

**New Vocabulary:** domain, range, maximum value, minimum value

**Mathematical concepts/topics:** area, function notation, domain, range, maximum value, minimum value, increasing, decreasing, rate of change, symmetry

**Prior knowledge:** linear functions, perimeter, domain, constant rate of change

**Essential Question(s):** How can a situation be modeled using tables, graphs, and formulas?

**Suggested materials:** graph paper

**Warm-up:**
Sketch and label a rectangle that meets the following conditions:

- the perimeter is between 20 inches and 100 inches;
- the perimeter is divisible by 8; and
- the length is more than twice the width.

**Opening:** Have students discuss the warm-up, sharing multiple solutions. Ask how many solutions they think there might be. Read the introduction to the problem for Day 1 (GaDOE #1). Ask students what they think are the important facts in the problem. Make sure they understand the situation, including the facts that Claire is using all 30 panels to create her garden and that she cannot cut a panel. Ask students to draw and label a panel to be sure that they understand the concept.

**Worktime:** Students should do each part of problem 1, answering all questions completely. The knowledge gained or reviewed in this part of the task, will lay the foundation for the mathematics to follow. Notes:

- Parts d and e are laying the foundation for restricted domains and limiting cases. Discuss them thoroughly during the closing.
- In part f, the formula relating the y dimension of the garden to the x dimension, should be a result of what the student is seeing happen in their table, not necessarily a result of the fact that they have memorized the formula for finding the perimeter of a rectangle.

When students have had an appropriate amount of time to finish problem 1, have a whole group discussion about the concepts in the problem based on student understanding at that point. Tell them that in the next problem they will consider the area of the garden. Ask what they know

about area. Discuss the concept of area, including units of measure. Compare and contrast the areas of plane figures with the surface areas of solids.
Questions you might ask:

- What is area?
- Give me a real example of a figure for which we might approximate the area.
- What units might we use to measure (student's example)?
- What kind of shape is that? Can you approximate the area? How might you calculate the area?
- Give me another real-life example that is a different shape. How might you find the area?
- Choose a solid figure in the classroom. Does it have area?

Have students do parts 2 and 3 of the task. Be sure that they have graph paper and colored tiles.
Possible areas of weakness, misconceptions, and important points:

- Students will need to refer to their work from part 1. Writing a formula to represent the area of the garden in terms of x will be difficult for some students.
- It is counterintuitive for many that the area can change even though the perimeter remains constant. This is why drawing the "gardens" on grid paper and calculating the area is so important.
- This is most students' first encounter with the graph of a quadratic function. Be sure that students answer and understand all the questions that are asked.

**Closing:** Have students present their work, making sure that ALL questions are explored and answered. (See teacher notes.)

**Homework:** Day 1 homework reinforces the ideas presented in class by asking the same or similar questions based on a different number (36) of panels.

**Differentiated support/enrichment:**

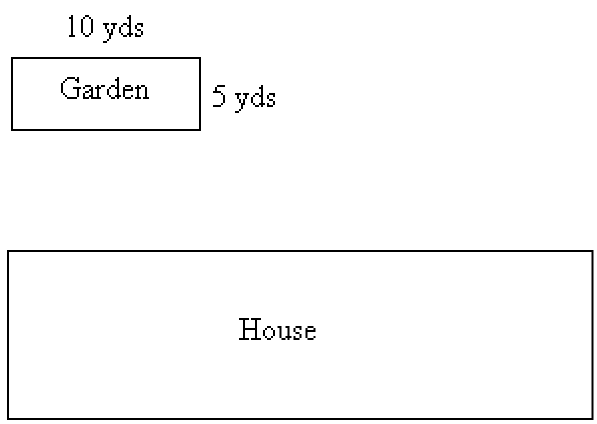
**Check for understanding:**

**Resources/materials for Math Support:**

# Mathematics I
# Fences and Functions
### Day 1 Student Task

Claire decided to plant a rectangular garden in her back yard using 30 pieces of fencing that were given to her by a friend. Each piece of fencing was a vinyl panel 1 yard wide and 6 feet high. Claire wanted to determine the possible dimensions of her garden, assuming that she would use all of the fencing and did not cut any of the panels. She began by placing ten panels (10 yards) parallel to the back side of her house and then calculated that the other dimension of her garden then would be 5 yards, as shown in the diagram below.

1. Claire looked at the 10 fencing panels lying on the ground and decided that she wanted to consider other possibilities for the dimensions of the garden. In order to organize her thoughts, she let $x$ be the garden dimension parallel to the back of her house, measured in yards, and let $y$ be the other dimension, perpendicular to the back of the house, measured in yards. She recorded the first possibility for the dimensions of the garden as follows: When $x = 10$, $y = 5$.




a. Explain why $y$ must be 5 when $x$ is 10.
b. Make a table showing the possibilities for $x$ and $y$.
c. Find the perimeter of each of the possible gardens you listed in part b. What do you notice? Explain why this happens.
d. Did you consider $x = 15$ in part b? If $x = 15$, what must y be? What would Claire do with the fencing if she chose $x = 15$?
e. Can $x$ be 16? What is the maximum possible value for $x$? Explain.
f. Write a formula relating the $y$-dimension of the garden to the $x$-dimension.
g. Make a graph of the possible dimensions of Claire's garden.
h. What would it mean to connect the dots on your graph? Does connecting the dots make sense for this context? Explain.
i. As the $x$-dimension of the garden increases by 1 yard, what happens to the $y$-dimension? Does it matter what $x$-value you start with? How do you see this in the graph? In the table? In your formula? What makes the dimensions change together in this way?

2. After listing the possible rectangular dimensions of the garden, Claire realized that she needed to pay attention to the area of the garden, because area determines how many plants can be grown.

a. Does the area of the garden change as the $x$-dimension changes? Make a prediction, and explain your thinking.
b. Use grid paper to make accurate sketches for at least three possible gardens. How is the area of each garden represented on the grid paper?
c. Make a table listing all the possible $x$-dimensions for the garden and the corresponding areas. (To facilitate your calculations, you might want to include the $y$-dimensions in your table or add an area column to your previous table.)
d. Make a graph showing the relationship between the $x$-dimension and the area of the garden. Should you connect the dots? Explain.
e. Write a formula showing how to compute the area of the garden, given its $x$-dimension.

3. Because the area of Claire's garden depends upon the $x$-dimension, we can say that the area is a function of the $x$-dimension. Let's use $G$ for the name of the function that uses each $x$-dimension an input value and gives the resulting garden area as the corresponding output value.

a. Use function notation to write the formula for the garden area function. What does $G(11)$ mean? What is the value of $G(11)$? What line of your table, from #2, part c, and what point on your graph, from #2, part d, illustrate this same information?
b. The set of all possible input values for a function is called the ***domain*** of the function. What is the domain of the garden area function $G$? How is the question about domain related to the question about connecting the dots on the graph you drew for #2, part d?
c. The set of all possible output values is called the ***range*** of the function. What is the range of the garden area function $G$? How can you see the range in your table? In your graph?
d. As the $x$-dimension of the garden increases by 1 yard, what happens to the garden area? Does it matter what $x$-dimension you start with? How do you see this in the graph? In the table? Explain what you notice.
e. What is the maximum value for the garden area, and what are the dimensions when the garden has this area? How do you see this in your table? In your graph?
f. What is the minimum value for the garden area, and what are the dimensions when the garden has this area? How do you see this in your table? In your graph?
g. In deciding how to lay out her garden, Claire made a table and graph similar to those you have made in this investigation. Her neighbor Javier noticed that her graph had symmetry. Your graph should also have symmetry. Describe this symmetry by indicating the line of symmetry. What about the context of the garden situation causes this symmetry?
h. After making her table and graph, Claire made a decision, put up the fence, and planted her garden. If it had been your garden, what dimensions would you have used and why?

# Mathematics I
# Fences and Functions
### Day 1 Homework

Bob had 36 pieces of fencing with which to enclose his rectangular garden. Like Claire, he cannot cut any of the panels.
.

1. Use function notation to write the formula for the area of the garden in terms of x.
2. Graph your function.
3. Did you connect the dots on your graph? Why or why not?
4. What is the domain of this function? What is the range?
5. If Bob wanted to build the garden with the largest possible area, what dimensions would he use? How do you know?

# Mathematics I

## Task 2: Fences and Functions Day 2/2

(GaDOE #4, #5, and #6)

**Standard(s): MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.**
d. Investigate and explain the characteristics of a function: domain, range, zeros, intercepts, intervals of increase and decrease, maximum and minimum values, and end behavior.
e. Relate to a given context the characteristics of a function, and use graphs and tables to investigate its behavior.

**New Vocabulary:** limiting case; open and closed circles to represent points that are not included and points that are included, respectively; degenerate rectangle; intervals of increase and decrease; equal functions

**Mathematical concepts/topics:** area, function notation, domain, range, maximum value, minimum value, increasing, decreasing, rate of change, symmetry, limiting case; open and closed circles to represent points that are not included and points that are included, respectively; intervals of increase and decrease; equal functions

**Prior knowledge:** area

**Essential Question(s):** How do we represent a situation using a graph?

**Suggested materials:** graph paper

**Warm-up:**
Have the following posted as students enter the room.

- Compare last night's homework with your partner.
- Discuss any differences in your work.
- Be prepared to discuss differences and/or ask any questions you have related to your work.

**Opening:** Allow one pair of students to present their solutions to the previous night's homework and address any questions. Have students read the first paragraph of problem 4 of the task silently and then ask a student to read the paragraph aloud. Ask students how they think Kenya's situation is like Claire's. Ask how it is different. Hopefully some students will discuss the fact that because Kenya is using chain-link fence rather than panels, her dimensions can include parts of a yard.

**Worktime:** Students should do problems #4 and #5 of the task during worktime. Problem #6 may be assigned for homework but should not be skipped.

Refrain from giving students too much information up-front. Much of this task is discovery. For students who struggle with the concepts of continuous versus discrete values, having manipulatives or visuals to illustrate the differences between the two kinds of materials used to enclose the garden and the pen would be useful-for example, toothpicks for the panels and string for the chain-link fence.

There is a great deal of important mathematics in these parts of the task. Be sure to pay close attention to the details of the teacher notes. New ideas include continuous domains and ranges; limiting cases; using open and closed circles to represent points that are not included and points that are included on a graph; and equal functions.

**Closing:** Have students present their work, making sure that ALL questions are explored and answered. (See teacher notes.)

**Homework:** Problem #6 of the task may be assigned for homework but should be discussed in class the following day. Make sure that students understand the concept of equal functions.

**Differentiated support/enrichment:**

**Check for understanding:**

**Resources/materials for Math Support:**

# Mathematics I
# Fences and Functions
### Day 2 Student Task

4. Later that summer, Claire's sister-in-law Kenya mentioned that she wanted to use 30 yards of chain-link fence to build a rectangular pen for her two pet pot bellied pigs. Claire experienced déjà vu and shared how she had analyzed how to fence her garden. As Claire explained her analysis, Kenya realized that her fencing problem was very similar to Claire's.
   a. Does the formula that relates the $y$-dimension of Claire's garden to the $x$-dimension of her garden apply to the pen for Kenya's pet pigs? Why or why not?
   b. Does the formula that relates the area of Claire's garden to its $x$-dimension apply to the pen for Kenya's pet pigs? Why or why not?
   c. Write a formula showing how to compute the area of the pen for Kenya's pet pigs given the $x$-dimension for the pen.
   d. Does the $x$-dimension of the pen for Kenya's pet pigs have to be a whole number? Explain.
   e. Let $P$ be the function which uses the $x$-dimension of the pen for Kenya's pet pigs as input and gives the area of the pen as output. Write a formula for $P(x)$.
   f. Make a table of input and output values for the function $P$. Include some values of the $x$-dimension for the pen that that could not be used as the $x$-dimension for Claire's garden.
   g. Make a graph of the function $P$.
   h. Does your graph show any points with x-value less than 1? Could Kenya have made a pen with ½ yard as the $x$-dimension? If so, what would the other dimension be? How about a pen with an $x$-dimension 0.1 of a yard? How big is a pot bellied pig? Would a pot bellied pig fit into either of these pens?
   i. Is it mathematically possible to have a rectangle with $x$-dimension equal to ½? How about a rectangle with $x$-dimension 0.1?
   j. Of course, Kenya would not build a pen for her pigs that did not give enough room for the pigs to turn around or pass by each other. However, in analyzing the function $P$ to decide how to build the pen, Kenya found it useful to consider all the input values that could be the $x$-dimension of a rectangle. She knew that it didn't make sense to consider a negative number as the $x$-dimension for the pen for her pigs, but she asked herself if she could interpret an $x$-dimension of 0 in any meaningful way. She thought about the formula relating the $y$-dimension to the $x$-dimension and decided to include $x = 0$. What layout of fencing would correspond to the value $x = 0$? What area would be included inside the fence? Why could the shape created by this fencing layout be called a "degenerate rectangle"?
   k. The value $x = 0$ is called a ***limiting case***. Is there any other limiting case to consider in thinking about values for the $x$-dimension of the pen for Kenya's pigs? Explain.
   l. Return to your graph of the function $P$. Adjust your graph to include all the values that could mathematically be the $x$-dimension of the rectangular pen even though some of these are not reasonable for fencing an area for pot bellied pigs.
   (<u>Note</u>: You can plot points corresponding to any limiting cases for the function using a small circle. To *include* the limiting case, *fill in the circle* to make a solid dot ●. To *not*

*include* the limiting case, but just use it to show that the graph does not continue beyond that point, *leave the circle open* **o**.)

5. Use your table, graph, and formula for the function *P* to answer questions about the pen for Kenya's pet pigs.
   a. Estimate the area of a pen of with an *x*-dimension of 10 feet (not yards). Explain your reasoning.
   b. Estimate the *x*-dimension of a rectangle with an area of 30 square yards. Explain your reasoning.
   c. What is the domain of the function *P*? How do you see the domain in the graph?
   d. What point on the graph corresponds to the pen with maximum area?
   e. What is the maximum area that the pen for Kenya's pigs could have? Explain. What do you notice about the shape of the pen?
   f. What is the range of the function *P*? How can you see the range in the graph?
   g. How should you move your finger along the *x*-axis (right-to-left or left-to-right) so that the *x*-value increases as your move your finger? If you move your finger along the graph of the function *P* in this same direction, do the *x*-values of the points also increase? As the *x*-dimension of the pen for Kenya's pigs increases, sometimes the area increases and sometimes the area decreases. Using your graph, determine the *x*-values such that the area increase as *x* increases. For what *x*-values does the area decrease as *x* increases?

When using tables and formulas, we often look at a function by examining only one or two points at a time, but in high school mathematics, it is important to begin to think about "the whole function," that is, all of the input-output pairs. We've started working on this idea already by using a single letter such as *f*, *G*, or *P* to refer to the whole collection of input-output pairs. We'll go further as we proceed with this investigation.

We say that **two functions are equal** (as whole functions) if they have exactly the same input-output pairs. In other words, two functions are equal if they have the *same domain* **and** the *output values are the same for each input value in the domain*. From a graphical perspective, two functions are equal if their graphs have exactly the same points. Note that the graph of a function consists of all the points which correspond to input-output pairs, but when we draw a graph we often can show only some of the points and indicate the rest. For example, if the graph of the function is a line, we show part of the line and use arrowheads to indicate that the line continues without end.

6. The possibilities for the pen for Kenya's pet pigs and for Claire's garden are very similar in some respects but different in others. These two situations involve different *functions*, even though the *formulas* are the same.
   a. If Kenya makes the pen with maximum area, how much more area will the pen for her pet pigs have than Claire's garden of maximum area? How much area is that in square feet?
   b. What could Claire have done to build her garden with the same area as the maximum area for Kenya's pen? Do you think this would have been worthwhile?
   c. Consider the situations that led to the functions *G* and *P* and review your tables, graphs, formulas related to the two functions. Describe the similarities and differences between

# Atlanta Public Schools

# Teacher's Curriculum Supplement

# Mathematics I: Unit 1

# Task 3: From Wonderland to Functionland

## Mathematics I

## Task 3: From Wonderland to Functionland Day 1/3

(GaDOE TE # 1-11)

**Standard(s): MM1G2. Students will understand and use the language of mathematical argument and justification.**
b. Understand and use the relationships among a statement and its converse, inverse, and contrapositive.
**MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.**
a. Represent functions using function notation.
b. Graph the basic functions $f(x) = x^n$, where $n = 1$ to 3, $f(x) = \sqrt{x}$, $f(x) = |x|$, and $f(x) = 1/x$.
d. Investigate and explain the characteristics of a function: domain, range, zeros, intercepts, intervals of increase and decrease, maximum and minimum values, and end behavior.

**New Vocabulary:** statement, compound statement (compound proposition), conditional statement, hypothesis, conclusion, converse of a conditional statement

**Mathematical concepts/topics:** propositional logic, absolute value function, domain, range, intervals of increase and decrease, function notation

**Prior knowledge:** statements using "and, "or", and "not" in the contexts of set theory and probability, operating with absolute value

**Essential Question(s):** What is logic? How can logic be used to help determine the truth values of mathematical statements?

**Suggested materials:** Youtube video, Alice in Wonderland audio dialog on-line, graph paper

**Warm-up:** Have the following posted on the board:

*What is a sentence? Give a definition that would please your English teacher!*

**Opening:** Show a short clip of the "Mad Tea Party" from "Alice in Wonderland". (There are several short versions on youtube. These versions do not include the passage in this task but they will set the stage for the audio version of the passage.) Play the audio discussion. ( "Alice in Wonderland" audio discussion (http://wiredforbooks.org/alice/chapter7.htm, chapter 7, 1:58 into clip) or read the passage from the beginning of the task. Begin discussion of the passage by asking questions like: Why is Alice being admonished? Is Alice right? Is the Hatter right? Why?

Ask students to read the section of the restructured task on basic definitions. When they have finished, ask them to share their definitions of a sentence. A sentence is defined by Webster's dictionary to be "a conventional unit of speech or writing, usually containing a subject or a predicate, beginning with a capital letter and ending with an end mark (a period, question mark, exclamation point, …)". After agreeing on a definition or the components of a sentence, have students describe a statement in their own words. Have them give examples and non-examples. Ask them whether their statements are true or false. Ask them what a conditional statement is and have them give examples.

**Worktime:** Students should do problems #1- #11 of the restructured task. Because the topic is logic, this task is naturally very wordy. This may be a problem for some students. We strongly suggest that students work in pairs, rather than groups, to complete the task. In the restructured task, the problems for this lesson are the same as those in the GaDOE TE but we have omitted or simplified some of the reading. After problem #3, have a group discussion about problems #1 - #3, including the use of the words "it" and "thing" to make the meanings of the statements more clear. Many students will insist that the phrase that comes first in a sentence is the hypothesis. This misconception needs to be corrected.

Since this is the first time that students have seen the absolute value function, the function should be graphed by hand and all questions related to the function should be answered.

**Closing:** Have students present their work, making sure that ALL questions are explored and answered. The teacher notes for this task are particularly important in helping teachers know how much logic should be taught in this unit and what should not be taught.

**Homework:** Have students write three of their own conditional statements. At least one of their statements should involve mathematics. Ask them to give the hypothesis and conclusion of each original statement and then write its converse.

**Differentiated support/enrichment:**

**Check for understanding:**

**Resources/materials for Math Support:** Previewing the vocabulary used in this task is very important. We strongly suggest that teachers use interactive word walls in Support classes. Students may need practice identifying the hypotheses and conclusions of conditional statements and writing converses.

# Mathematics I
# From Wonderland to Functionland
### Day 1 Student Task

Consider the following passage from Lewis Carroll's *Alice's Adventures in Wonderland*, Chapter VII, "A Mad Tea Party."

> "Then you should say what you mean." the March Hare went on.
>
> "I do," Alice hastily replied; "at least -- at least I mean what I say -- that's the same thing, you know."
>
> "Not the same thing a bit!" said the Hatter, "Why, you might just as well say that 'I see what I eat' is the same thing as 'I eat what I see'!"
>
> "You might just as well say," added the March Hare, "that 'I like what I get' is the same thing as 'I get what I like'!"
>
> "You might just as well say," added the Dormouse, who seemed to be talking in his sleep, "that 'I breathe when I sleep' is the same thing as 'I sleep when I breathe'!"
>
> "It is the same thing with you," said the Hatter, and here the conversation dropped, and the party sat silent for a minute.

Lewis Carroll, the author of *Alice in Wonderland* and *Through the Looking Glass*, was a mathematics teacher who had fun playing around with logic. In this activity, you'll investigate some basic ideas from logic and perhaps have some fun too.

We need to start with some basic definitions.

A ***statement*** is a sentence that is either true or false, but not both.

A ***conditional statement*** is a statement that can be expressed in "if … then" form.

A few examples should help clarify these definitions. The following sentences <u>are statements.</u>

- Atlanta is the capital of Georgia. (*This sentence is true.*)
- Jimmy Carter was the thirty-ninth president of the United States and was born in Plains, Georgia. (*This sentence is true.*)
- The Atlanta Falcons are a professional basketball team. (*This sentence is false.*)
- George Washington had eggs for breakfast on his fifteenth birthday. (*Although it is unlikely that we can find any source that allows us to determine whether this sentence is true or false, it still must either be true or false, and not both, so it is a statement.*)

Here are some sentences that are <u>not statements</u>.

- What's your favorite music video? (*This sentence is a question.*)

- Turn up the volume so I can hear this song. (*This sentence is a command.*)
- This sentence is false. (*This sentence is a very peculiar object called a self-referential sentence. It creates a logical puzzle that bothered logicians in the early twentieth century. If the sentence is true, then it is also false. If the sentence is false, then it is also true. Logicians finally resolved this puzzling issue by excluding such sentences from the definition of "statement" and requiring that statements must be either true or false, but not both.*)

The last example discussed a sentence that puzzled logicians in the last century. The passage from *Alice in Wonderland* contains several sentences that may have puzzled you the first time you read them. The next part of this activity will allow you to analyze the passage while learning more about conditional statements.

Near the beginning of the passage, the Hatter responds to Alice that she might as well say that "I see what I eat" means the same thing as "I eat what I see." Let's express each of the Hatter's example sentences in "if … then" form.

"I see what I eat" has the same meaning as the conditional statement "If I eat a thing, then I see it." On the other hand, "I eat what I see" has the same meaning as the conditional statement "If I see a thing, then I eat it."

1. Express each of the following statements from the Mad Tea Party in "if … then" form.
   a. I like what I get. ________________________________
   b. I breathe when I sleep. ________________________________

2. We use specific vocabulary to refer to the parts of a conditional statement written in "if … then" form. The ***hypothesis*** of a conditional statement is the statement that follows the word "if." So, for the conditional statement "If I eat a thing, then I see it," the hypothesis is the statement "I eat a thing." Note that the hypothesis does **not** include the word "if" because the hypothesis is the statement that occurs after the "if."

   Give the hypothesis for each of the conditionals in 1a and 1b above.
   a. ________________________________
   b. ________________________________

3. The ***conclusion*** of a conditional statement is the statement that follows the word "then." So, for the conditional statement "If I eat a thing, then I see it," the conclusion is the statement "I see it." Note that the conclusion does **not** include the word "then" because the conclusion is the statement that occurs after the word "then."

   Give the conclusion for each of the conditionals in 1a and 1b.
   a. ________________________________
   b. ________________________________

Notice that we used the words "it" and "thing" to make the meaning of the conditional statements above more clear. This replacement doesn't change the meaning, and it does help us analyze the relationship between hypotheses and conclusions.

4. a. List the hypothesis and conclusion for the revised version of each of the Hatter's conditional statements given below.

| | Hypothesis | Conclusion |
|---|---|---|
| If I eat a thing, then I see the thing. | ________________ | ________________ |
| If I see a thing, then I eat the thing. | ________________ | ________________ |

   b. Explain how the Hatter's two conditional statements are related.

5. There is a term for the new statement you get by swapping the hypothesis and conclusion in a conditional statement. This new statement is called the ***converse*** of the first

   a. Write the **converse** of each of the conditional statements in 1a and 1b.
      1a. ________________________________________________
      1b. ________________________________________________

   b. What is the converse of each of the converses in 5a?
      1a. ________________________________________________
      1b. ________________________________________________

6. The March Hare, Hatter, and Dormouse did not use "if … then" form when they stated their conditionals. Write the converse for each conditional statement below without using "if … then" form.
   Conditional: I breathe when I sleep. Converse: ______________________
   Conditional: I like what I get. Converse: ______________________
   Conditional: I see what I eat. Converse: ______________________
   Conditional: I say what I mean. Converse: ______________________

7. a. What relationship between breathing and sleeping is expressed by the conditional statement "I breathe when I sleep"? If you make this statement, is it true or false? Explain.

   b. What is the relationship between breathing and sleeping expressed by the conditional statement "I sleep when I breathe"? If you make this statement, is it true or false? Explain.

8. The conversation in the passage from *Alice in Wonderland* ends with the Hatter's response to the Dormouse "It is the same thing with you." The Hatter was making a joke. Do you get the joke? If you aren't sure, you may want to learn more about Lewis Carroll's characterization of the Dormouse in Chapter VII of *Alice in Wonderland*.

We have looked at conditional statements from *Alice in Wonderland*. Now we want to look at some conditional statements involving mathematics. In order to do so, we will first investigate a function that may be new to you, the ***absolute value function.***

The absolute value function, $f$, is defined as follows:

$f$ is the function with domain all real numbers such that $f(x) = | x |$.

(Note. To give a complete definition of a function, we must specify the domain and a formula for obtaining the unique output for each input. It is not necessary to specify the range because the domain and the formula determine the set of outputs.)

9. We'll explore the graph of the absolute value function $f$ and then consider some related conditional statements.

   a. Complete the table of values given below.

| $x$ | 0 | $\frac{1}{2}$ | $-\frac{1}{2}$ | 1 | –1 | 3.8 | –3.8 | | –5 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $f(x) = \| x \|$ | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | | | | | 5 | | $\frac{15}{2}$ | $\frac{15}{2}$ | 10 |

   b. Most graphing calculators have a standard graphing window which shows the portion of the graph of a function corresponding to $x$-values from – 10 to 10 and $y$-values from –10 to 10. On grid paper, set up such a standard viewing window and then use the table of values above to draw the part of the graph of the function $f$ for this viewing window.

      Does your graph show all of the input/output pairs listed in your table of values?

      Does your graph show the input/output pairs for those $x$-values from –10 to 10 that were not listed in the table? Explain.

   c. Think of your graph as a picture. Do you see a familiar shape? Describe the graph to someone who cannot see it.

      If you extended your graph to include points corresponding to additional values of $x$, say to include $x$-values from –1000 to 1000, would the shape change?

      What can you do to your sketch to indicate information about the points outside of your "viewing window"?

   d. For what $x$-values shown on your graph of $f$ does the $y$-value increase as $x$ increases, that is, for what $x$-values is it true that, as you move your finger along your graph so that the $x$-values increase, the $y$-values also increase? (Be sure to give the $x$-values where this happens.)

      For what $x$-values does the $y$-value decrease as $x$ increases, that is, for what $x$-values is it true that, as you move your finger along your graph so that the $x$-values increase, the $y$-values decrease? (Be sure to give the $x$-values where this happens.)

Considering your answers to the questions in part c, <u>for the whole function *f*</u>, determine:
(*i*) those *x*-values such that the *y*-value increases as *x* increases

(*ii*) those *x*-values such that the *y*-value decreases as *x* increases.

10. Evaluate each of the following expressions written in function notation. Be sure to simplify so that there are no absolute value signs in your answers. Use your graph to verify that each of your statements is true.

a. $f(0) =$ ___ b. $f(-5) =$ ___ c. $f(-\pi) =$ ___ d. $f(\sqrt{3}) =$ ___

11. The statements in 10, a – d, are written using the equals sign. The same ideas can be expressed with conditional statements. Fill in the blanks to form such equivalent statements.

a. If $x = 0$, then $f(x) =$ _____ . b. If $x = -5$, then $f(x) =$ _____ .

c. If the input for the function *f* is $-\pi$, then the output for the function *f* is _____ .

d. If the input for the function *f* is $\sqrt{3}$, then the output for the function *f* is _____ .

# Mathematics I
## Task 3: From Wonderland to Functionland Day 2/3
(GaDOE TE # 12, #13, and #14)

**Standard(s): MM1G2. Students will understand and use the language of mathematical argument and justification.**
b. Understand and use the relationships among a statement and its converse, inverse, and contrapositive.
**MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.**
a. Represent functions using function notation.
c. Graph transformations of basic functions including vertical shifts, stretches, and shrinks, as well as reflections across the $x$- and $y$-axes.
d. Investigate and explain the characteristics of a function: domain, range, zeros, intercepts, intervals of increase and decrease, maximum and minimum values, and end behavior.

**New Vocabulary:** truth value of conditional statements, statement (propositional) variables, propositional form of a conditional statement, logical equivalence, negation of a statement, inverse of a conditional statement

**Mathematical concepts/topics:** logical equivalence of propositional forms of conditional statements, the inverse of a conditional statement, vertical translation of the absolute value function, domain, range, intervals of increase and decrease, function notation

**Prior knowledge:** statements using "and, "or", and "not" in the contexts of set theory and probability, operating with absolute value

**Essential Question(s):** What is the inverse of a conditional statement? Which propositional forms of conditional statements are logically equivalent?

**Suggested materials:** graph paper

**Warm-up:** Have the following posted on the board:

*Share the statements you wrote for homework with your partner. Are your partner's statements true or false? What do you think makes a conditional statement true? What do you think makes a conditional statement false?*

**Opening:** Allow students to share their statements. Ask them whether they think their statements are true or false. Have them explain their thinking. Establish the fact that conditional statements are true if when the hypothesis is true, the conclusion is always true. A conditional statement is false if when the hypothesis is true (or can be true), the conclusion is false. (Note: The case when the hypothesis of a conditional statement is false is not discussed in this course. See teacher notes.)

**Worktime:** Students should do problems #12-#14 of the restructured task. (Note that the reading in problem 12 and much of the work in problem 14 have been changed. The definitions of propositional statements and propositional form that exist in the GaDOE TE before problem 9 have been simplified and moved to problem 12, where they are used, and simplified to some extent.)

When students have completed problem 12, stop and discuss the use of p and q to represent statements and to write conditional statements in propositional form. Have students discuss solutions to problem 12 before moving on to graph g(x).

Students should graph g(x) by hand. This is the first vertical translation of a function that they have seen.

Parts a-c of problem 14 have been re-written to introduce the negation of a statement and the inverse of a conditional statement with simple sentences using common language rather than mathematical language. Students then apply these concepts to the function g(x) in the table in part d.

**Closing:** Have students present their work, making sure that ALL questions are explored and answered.

**Homework:** GaDOE TE #15. Students should graph h(x) on graph paper for homework. They will use their graphs to complete problem 16 in class.

**Differentiated support/enrichment:**

**Check for understanding:**

**Resources/materials for Math Support:** Previewing all vocabulary in this task is important. Students will continue to need practice with the concepts addressed in class including graphing transformations of the absolute value function, writing conditional statements involving function notation and input/output pairs, determining the truth values of statements, and writing inverses of conditional statements. Problems 1 and 2 of the supplemental homework which follows the student task for this lesson could be used in Math Support.

In the remainder of this task, we will often consider whether a conditional statement is true or false. To say that a **conditional is true** means that, whenever the hypothesis is true, then the conclusion is also true; and to say that **a conditional is false** means that the hypothesis is, or can be, true while the conclusion is false.

12. a. Write the converse of each of the **true** conditional statements from item 11. For each converse, use the graph of *f* to determine whether the statement is true or false. Organize your work in a table such as the one shown below. For the statements in the table that you classify as false, specify a value of *x* that makes the hypothesis true and the conclusion false.

| Conditional statement | Truth value | Converse statement | Truth value |
|---|---|---|---|
| If $x = 0$, then $f(x) =$ | True | | |
| | True | | |
| | True | | |
| | True | | |

b. Whether a statement is true or false is called the ***truth value*** of the statement. Our goal for this item is to decide whether there is a general relationship between the truth value of a conditional statement and the truth value of its converse. Any particular conditional statement can be true or false, so we need to consider examples for both cases. Add lines to your table from part a for the converses of the following **false** conditional statements. For these statements, and for any converse that you classify as false, give a value of *x* that makes the hypothesis true and the conclusion false.

(i) If $f(x) = 7$, then $x = 5$.

(ii) If $f(x) = 2$, then $x = 2$.

| Conditional statement | Truth value | Converse statement | Truth value |
|---|---|---|---|
| | | | |
| | | | |

c. Complete the following sentence to make a true statement. Explain your reasoning. Is your answer choice consistent with **all** of the examples of converse in the table above?

The converse of a true conditional statement is_____ .

A) always also true B) always false

C) sometimes true and sometimes false

d. Complete the following sentence to make a true statement. Explain your reasoning. Is your answer choice consistent with **all** of the examples of converse in the table above?

The converse of a false conditional statement is ______ .

A) always also false B) always true

C) sometimes true and sometimes false

In order to talk about statements in general, without regard to the particular statements used for the hypothesis and the conclusion, we can use variables to represent statements as a whole. This use of variables is demonstrated in the formal definition that follows.

**Definition:** If $p$ and $q$ are statements, then the statement "if $p$, then $q$" is the ***conditional statement,*** or ***implication,*** with hypothesis $p$ and conclusion $q$.

We call the variables used above, ***statement***, or ***propositional***, ***variables***.

We are seeking a general conclusion about the logical relationship between a conditional statement and its converse; we are looking for a relationship that is true no matter what particular statements we substitute for the statement variables $p$ and $q$.

For the propositional form "if $p$, then $q$", the converse propositional form is "if $q$, then $p$."

If two propositional forms result in statements with the same truth value for *all possible cases* of substituting statements for the propositional variables, we say that the forms are ***logically equivalent.*** If there exist statements that can be substituted into the propositional forms so that the resulting statements have different truth values, we say that the propositional forms are ***not logically equivalent***

e. Consider your answers to parts a and b, and decide how to complete the following statement to make it true. Justify your choice.

The converse propositional form "if $q$, then $p$" is/ **is not** (*choose one*) logically equivalent to the conditional statement "if $p$, then $q$."

f. *Multiple choice:* If you learn a new mathematical fact in the form "if $p$, then $q$", what can you immediately conclude, without any additional information, about the truth value of the converse? Explain your choice.

A) no conclusion B) conclude that the converse is true
C) conclude that the converse is false

g. Look back at the opening of the passage from *Alice in Wonderland*, when Alice hastily replied "I do, at least -- at least I mean what I say -- that's the same thing, you know." What statements did Alice think were logically equivalent? What was the Hatter saying about the equivalence of these statements when he replied to Alice by saying "Not the same thing a bit!"?

There are two other important propositional forms related to any given conditional statement. We introduce these by exploring other inhabitants of the land of functions.

13. Let $g$ be the function with domain all real numbers such that $g(x) = |\,x\,| + 3$ .

a. Complete table of values given below.

| $x$ | 0 | $\frac{1}{2}$ | $-\frac{1}{2}$ | 1 | –1 | 3.8 | –3.8 | | –5 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $g(x) = \|\,x\,\| + 3$ | 3 | $\frac{7}{2}$ | $\frac{7}{2}$ | | | | | | | $\frac{21}{2}$ | $\frac{21}{2}$ | 13 |

b. On grid paper, draw the portion of the graph of $g$ for all $x$-values such that $-10 \le x \le 10$. To show all of the points corresponding to input/output pairs shown in the table, how much of the $y$-axis should your viewing window include?

Is the part of the graph that you have drawn representative of the whole graph? Explain.

c. For what $x$-values shown on your graph of $g$ does the $y$-value increase as $x$ increases? For what $x$-values does the $y$-value decrease as $x$ increases?

For <u>the whole function $g$</u>, determine:
($i$) those $x$-values such that the $y$-value increases as $x$ increases
($ii$) those $x$-values such that the $y$-value decreases as $x$ increases.

d. What is the relationship between the graphs of $f(x) = |x|$ and $g(x) = |x| + 3$?

What in the formulas for $f(x)$ and $g(x)$ tells you that the graphs are related in this way?

In logic, we form the ***negation*** of a statement $p$ by forming the statement "It is not true that $p$." For convenience, we use "not $p$" to refer to the negation of the statement $p$. For a specific choice of statement, when we translate "not $p$" into English, we can usually state the negation in a more direct way. For example:

- when $p$ represents the statement "A person drives a Mercedes"," then "not $p$" represents "A person does not drive a Mercedes"; and
- when $q$ represents the statement "A person does not drive an American-made car", then "not $q$" represents "A person does not, not drive an American-made car" or more simply "A person drives an American-made car."

A statement of the form "If not $p$, then not $q$" is called the ***inverse*** of the conditional statement "if $p$, then $q$." Note that the inverse is formed by negating the hypothesis and conclusion of a conditional statement

14. If $p$ and $q$ represent the statements indicated above:

a. What statement is represented by "If p, then q"? Is the statement true?

b. What statement is represented by "If not $p$, then not $q$"? Is this inverse statement true? Explain your reasoning.

c. The table below includes statements about the functions $f(x) = |x|$ and $g(x) = |x| + 3$. Fill in the blanks in the table. Be sure that your entries for the truth value columns agree with the graphs for $f$ and $g$. For the statements in the table that are false, give a value of $x$ that makes the hypothesis true and the conclusion false.

| Conditional statement | Truth value | Inverse statement | Truth value |
|---|---|---|---|
| If $x = 4$, then $f(x) \neq 9$. | True | If $x \neq 4$, then $f(x) = 9$. | |
| If $g(x) \neq 3$, then $x \neq 0$. | | | |
| If $x = 0$, then $g(x) \neq 3$. | False, | | |
| | | If $g(x) \neq 6$, then $x \neq 3$. | True |

d. Consider the results in the table above, and then decide how to complete the following statement to make it true. Justify your choice.

The inverse propositional form "if not $p$, then not $q$" is/ ***is not*** (*choose one*) logically equivalent to the conditional statement "if $p$, then $q$."

e. *Multiple choice:* If you learn a new mathematical result in the form "if $p$, then $q$", what can you immediately conclude, without any additional information, about the truth value of the inverse? Explain your choice.

A) no conclusion
B) conclude that the inverse is true
C) conclude that the inverse is false

# Mathematics I
# From Wonderland to Functionland
### Day 2 Homework

15. Let $h$ be the function with domain all real numbers such that $h(x) = 2|x|$.
 On grid paper, draw the portion of the graph of $h$ for all $x$-values such that $-10 \le x \le 10$.

 For $x = -7, -2, 0, 5, 8$, compare $f(x)$ and $h(x)$.

 What is the relationship between the values? Does this relationship hold for every real number?
 How do the graphs of $f$ and $h$ compare? Explain why the graphs have this relationship.

# Mathematics I

## Task 3: From Wonderland to Functionland — Day 3/3

(GaDOE TE #16, #17, and #18)

**Standard(s): MM1G2. Students will understand and use the language of mathematical argument and justification.**
b. Understand and use the relationships among a statement and its converse, inverse, and contrapositive.
**MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.**
a. Represent functions using function notation.
c. Graph transformations of basic functions including vertical shifts, stretches, and shrinks, as well as reflections across the *x*- and *y*-axes.
d. Investigate and explain the characteristics of a function: domain, range, zeros, intercepts, intervals of increase and decrease, maximum and minimum values, and end behavior.
e. Relate to a given context the characteristics of a function, and use graphs and tables to investigate its behavior.

**New Vocabulary:** the contrapositive of a conditional statement, biconditional statement

**Mathematical concepts/topics:** the contrapositive of a conditional statement, logical equivalence of propositional forms of conditional statements, biconditional statements, vertical stretch of the absolute value function, domain, range, intervals of increase and decrease, function notation, linear functions

**Prior knowledge:** linear functions

**Essential Question(s):** What is the contrapositive of a conditional statement? Which propositional forms of conditional statements are logically equivalent?

**Suggested materials:** graph paper

**Warm-up:** Have the following posted on the board:

*Share your graph of* $h(x) = 2|x|$ *with your partner. How do your graphs compare? Discuss the answers to parts a and b of question 15 and be prepared to share your answers with the class.*

**Opening:** Ask a student to graph h(x) on the board and discuss answers to the homework with the class. Allow students to read the definition of the contrapositive of a conditional statement given at the beginning of today's task. Ask them to define the contrapositive and write the definition on the board. Review the definitions of the converse and the inverse of a conditional statement and leave all three definitions up for students to refer to during today's lesson.

**Worktime:** Students should do problems #16 and #17 of the restructured task. (Note that problem 16 has been simplified considerably.)

**Closing:** When students have finished problems 16 and 17, allow students to present their work. Problem 17 summarizes the work on propositional forms and logical equivalence. When you are sure that all students understand the concepts in 16 and 17, assign problem 18 to be done individually in class as a check for understanding of the concepts in this task. This problem also allows an opportunity for students to use what they already know about linear functions. Although biconditional statements have not been discussed to this point, give students an opportunity to answer part f of this problem. This is an extension and an application of what they should have learned from the task.

**Homework:**

**Differentiated support/enrichment:**

**Check for understanding:**

**Resources/materials for Math Support:** Previewing all vocabulary in this task is important. Students will continue to need practice with the concepts addressed in class including graphing transformations of the absolute value function; writing conditional statements involving function notation and input/output pairs; determining the truth values of statements; writing converses, inverses, and contrapositives of conditional statements; and determining the logical equivalence of propositional forms.

A statement of the form "If not $q$, then not $p$" is called the ***contrapositive*** of the conditional statement "if $p$, then $q$." Note that the contrapositive is formed by both negating and exchanging the hypothesis and conclusion.

16.a. The table below includes statements about the function $h$. Fill in the blanks in the table. Be sure that your answers for the truth value columns are consistent with the graph of $h$. For the statements in the table classified as false, give a value of $x$ that makes the hypothesis true and the conclusion false.

| Conditional statement | Truth value | Contrapositive statement | Truth value |
|---|---|---|---|
| If $x = 4$, then $h(x) = 3$. | | If $h(x) \neq 3$, then $x \neq 4$. | |
| If $x = 6$, then $h(x) = 12$. | | | |
| If $h(x) = 0$, then $x = 0$. | | | |
| If $x < 3$, then $h(x) < 6$. | False, let $x = -4$ | | |

b. On each line of the table, how do the truth value of the conditional statement and its contrapositive compare?

c. Based on your reasoning in part a above, if a conditional statement is true, could the contrapositive be false? If a conditional statement is false, could the contrapositive statement be true?

d. Consider your answer to part c, and decide how to complete the following statement to make it true. Justify your choice.

The propositional form "if $p$, then $q$" ***is***/ is not (*choose one*) logically equivalent to its contrapositive "if not $q$, then not $p$."

e. *Multiple choice:* If you learn a new mathematical result in the form "if $p$, then $q$," what can you immediately conclude, without any additional information, about the truth value of the contrapositive? Explain.

A) no conclusion

B) conclude that the contrapositive is true C) conclude that the contrapositive is false

17. Summarizing the information about forming related conditional statements, we see that the conditional "if $p$, then $q$," has three related conditional statements:

**converse**: "if $q$, then $p$" (swaps the hypothesis/conclusion)

**inverse**: "if not $p$, then not $q$" (negates the hypothesis/conclusion)

**contrapositive**: "if not $q$, then not $p$" (negates hypothesis/conclusion, then swaps)

Which, if any, of these is logically equivalent to the original conditional statement and always has the same truth value as the original?

The absolute value function $f$ and the functions $g$ and $h$ that you have worked with in this investigation are not linear. However, in your study of functions prior to Mathematics I, you have worked with many linear functions. We conclude this investigation with discussion about converse, inverse, and contrapositive using a linear function.

18. Consider the linear function $F$ which converts a temperature of $c$ degrees Celsius to the equivalent temperature of $F(c)$ degrees Fahrenheit. The formula is given by

$$F(c) = \frac{9}{5}c + 32, \text{ where } c \text{ is a temperature in degrees Celsius.}$$

a. At what temperature, in degrees Celsius, does water freeze? in degrees Fahrenheit? Verify that the formula for $F$ converts correctly for freezing temperatures.

b. At what temperature, in degrees Celsius, does water boil? in degrees Fahrenheit? Verify that the formula for $F$ converts correctly for boiling hot temperatures.

c. Draw the graph of $F$ for values of $c$ such that $-100 \le c \le 400$. What is the shape of the graph you drew? Is this the shape of the whole graph?

d. Verify that "if $c = 25$, then $F(c) = 77$" is true. What is the contrapositive of this statement? How do you know that the contrapositive is true without additional verification?

e. What is the converse of "if $c = 25$, then $F(c) = 77$"? How can you use the formula for $F$ to verify that the converse is true? What is the contrapositive of the converse? How do you know that this last statement is true without additional verification?

f. There is a statement that combines a statement and its converse; it's called a ***biconditional***.

**<u>Definition</u>:** If $p$ and $q$ are statements, then the statement "$p$ if and only if $q$" is called a ***biconditional statement*** and is logically equivalent to "if $q$, then $p$" **and** "if $p$, then $q$."

Write three true biconditional statements about values of the function $F$. Explain how you know that the statements are true.

# Mathematics I
# From Wonderland to Functionland
### Extra Problems

1. Complete the table below with input/output pairs for the function t(x) = -2|x|. An x-value of zero has been chosen for you. Choose other values for x between -10 and 10. Be sure to choose different kinds of values for x including positive numbers, negative numbers, whole numbers, and fractions.

| $x$ | 0 | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $t(x) = -2\| x \|$ | | | | | | | | | | | |

b. Use the values in your table to help you draw the graph of the function *t* above. Use a "viewing window" with *x*-values from – 10 to 10. Choose an appropriate window for y.

c. If the domain of the function *t* is all real numbers, what is the range?

d. For what x-values is the function *t* increasing? For what x-values is the function *t* decreasing?

e. Compare the graph of *t* to the graph of $f(x) = |x|$ drawn in class. How are the graphs of the two functions alike? How are they different? How do you account for the similarities and the differences? Explain.

2. Use input/output pairs from your table in problem 1 to complete the table below. In each row, write a conditional statement and its converse, and give the truth value of each statement. In the first row, a conditional statement has been started for you.

| Conditional statement | Truth value | Converse statement | Truth value |
|---|---|---|---|
| a. If $x = 0$, then $t(x) =$ ______ . | | | |
| b. | | | |
| c. | | | |
| d. | | | |

3. Challenge: You learned that the propositional form of a conditional statement, "If p, then q", and its converse, "If q, then p", are not logically equivalent. In other words, you cannot guarantee that a conditional statement and its converse always have the same truth value. They might but they might not!

You have investigated the absolute value function $f(x) = |x|$ and, in this homework, you have graphed the function $t(x) = -2|x|$. You have written conditional statements and their converses based on input/output pairs for each of these functions. Sometimes a statement and its converse had the same truth value. Other times the statements had opposite truth values.

Your study of functions began in middle school and will continue throughout your high school career. Think of the functions you have studied so far. Can you think of a function for which conditional statements expressing input/output pairs and their converses will always have the same truth values? If so, represent the function using function notation and explain why you think this would be true. Be sure to include the graph of the function in your explanation.

# Atlanta Public Schools

# Teacher's Curriculum Supplement

# Mathematics I: Unit 1

# Task 4: Sequences as Functions

# Mathematics I
## Task 4: Sequences as Functions Day 1/2
(GaDOE TE # 1- #4)

**Standard(s): MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.**
a. Represent functions using function notation.
f. Recognize sequences as functions with domains that are whole numbers.

**New Vocabulary:** sequence, term, infinite sequence, finite sequence, ellipses, recursive and closed (explicit) definitions for sequences

**Mathematical concepts/topics:** finite and infinite sequences, recursive and explicit (closed) definitions for sequences

**Prior knowledge:** representation of arithmetic sequences as linear functions with whole number domains

**Essential Question(s):** How can sequences be defined using recursive and closed (explicit) formulas?

**Suggested materials:** no special materials

**Warm-up:** Write the nine sequences in problem 1 on the board. Have students work alone to find the pattern for as many of the sequences as possible.

**Opening:** Allow students to share the patterns they have found. Once all existing patterns have been established, allow students to read the definitions in the task. As a class, define *sequence, term, infinite sequence, finite sequence, and ellipses*. Make sure students are familiar with the notation for sequences. Although this should have been addressed in the eighth grade, many students may still be unfamiliar with the fact that $t_n$ refers to the value of the $n^{th}$ term in a sequence.

**Worktime:** Students should do problems #2- #4 of the task. Although most of these sequences are simple, finding recursive and explicit formulas may be difficult for students

**Closing:** Have students present their work, making sure that ALL questions are explored and answered.

**Homework:** Practice in writing recursive and closed definitions for sequences.

**Differentiated support/enrichment:**

**Check for understanding:**

**Resources/materials for Math Support:** Previewing the vocabulary used in this task is very important. Students may need extra practice in using the appropriate notation for sequences and in writing recursive and explicit definitions.

# Mathematics I
# Sequences as Functions
### Day 1 Student Task

In your previous study of functions, you have seen examples that begin with a problem situation. In some of these situations, you needed to extend or generalize a pattern. In the following activities, you will explore patterns as sequences and view sequences as functions.

A *sequence* is an ordered list of numbers, pictures, letters, geometric figures, or just about any object you like. Each number, figure, or object is called a *term* in the sequence. For convenience, the terms of sequences are often separated by commas. In Mathematics I, we focus primarily on sequences of numbers and often use geometric figures and diagrams as illustrations and contexts for investigating various number sequences.

1. Some sequences follow predictable patterns, though the pattern might not be immediately apparent. Other sequences have no pattern at all. Explain, when possible, patterns in the following sequences:

   a. 5, 4, 3, 2, 1
   b. 3, 5, 1, 2, 4
   c. 2, 4, 3, 5, 1, 5, 1, 5, 1, 5, 1, 7
   d. S, M, T, W, T, F, S
   e. 31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30, 31
   f. 1, 2, 3, 4, 5, …, 999, 1000
   g. 1, -1, 1, -1, 1, -1, …
   ***h.*** 4, 7, 10, 13, 16, …
   i. 10, 100, 1000, 10000, 100000, …

The first six sequences above are ***finite sequences***, because they contain a finite number of terms. The last three are ***infinite sequences*** because they contain an infinite number of terms. The three dots, called ***ellipses***, indicate that some of the terms are missing. Ellipses are necessary for infinite sequences, but ellipses are also used for large finite sequences. The sixth example consists of the counting numbers from 1 to 1000; using ellipses allows us to indicate the sequence without having to write all of the one thousand numbers in it.

2. In looking for patterns in sequences it is useful to look for a pattern in how each term relates to the previous term. If there is a consistent pattern in how each term relates to the previous one, it is convenient to express this pattern using a ***recursive definition*** for the sequence. A recursive definition gives the first term and a formula for how the $n^{th}$ term relates to the $(n-1)^{th}$ term. For each sequence below, match the sequence with a recursive definition that correctly relates each term to the previous term, and then fill in the blank in the recursive definition.

   a. 1, 2, 3, 4, 5, …      I. $t_1 = \underline{\ 5\ }$, $t_n = t_{n-1} + 5$

   b. 5, 10, 15, 20, 25, …      II. $t_1 = \underline{3 + \sqrt{2}}$, $t_n = t_{n-1} + 2$

c. 1.2, 11.3, 21.4, 31.5, 41.6, 51.7, … III. $t_1 = 1,\ t_n = t_{n-1} + 1$

d. $3+\sqrt{2}$, $5+\sqrt{2}$, $7+\sqrt{2}$, $9+\sqrt{2}$, $11+\sqrt{2}$, … IV. $t_1 = -1,\ t_n = (-2)t_{n-1}$

e. – 1, 2, –4, 8, –16, … V. $t_1 = 1.2,\ t_n = t_{n-1} + 10.1$

3. Some recursive definitions are more complex than those given in item 2. A recursive definition can give two terms at the beginning of the sequence and then provide a formula for the *n*th term as an expression involving the two preceding terms, $n - 1$ and $n - 2$. It can give three terms at the beginning of the sequence and then provide a formula for the *n*th term as an expression involving the three preceding terms, $n - 1$, $n - 2$, and $n - 3$; and so forth. The sequence of Fibonacci numbers, 1, 1, 2, 3, 5, 8, 13, 21, …, is a well known sequence with such a recursive definition.

   a. What is the recursive definition for the Fibonacci sequence?

   b. The French mathematician Edouard Lucas discovered a related sequence that has many interesting relationships to the Fibonacci sequence. The sequence is 2, 1, 3, 4, 7, 11, 18, 29, …, and it is called the Lucas sequence. For important reasons studied in advanced mathematics, the definition of the Lucas numbers starts with the 0th term. Examine the sequence and complete the recursive definition below.

   $t_0 = 2,\ t_1 = 1,$

   $t_n = t_{n-1} + t_{n-2},$ for all integers $n = 2, 3, 4, ...$

   c. What is the 8th term of the Lucas sequence, that is, the term corresponding to $n = 8$?

   d. What is the 15th term?

4. Recursive formulas for sequences have many advantages, but they have one disadvantage. If you need to know the 100th term, for example, you first need to find all the terms before it. An alternate way to define a sequence uses a ***closed form*** definition that indicates how to determine the *n*th term directly, without the need to calculate other terms. Match each sequence below with a definition in closed form. Verify that the closed form definitions you choose agree with the five terms given for each sequence.

   a. 5, 10, 15, 20, 25, … I. $t_n = n^3,\ \textit{for } n = 1, 2, 3, ..$

   b. 1, 0, 2, 0, 3, 0, 4, … II. $t_n = 5n,\ \textit{for } n = 1, 2, 3, ..$

   c. 5, 9, 13, 17, 21, 25, … III. $t_n = 4n + 1,\ \textit{for } n = 1, 2, 3, ..$

   d. 1, 8, 27, 64, 125, . . . IV. $t_n = \begin{cases} \dfrac{n+1}{2}, & \text{if } n = 1, 3, 5, ... \\ 0, & \text{if } n = 2, 4, 6, ... \end{cases}$

# Mathematics I
## Task 4: Sequences as Functions Day 2/2
(GaDOE TE # 5- #8)

**Standard(s): MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.**
a. Represent functions using function notation.
f. Recognize sequences as functions with domains that are whole numbers.

**New Vocabulary:**

**Mathematical concepts/topics:** representing sequences as functions with domains that are whole numbers, function notation, domain, range, limiting cases, equal functions, linear functions

**Prior knowledge:** representation of arithmetic sequences as linear functions with whole number domains

**Essential Question(s):** How can sequences be represented as functions?

**Suggested materials:** no special materials

**Warm-up:** Have the following posted as students enter the room.

- *Compare last night's homework with your partner.*
- *Discuss any differences in your work.*
- *Be prepared to discuss differences and/or ask any questions you have related to your work.*

**Opening:** Review vocabulary from previous lesson and answer any questions students may have related to homework.

**Worktime:** Students should do problems #5- #8 of the task.

**Closing:** Have students present their work, making sure that ALL questions are explored and answered. (See teacher notes.)

**Homework:**

**Differentiated support/enrichment:**

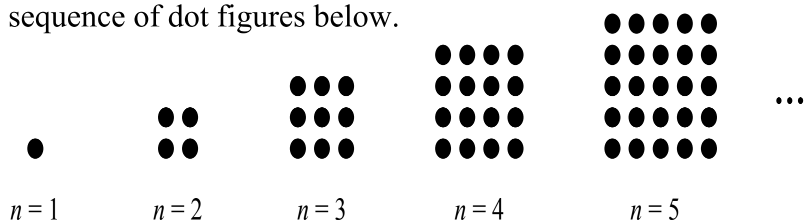
**Check for understanding:**

**Resources/materials for Math Support:**

# Mathematics I
# Sequences as Functions
### Day 2 Student Task

Sequences do not need to be specified by formulas; in fact, some sequences are impossible to specify with formulas. When sequences are given by formulas, the two types of formulas most commonly used are the ones explained above: recursive and closed form. In item 4, you probably noticed that a closed form definition for a sequence looks very much like a formula for a function. If you think of the index values as inputs and the terms of the sequence as outputs, then any sequence can be considered to be a function, whether or not you have a formula for the *n*-th term of the sequence.

5. Consider the sequence of dot figures below.




   a. The sequence continues so that the number of dots in each figure is the next *square number*. Why are these numbers called square numbers?

   b. What is the next figure in the sequence? What is the next square number?

   c. What is the $25^{th}$ square number? Explain.

   d. Is 156 a square number? Explain.

   e. Let $g$ be the function that gives the number of dots in the $n^{th}$ figure above. Write a formula for this function.

   f. Make a table and graph for the function $g$.

   g. What is the domain of the function $g$?

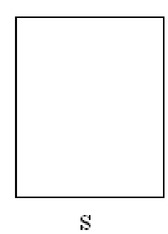
   h. What is the range of the function $g$?

6. Consider a function $A$ that gives the area of a square of side-length $s$, shown below (and to the right).

   a. What is the area of a square of side length 6?

   b. Find a formula for the function $A$.

   c. Make a table and graph for the function $A$.



d. Use your formula to find the area of squares with the following side lengths: 7, ½ , 4.2, $\sqrt{3}$ , and 200.

e. What is the domain of *A* (in the given context)?

f. What are the limiting case(s) in this context? Explain.

g. Change the graph to extend the domain of *A* to include the limiting case(s). What change did you make in the graph?

7. Compare the functions *g* and *A* from items 5 and 6.

a. Explain how similarities and differences in the formulas, tables, and graphs arise from similarities and differences in the contexts.

b. Are these functions equal? Explain.

8. Look back at the finite sequences in 1a and 1b above. Consider the sequence in item1, part a, to be a function named *h* and the sequence in item 1, part b to be a function named *k*.

a. Make tables and graphs for the functions *h* and *k*.

b. How many points are included in each graph?

c. Determine the following:

domain of *h* ________ range of *h* ___________
domain of *k* ________ range of *k* ____________

d. Are these functions equal? Explain.

e. Write a formula for that gives *h*(*n*) for each number *n* in the domain of *h*. What kind of function did you write?

f. Can you write a linear function formula for the function *k*? Explain why or why not?

g. Consider the linear function *d* given by the formula $d(x) = 10 - x$, for each real number *x*. Make a table and graph for the function *d*.

h. Compare the tables, graphs, and formulas for the functions *h* and *d*. In your comparison, pay attention to domains, ranges, and the shapes of the graphs.

# Atlanta Public Schools

# Teacher's Curriculum Supplement

# Mathematics I: Unit 1

# Task 5: Walking, Falling and Making Money

## Mathematics I
## Task 5: Walking, Falling, and Making Money — Day 1/2
(GaDOE #1 and #2)

**Standard(s): MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.**
**g.** Explore rates of change, comparing constant rates of change (i.e. slope) verses variable rates of change. Compare rates of change of linear, quadratic, square root, and other function families.

**New vocabulary:** rate of change**,** variable rates of change, average rate of change, constant rate of change, speed, interval

**Mathematical concepts/topics:** average rate of change, constant vs variable rate of change, calculation of the average rate of change of y with respect to x as the ratio of the change in y to change in x, linear functions, quadratic functions

**Prior knowledge:** distance = rate x time, speed as linear units per unit of time, linear functions, constant rate of change (slope)

**Essential question(s):** How are characteristics of a function important in interpreting its graph? How can you determine rates of change by using the graph of a function? What is the difference between constant rates of change and variable rates of change?

**Suggested materials:** graph paper, CBR Units

**Warm-up/Opening:** Divide the class into 5 groups. Give each group one of the graphs provided with this lesson. Explain to students that the graphs show the distance from an established starting point at time t. Ask students to talk about how they would have to walk in order to obtain their graph. Demonstrate the use of the CBR. Groups should choose a representative and then decide how the representative should walk with the CBR to create their graph. As students talk, be sure to remind them to discuss the starting point, and the movement required to obtain the given graphs. Prizes might be given to the group that best replicates their graph.
.
**Alternative Warm-up/Opening:** Choose one of the graphs provided with this lesson and share with students a scenario that could be represented by the graph. Be sure to discuss rate of change in your scenario. Divide the class into groups and give each group a different graph. Ask the groups to write a story related to their graph. Allow students to share their stories making sure that their ideas related to the characteristics of the graph, including rates of change, are correct.

**Worktime:** Students should complete GaDOE TE problems 1 and 2. Be sure to read all teacher notes before beginning the task. It is not necessary to introduce the formal notation for calculating average rate of change, $\frac{f(x_2)-f(x_1)}{x_2-x_1}$, at this point. Rather, *average rate of change in y with respect to x* should be described and calculated as the ratio of the *change in y to the change in x.* Be sure that students understand the concepts addressed in problem 1 before allowing them to begin problem 2. Students investigated the function in problem 2, $y = 5x^2$, in the first task in this unit. Have them begin the problem by reviewing their table and graph in that lesson.

**Closing:** Have students present their work, making sure that ALL questions are explored and answered. (See teacher notes.)

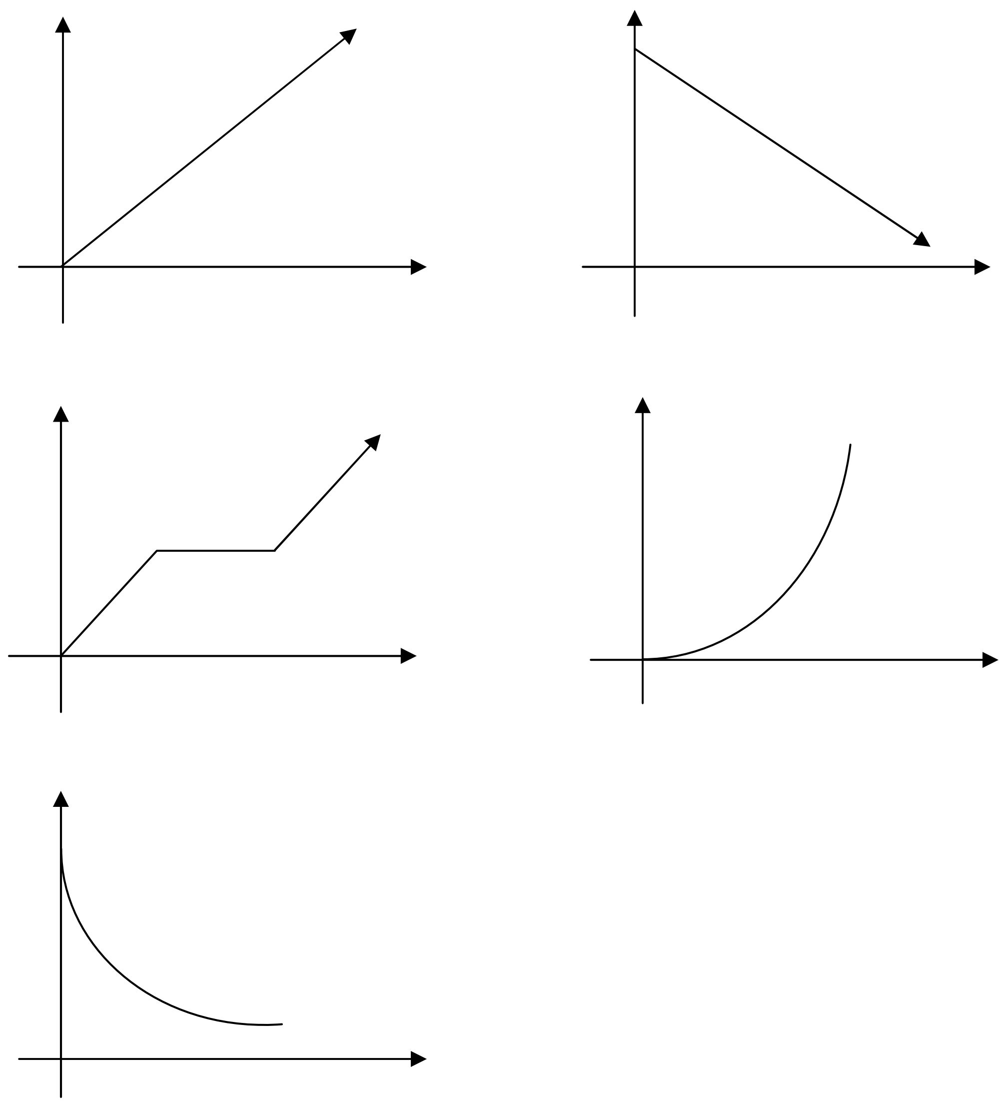
**Homework:** Work should focus on reading graphs and calculating average rate of change.

**Differentiated support/enrichment:**

**Check for understanding:**

**Math Support:**

# Mathematics I
# Walking, Falling, and Making Money
### Graphs for Day 1 Warm-up



# Mathematics I
# Walking, Falling, and Making Money
### Day 1 Student Task

Walking, Falling, and Making Money

In previous mathematics courses, you studied the formula *distance* = *rate* x *time*, which is usually abbreviated $d = rt$. If you and your family take a trip and spend 4 hours driving 200 miles, then you can substitute 200 for *d*, 4 for *t*, and solve the equation 200 = *r**4 to find that *r* = 50. Thus, we say that the average speed for the trip was 50 miles per hour. In this task, we develop the idea of *average rate of change* of a function, and see that it corresponds to average speed in certain situations.

1. To begin a class discussion of speed, Dwain and Beth want to stage a walking race down the school hallway. After some experimentation with a stop watch, and using the fact that the flooring tiles measure 1 foot by 1 foot, they decide that the distance of the race should be 40 feet and that they will need about 10 seconds to go 40 feet at a walking pace. They decide that the race should end in a tie, so that it will be exciting to watch, and finally they make a table showing how their positions will vary over time. Your job is to help Dwain and Beth make sure that they know how they should walk in order to match their plans as closely as possible.

| Time (sec.) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Dwain's position (ft.) | 0 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 |
| Beth's position (ft.) | 0 | 1 | 3 | 6 | 10 | 15 | 20 | 25 | 30 | 35 | 40 |

a. Draw a graph for this data. Should you connect the dots? Explain.

b. How can you tell that the race is supposed to end in a tie? Provide two explanations.

c. Who is ahead 5 seconds into the race? Provide two explanations.

d. Describe how Dwain should walk in order to match his data. In particular, should Dwain's speed be constant or changing? Explain how you know, using observations from both the graph and the table.

e. Describe how Beth should walk in order to match her data. In particular, should Beth's speed be constant or changing? Explain how you know, using observations from both the graph and the table.

f. In your answers above, sometimes you paid attention to the actual data in table. At other times, you looked at how the data change, which involved computing differences between values in the table. Give examples of each. How can you use the graph to distinguish between actual values of the data and differences between data values?

g. Someone asks, “What is Beth’s speed during the race?” Kellee says that this question does not have a specific numeric answer. Explain what she means.

h. Chris says that Beth went 40 feet in 10 seconds, so Beth’s speed is 4 feet per second. But Kellee thinks that it would be better to say that Beth’s *average speed* is 4 feet per second. Is Chris’s calculation sensible? What does Kellee mean?

i. Taylor explains that to compute average speed over some time interval, you divide the distance during the time interval by the amount of time. Compute Dwain’s average speed over several time intervals (e.g., from 2 to 5 seconds; from 3 to 8 seconds). What do you notice? Explain the result.

j. What can you say about Beth’s speed during the first five seconds of the race? What about the last five seconds? Explain.

k. Trey wants to race alongside Dwain and Beth. He wants to travel at a constant speed during the first five seconds of the race so that he will be tied with Beth after five seconds. At what speed should he walk? Explain how Trey’s walking can provide an interpretation of Beth’s average speed during the first five seconds.

In describing relationships between two variables, it is often useful to talk about the rate of change of one variable with respect to the other. When the rate of change is not constant, we often talk about average rates of change. If the variables are called $x$ and $y$, and $y$ is the dependent variable, then

$$\text{Average rate of change of } y \text{ with respect to } x = \frac{\text{the change in } y}{\text{the change in } x}.$$

When $y$ is distance and $x$ is time, the average rate of change can be interpreted as an average speed, as we have seen above.

2. Earlier you investigated the distance fallen by a ball dropped from a high place, such as the Tower of Pisa. In that problem, $y$, measured in meters, is the distance the ball has fallen and $x$, measured in seconds, is the time since the ball was dropped. You saw that $y$ is a function of $x$, and the relationship can be approximated by the formula $y = f(x) = 5x^2$. You completed a table like the one below.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | … |
| $x^2$ | 0 | 1 | 4 | 9 | | | | … |
| $y = f(x) = 5x^2$ | 0 | 5 | 20 | | | | | … |

a. What is the average rate of change of the height of the ball with respect to time during the first second after the ball is dropped? In what units is the average rate of change measured?

b. What is the average rate of change of the height of the ball during the second second after the ball is dropped? after the third, fourth, and fifth? What do these answers say about to the speed of the ball?

c. Find the average rate of change over the first five seconds. Is this the same average speed as the average speed for the fifth second during which the ball is falling? Explain.

d. What is the average rate of change over the first two seconds?

e. What is the average rate of change from 2 to 5 seconds after the ball is dropped?

f. What is the average rate of change from $t = 1.5$ to $t = 2$?

g. Why is it important to use the phrase “average rate of change” or “average speed” for your calculations in this problem?

# Mathematics I
## Task 5: Walking, Falling, and Making Money Day 2/2
(GaDOE #3 and #4)

**Standard(s): MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.**
**g.** Explore rates of change, comparing constant rates of change (i.e. slope) verses variable rates of change. Compare rates of change of linear, quadratic, square root, and other function families.

**New vocabulary:** supply, demand, price, cost, revenue, profit

**Mathematical concepts/topics:** average rate of change, constant vs variable rate of change, calculation of the average rate of change of y with respect to x as the ratio of the change in y to change in x, linear functions, quadratic functions, mathematical analysis of simple business situations, sequences

**Prior knowledge:** linear functions, constant rate of change (slope)

**Essential question(s):** How can we use functions to analyze simple business situations? How can you determine rates of change by using the graph of a function? What is the difference between constant rates of change and variable rates of change?

**Suggested materials:** graph paper

**Warm-up:** Have students compare their homework.

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**Opening:** Discuss the terminology that will be used in this lesson. Make sure students understand the terms *price, cost, revenue, profit, supply and demand.* A discussion of today's gas prices may be interesting to students.

Have students read the introduction to this part of the task and the explanation of the information in the table for problem 3. Ask them for examples of pricing similar to that of the Zingo games- the more items you buy, the less the price per item.

Make sure students understand that in the table in problem 3, x represents the number of *cartons* of games ordered per week.

**Worktime:** Students should complete GaDOE TE problems 3 and 4. As stated in the teacher notes, it is important in working problem 3 that students understand that values of the input variable give the number of **cartons** that the toy store chain will order while values of the output give the price, in dollars, for a **single Zingo game**, not the price of a whole carton of 24 games.

Make sure students understand the concepts in problem 3 before allowing them to move on to problem 4. Both of these problems are lengthy. If students thoroughly understand the concepts previously discussed in this task, it is feasible that they can finish any uncompleted parts of problem 4 for homework.

**Closing:** Have students present their work, making sure that ALL questions are explored and answered. (See teacher notes.)

**Homework:**

**Differentiated support/enrichment:**

**Check for understanding:**

**Math Support:** Preview terminology used in this task. In addition to practice in reading graphs, and calculating average rates of change, students in Math Support may need to preview a scenario with less complex units than those used in problem 3 of this task.

# Mathematics I
# Walking, Falling, and Making Money
### Day 2 Student Task

Average speed is a common application of the concept of average rate of change but certainly not the only one. There are many applications to analyzing the money a company can make from producing and selling products. The rest of this task explores average rate of change for functions related to the Vee Company and its production and sale of a game called Zingo. The Vee Company is a small privately owned manufacturing company which sells to exclusively to a national chain of toy stores. Zingo games are packaged and sold in cartons holding 24 games each. Due to the size of the Vee Company work force, the maximum number of games per week that can be produced is 6000, which is enough to fill 250 cartons.

3. The table below shows data that the Vee Company has collected about the relationship between the wholesale price per game and the number of cartons of Zingo that the toy store chain will order each week. In the business world, it generally happens that lowering the price of a product increases the number that will be bought; this holds true for the Zingo sales data. Also, it may seem a bit backwards, but in business analysis, price is usually expressed as a function of the number sold, as indicated in the table.

| no. cartons ordered per week, $x$ | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
|---|---|---|---|---|---|---|---|---|---|---|
| $ price per Zingo game, $y = p(x)$ | 14.50 | 14.00 | 13.50 | 13.00 | 12.50 | 12.00 | 11.50 | 11.00 | 10.50 | 10.00 |

a. When the number of cartons ordered increases from 10 per week to 20 per week, the price per game changes. If we subtract the 10 cartons-per-week price per game from the 20 cartons-per-week price per game, is the difference positive or negative? What is the difference? What does the positive or negative sign on the number tell you about how the price per game changes as the number of cartons ordered increases from 10 to 20?

b. Calculate the average rate of change of the price per game with respect to the number of cartons ordered as $x$ increases from 10 to 20 . What are the units of measure for this average rate of change?

c. Calculate the average rate of change of the price per game with respect to the number of cartons ordered for the increase from 20 to 40 cartons ordered per week, for the increase from 40 to 70 cartons ordered per week, and for the increase from 50 to 100 cartons ordered per week.

d. Based on the data in the table, is the average rate of change of price per game constant for all these increases in the number of cartons ordered, or does it change depending on the particular numbers of cartons ordered?

e. Based on your calculations of average rate of change, determine whether the relationship between $x$ and $p(x)$ is a linear relationship or a non-linear relationship. Explain.

f. How much does the Vee Company need to change the price of a Zingo game to sell one more carton per week? Does your answer depend on how many cartons are currently being sold? Explain.

g. The function $p$ can be viewed as a finite sequence with 250 terms. Explain why and relate this observation to the domain of $p$. What does the value of the $n^{th}$ term mean?

h. Write a formula to calculate $p(x)$. What information from your answers above do you need in order to find this formula? To what values of $x$ does the formula apply?

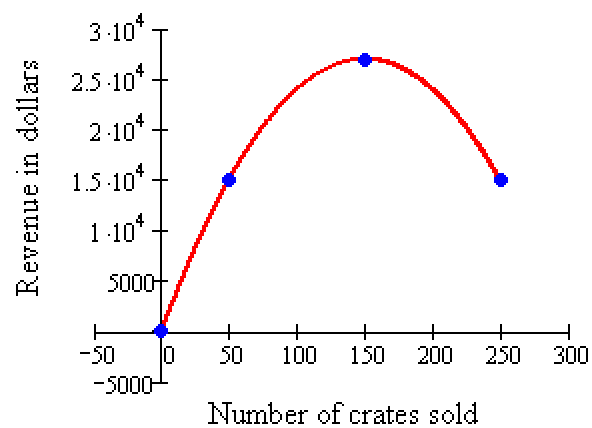
i. Graph the equation $y = p(x)$ for the domain $0 \le x \le 250$. Is this the graph of the function $p$? Explain why or why not.

4. In business, the term *revenue* is used to indicate the money a company receives for sales of its products. In this context, revenue is a function of the number of items sold. The Vee Company's financial analyst has determined that its revenue, in dollars, for sales of the Zingo game is given by the function $R$ with the formula

$$R(x) = 360x - 1.20x^2,$$

where $x$ is the number of cartons sold, and $R(x)$ is revenue measured in dollars.

a. The graph of the function $R$ is shown at the right. Is the relationship between $x$ and $R(x)$ a linear relationship or a non-linear relationship? Explain.




b. Is the average rate change of revenue with respect to number of cartons sold per week constant or changing?

c. Using the formula for the price function in item 3, part h, find the price per Zingo game when 150, 200, and 250 cartons are sold per week. Then, find the price per carton when 150, 200, and 250 cartons are sold per week. Do these values agree with the values of the revenue function when 150, 200, and 250 cartons are sold per week? Explain.

d. When the number of cartons sold per week increases from 50 to 150, is the average rate of change of revenue positive or negative? Explain how to use the graph to find the answer without actually calculating this average rate of change.

e. When the number of cartons sold per week increases from 150 to 250, is the average rate of change of revenue positive or negative? Explain how to use the graph to find the answer without calculating the value.

f. What are the units of measure for the average rate of change of revenue with respect to number of cartons sold per week?

g. Calculate the average rate of change of revenue as $x$ increases from 50 to 150 and the average rate of change of revenue as $x$ increases from 150 to 250. What is the relationship between these values? How is this shown in the graph?

h. The average rate of change of revenue as $x$ increases from 100 to 200 is 0. What feature of the graph causes this to be so? Explain and give other examples of the same phenomenon.

# Atlanta Public Schools

# Teacher's Curriculum Supplement

# Mathematics I: Unit 1

# Task 6: Southern Yard and Garden

## Task 6: Southern Yard and Garden Day 1/2

(GaDOE TE #1, #2, and #3)

**MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.**

b. Graph the basic functions $f(x) = x^n$ where $n = 1$ to 3, $f(x) = \sqrt{x}$ , $f(x) = |x|$, and $f(x) = 1/x$.

c. Graph transformations of basic functions including vertical shifts, stretches, and shrinks, as well as reflections across the *x*- and *y*-axes. [*Previewed in this unit*.]

d. Investigate and explain the characteristics of a function: domain, range, zeros, intercepts, intervals of increase and decrease, maximum and minimum values, and end behavior.

e. Relate to a given context the characteristics of a function, and use graphs and tables to investigate its behavior.

**New Vocabulary:** inverse function

**Mathematical concepts/topics:** $f(x) = \frac{1}{x}$, perimeter, area, symmetry, domain, range, intervals of increase and decrease

**Prior knowledge:** perimeter, area

**Essential Question(s):** How can functions and their graphs help us analyze real-life situations?

**Suggested materials:** graph paper, graphing calculators

**Warm-up:** Sketch and label three different rectangular gardens that have an area of 1,000 square feet. Label the length and width of each garden and find the perimeter.

**Opening:** Have students discuss the warm-up, sharing multiple solutions. Ask how many solutions they think there might be. Read the introduction to the task and to problem 1. Ask students what they think are the important facts in the problem. Make sure they understand the situation.

**Worktime:** Students should do 1-3 of the GaDOE TE. Note that in Fences and Functions, students were given a fixed perimeter and asked to investigate area using a quadratic function. In this task, students are given a fixed area and are investigating perimeter using an inverse function, $f(x) = k/x$.

Encourage students to use grid paper to represent the situations in various parts of the problem.

**Closing:** Have students present their work, making sure that ALL questions are explored and answered. (See teacher notes.)

**Homework:** Homework follows the Day 1 Student Task and reinforces the graphing of the inverse function and transformations of the inverse function. It is very important that students understand why zero cannot be included in the domain of the inverse function. Be sure students understand why division by zero is undefined!

**Differentiated support/enrichment:**

**Resources/materials for Math Support:**

# Southern Yard and Garden

**Day 1 Student Task**

In this task, you will investigate functions whose formulas involve the algebraic expressions $\frac{1}{x}$ and $\sqrt{x}$. You will do so by considering activities of Southern Yard and Garden, a Georgia-owned business that produces grass sod, garden plants, and trees for sale to nurseries and landscaping companies.

Last year, the Research and Development (R&D) Team of the Grass Division at Southern Yard and Garden (SYnG – pronounced "sin gee") was ready to plant test plots of several new drought resistant grasses. The R&D Team wanted each new grass variety planted in five different test plots at the SYnG experiment farm.

The R&D Team had specific requirements for the plots.

- To assure comparable data, each plot must be the same area, 1200 square feet.
- To assure consideration of a variety of soil and sun/shade conditions, the plots must not be adjacent.
- To prevent any grazing by wildlife in the area, each plot must be surrounded by a special fence.

When the plans were sent to the Chief Financial Officer (CFO) for approval, she concluded that a separate fence for each grass test plot would be a major expense and asked the R&D Team how much fencing would be needed. The R&D Team sent a quick email that the length of fencing needed would depend on the perimeter of the plot, and the CFO replied with a request that they provide a detailed analysis of the possible perimeters for the test plots.

1. Before the CFO's request for an analysis of possible perimeters, the R&D Team had planned to use rectangular plots 24 feet wide and 50 feet long. They chose these dimensions because they would facilitate cutting of the sod for sale. All SYnG machinery is set to cut sod in 3' by 1' rectangular sections for sale to nurseries and landscaping companies.

   a. Verify that a 24' by 50' plot has the required area.
   b. How can such a plot be cut into 3' by 1' rectangular sections so that there are no leftover pieces of sod? How many of these sections will be produced in a 24' by 50' plot?
   c. How much fencing is needed for a 24' by 50' grass test plot?

2. The Maintenance Division of SYnG is responsible for maintaining all experimental plots of plants, grass, and tree varieties. The R&D Team decided to consult the Maintenance Division as a first step in developing the analysis for the CFO. The mowing crew cuts the grass in the test plots several times before the sod is established and the gardeners monitor the use of water, fertilizer, and herbicides. So R&D asked the mowers and gardeners for input. After working with diagrams of mowing patterns for their 60-inch blade mowers, the mowing crew recommended that the plots be by 25' by 48' and that the fences have gates in opposite corners so that they could enter in one corner, cut five 60-inch strips and exit at the

opposite corner. The gardeners suggested that the plots be 12' by 100' to simplify use of existing 12-foot wide equipment currently in use for watering and applying liquid fertilizers.

a. Use grid paper to represent a 25' by 48' rectangle and the mowing crew's plans for cutting the grass.

b. Verify that a 25' by 48' rectangle has the required area and find its perimeter.

c. Using the same scale from part a, represent a 12' by 100' rectangle on your grid paper.

d. Verify that a 12' by 100' rectangle has the required area and find its perimeter.

3. The R&D Team at SYnG let $x$ represent one dimension of a rectangle that has area 1200 square feet, let $y$ represent the other dimension, and explored the relationship between the two variables.

a. Why is the relationship a functional relationship? Does it matter which dimension you consider to be the input and which to be the output? Explain.

b. Find a formula to write $y$ as a function of $x$.

c. Use your formula to find the value of $y$ when $x$ = 5, 10, 15, 20, 30, 35, 40, 60, 120, 240. Organize your results in a table, and add the possible values for $x$ and $y$ from item 2, parts b and d.

d. Because the 60-inch mowers must be able to cut the test plots, the smaller dimension of the rectangle must be at least 5'. In this context, what is the domain for the function from part b? Explain your reasoning.

e. **Use technology** to draw a graph of the function from part b. Use the same scale on each axis. The graph has a line of symmetry that is neither horizontal nor vertical. Where is the line?

f. The endpoints of the graph drawn for part e are reflections through the line of symmetry. What do you notice about the relationship between the coordinates of these points? Examine your table of values. Find another pair of points with this relationship between the coordinates. Are these points reflections of each other through the line of symmetry?

g. There, a value of $x$ such that $x$ and $y$ are the same. What is the corresponding point on your graph? What special shape does the rectangle have when $x = y$?

h. For what $x$-values does the $y$-value increase as $x$-increases? For what $x$-values does the $y$-value decrease as $x$-increases? Explain you answer based on the graph and the formula.

# Southern Yard and Garden
**Day 1 Homework**

In the case of each function below, the domain is all real numbers for which the function is defined. For each function, you are to do the following:

- Complete a table of 10 input/output pairs. When choosing ordered pairs to use in graphing a function, it is always good to choose different kinds of numbers in the domain of the function as input values, including positive numbers, negative numbers, integers, and fractions. After you have found 10 ordered pairs for a function, graph the function on graph paper.
- When you have finished graphing all of the functions, state the domain and the range of each function.
- Compare the graphs of the functions. In your comparison, discuss how the graphs are alike and how they are different. Be specific.
- Explain *what* about the algebraic representations for the functions accounts for the differences in their graphs.

1. $f(x) = 1/x$
2. $f(x) = 8/x$
3. $f(x) = 1/x + 8$

# Task 6: Southern Yard and Garden Day 2/2

(GaDOE TE #6 and #7)

**MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.**

b. Graph the basic functions $f(x) = x^n$ where $n = 1$ to 3, $f(x) = \sqrt{x}$, $f(x) = |x|$, and $f(x) = 1/x$.

c. Graph transformations of basic functions including vertical shifts, stretches, and shrinks, as well as reflections across the $x$- and $y$-axes. [*Previewed in this unit.*]

d. Investigate and explain the characteristics of a function: domain, range, zeros, intercepts, intervals of increase and decrease, maximum and minimum values, and end behavior.

e. Relate to a given context the characteristics of a function, and use graphs and tables to investigate its behavior.

**New Vocabulary:** square root function

**Mathematical concepts/topics:** $f(x) = \sqrt{x}$, domain, range, function notation, transformations of $f(x) = \sqrt{x}$, equal functions, sequences

**Prior knowledge:** finding and estimating the square roots of positive real numbers

**Essential Question(s):** How can functions and their graphs help us analyze real-life situations?

**Suggested materials:** graph paper, calculators

**Warm-up:** Have students find or estimate the following square roots, without a calculator.

$\sqrt{25}$, $\sqrt{144}$, $\sqrt{(81/16)}$, $\sqrt{.0036}$, $\sqrt{10}$, $\sqrt{60}$

**Opening:** Have students discuss their solutions to the warm-up. Ask them to share their methods for estimating $\sqrt{10}$ and $\sqrt{60}$.

Read the introduction to problems 6 and 7 and the scenario for problem 6. Make sure students understand the situation.

**Worktime:** Students should do 6 and 7 of the GaDOE TE. In problem 7, students are asked to find the length of each cross tie. This is a bit tricky. As you monitor student work, if students are struggling, encourage them to look carefully at the picture. (See teacher notes.)

**Closing:** Have students present their work, making sure that ALL questions are explored and answered. (See teacher notes.)

**Homework:**

**Differentiated support/enrichment:**

**Resources/materials for Math Support:**

# Southern Yard and Garden

**Day 2 Student Task**

The R&D Team at SYnG is also trying to develop new hardier and more drought tolerant varieties of flowering plants for use in landscaping. The experimental plants are studied for one growing season and are set in square raised beds with one plant for every square foot of area within the beds. Previously, the number of plats had been restricted to those sizes which allowed for a square arrangement of the plants, similar to the pattern of dots considered in item 5 of the "Sequences as Functions" learning task. Last summer, the Maintenance Division, which is responsible for building the raised beds as well as maintaining them, asked for permission to experiment with a variety of sizes for the beds. They pointed out that as long as the number of plants was the same as the number of square feet enclosed by the raised box, they could find an arrangement that gave each plant a square foot of area, although that area for each plant would not necessarily have a square shape.

6. The R&D Team agreed to allow the Maintenance Division to experiment with the size of the raised beds. Pleased with the chance to experiment, the Maintenance Division decided to build beds for every number of plants from 4 to 400. The length, in feet, of a side of a raised bed is a function of the number of plants that Maintenance will plant in the bed. Name this function *S*. If *n* represents the number of plants in a raised bed, then $S(n) = \sqrt{n}$ .

   a. What domain of values does Maintenance plan to use for the function *S*?

   b. Let *f* be the function defined for all real numbers $x \geq 0$ by the formula
   $$f(x) = \sqrt{x}\ .$$
   Make a table of values for *f* and draw a graph that includes domain values for $0 \leq x \leq 400$ and indicates the shape of the whole graph.

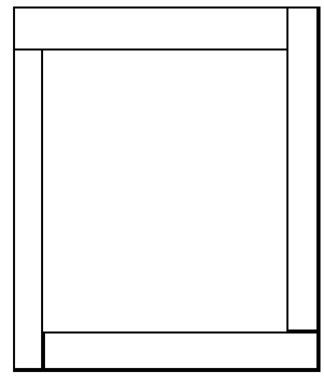
   c. The functions *f* and *S* both take the square root of the input to find the output. Are their domains the same? Are they equal functions? Explain.

   d. How can you use the graph of *f* to obtain the graph of *S*?

   e. The function *S* could be considered as a sequence. Do you think it is helpful to think of the function this way? Why or why not?

7. The Maintenance Division uses recycled railroad cross ties to build the raised beds. They arrange the cross ties as shown in the figure at the right.

a. The end view of each cross tie is a square approximately nine inches on a side. What is the length of the side of a square bed needed for 4 plants? What is the length of each cross tie, with length measured in feet, needed to make the box for this bed?



b. Answer the questions from part a for 9 plants, 16 plants, and 23 plants.

c. The Maintenance Division needs a cross-tie length function that inputs the number of plants, $n$, and outputs the length $L(n)$ of the cross ties needed to build the raised bed. Write the formula to calculate $L(n)$ given $n$.

d. How can you use the graph of $f$ from item 6, part b, to draw the graph of $L$, the cross-tie length function?

e. Draw a plan for a raised bed for 24 plants. Specify the lengths of the cross-ties and how you would arrange the plants. Explain how you know that your arrangement gives each plant one square foot of growing area.

# Atlanta Public Schools

# Teacher's Curriculum Supplement

# Mathematics I: Unit 1

# Task 7: The Six Basic Functions of Mathematics I

# Mathematics I
# Task 7: The Six Basic Functions of Mathematics I
# Graphing Basic Functions Day 1/1

(Teacher-made activity)

**Standard(s): MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.**

b. Graph the basic functions $f(x) = x^n$ where $n = 1$ to 3, $f(x) = \sqrt{x}$ , $f(x) = |x|$, and $f(x) = 1/x$.

c. Graph transformations of basic functions including vertical shifts, stretches, and shrinks, as well as reflections across the $x$- and $y$-axes. [*Previewed in this unit*.]

**New vocabulary:** transformation, vertical shift, stretch, shrink, reflection

**Mathematical concepts/topics:** graphs of the six basic functions studied in Math I; transformations of the six basic functions, including vertical shifts, stretches, and shrinks; function notation; domain; range

**Prior knowledge:** $y = kx$, $y = k/x$, linear functions, transformations of figures,

**Essential question(s):** How does the algebraic representation (or rule) for a function affect its graph?

**Suggested materials:** graph paper

**Warm-up/Opening:** Have the following posted as students enter the room.

> *Write your own linear function, using function notation. Graph your function. How does the graph of your function compare to the graph of $f(x) = x$ ?*

**Opening:** Allow several students to share the linear functions they chose during the warm-up. Ask them to compare the graphs of their functions to the graph of f(x) = x. Ask them what about their functions cause the similarities and the differences between the graphs.

**Worktime:** Students should complete the teacher-made activity provided with this lesson. Encourage students to use zero, positive numbers, negative numbers, integers and fractions as input values when possible. It is important that students graph the functions given in this task by hand.

**Closing:** As students share their graphs, be sure they discuss the fact that all the graphs in a group have the same basic shape. They should state that graphs are shifted up or down a specific number of units. They should also state that they are stretched or shrunk by a specific factor. Ask students for the domains and ranges of their graphs as they share. Make sure students understand *why* zero is not included in the domains of functions of the form $f(x) = k/x$ and negative numbers are not included in the domains of functions of the form $f(x) = \sqrt{x}$.

Students should be able to describe the sets of functions as being linear, quadratic, cubic, and as transformations of the absolute value function, the square root function, or the inverse function. Transformations will be addressed again in Unit 5 of this course. It is not necessary to drill students excessively on this material at this point. Please note that horizontal shifts, stretches, and shrinks are not tested in Mathematics I.

**Homework:** Work should address basic functions and their transformations.

**Differentiated support/enrichment:**

**Check for understanding:**

**Math Support:** Work should address basic functions and their transformations.

# Mathematics I

## The Six Basic Functions of Mathematics I

In Mathematics I, you will study six basic functions. You have already been introduced to most of these functions by completing the tasks in this unit. In each of the problems below, you are given one of these basic functions and one or more related functions. For each group of functions, you are to do the following:

- Complete a table of 10 input/output pairs for each function. When choosing ordered pairs to use in graphing a function, it is always good to choose different kinds of numbers from the domain of the function as input values, including zero, positive numbers, negative numbers, integers and fractions, if possible. After you have found 10 ordered pairs for a function, graph the function on graph paper.

- When you have finished graphing all of the functions in a set, compare the graphs of the functions. In your comparison, include the domain and range for each function. Discuss how the graphs are alike and how they are different. Be specific.

- Explain *what* about the algebraic representations for the functions in each set accounts for the differences in their graphs.

1. a. $f(x) = x$ b. $g(x) = x + 3$ c. $h(x) = 3x$
2. a. $f(x) = x^2$ b. $g(x) = x^2 – 2$ c. $h(x) = 2x^2$
3. a. $f(x) = x^3$ b. $g(x) = x^3 + 4$ c. $h(x) = .5x^3$
4. a. $f(x) = 1/x$ b. $g(x) = 4/x$
5. a. $f(x) = |x|$ b. $g(x) = - 3 |x|$
6. a. $f (x) = \sqrt{x}$ b. $g(x) = 2\sqrt{x} + 3$