Atlanta Public Schools

Teacher's Curriculum Supplement

Mathematics I: Unit 2

This document has been made possible by funding from the GE Foundation Developing Futures grant, in partnership with Atlanta Public Schools. It is derived from the Georgia Department of Education Math I Frameworks and includes contributions from Georgia teachers. It is intended to serve as a companion to the GA DOE Math I Frameworks Teacher Edition. Permission to copy for educational purposes is granted and no portion may be reproduced for sale or profit.

Preface

We are pleased to provide this supplement to the Georgia Department of Education's Mathematics I Framework. It has been written in the hope that it will assist teachers in the planning and delivery of the new curriculum, particularly in this first year of implementation. This document should be used with the following considerations.

- The importance of working the tasks used in these lessons cannot be overstated. In planning for the teaching of the Georgia Performance Standards in Mathematics teachers should work the task, read the teacher notes provided in the Georgia Department of Education's Mathematics I Framework Teacher Edition, and *then* examine the lessons provided here.

- This guide provides day-by-day lesson plans. While a detailed scope and sequence and established lessons may help in the implementation of a new and more rigorous curriculum, it is hoped that teachers will assess their students informally on an on-going basis and use the results of these assessments to determine (or modify) what happens in the classroom from one day to the next. Planning based on student need is much more effective than following a pre-determined timeline.

- It is important to remember that the Georgia Performance Standards provide a balance of concepts, skills, and problem solving. Although this document is primarily based on the tasks of the Framework, we have attempted to help teachers achieve this all important balance by embedding necessary skills in the lessons and including skills in specific or suggested homework assignments. The teachers and writers who developed these lessons, however, are not in your classrooms. It is incumbent upon the classroom teacher to assess the skill level of students on every topic addressed in the standards and provide the opportunities needed to master those skills.

- In most of the lesson templates, the sections labeled *Differentiated support/enrichment* have been left blank. This is a result of several factors, the most significant of which was time. It is also hoped that as teachers use these lessons, they will contribute their own ideas, not only in the areas of differentiation and enrichment, but in other areas as well. Materials and resources abound that can be used to contribute to the teaching of the standards.

On the topic of differentiation, it is critical to reiterate that many of the strategies used in a standards-based mathematics classroom promote differentiation. These strategies include

- the use of rich tasks with multiple points of entry and more than one path to a solution
- flexible grouping of students
- multiple representations of mathematical concepts
- writing in mathematics
- monitoring of progress through on-going informal and formative assessments
- and analysis of student work.

We hope that teachers will incorporate these strategies in each and every lesson.

It is hoped that you find this document useful as we strive to raise the mathematics achievement of all students in our district and state. Comments, questions, and suggestions for inclusions related to the document may be emailed to Dr. Dottie Whitlow, Executive Director, Mathematics and Science Department, Atlanta Public Schools, dwhitlow@atlantapublicschools.us

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Explanation of the Terms and Categories Used in the Lesson Template

Task: This section gives the suggested number of days needed to teach the concepts addressed in a task, the task name, and the problem numbers of the task as listed in the Georgia Department of Education's Mathematics I Framework Teacher Edition (GaDOE TE).

In some cases new tasks or activities have been developed. These activities have been named by the writers.

Standard(s): Although each task addresses many Math I standards and uses mathematics learned in earlier grades, in this section, only the key standards addressed in the lesson are listed.

New Vocabulary: Vocabulary is only listed here the first time it is used. It is strongly recommended that teachers, particularly those teaching Math Support, to use interactive word walls. Vocabulary listed in this section should be included on the word walls.

Mathematical concepts/topics: Here are listed the major concepts addressed in the lesson whether they are Math I concepts or were addressed in earlier grades.

Prior knowledge: Prior knowledge includes only those topics studied in previous grades. It does not include Math I content taught in previous lessons.

Essential Question(s): Essential questions may be daily and/or unit questions.

Suggested materials: In an attempt to list all materials that will be needed for the lesson, including consumable items, such as graph paper, and tools, such as graphing calculators and compasses. This list did not include those items that should always be present in a standards-based mathematics classroom such as markers, chart paper and rulers.

Warm-up: A suggested warm-up is included with every lesson. Warm-ups should be brief and should focus student thinking on the concepts that are to be addressed in the lesson.

Opening: Openings should set the stage for the mathematics to be done during the work time. The amount of class time used for an opening will vary from day-to-day but this should not be the longest part of the lesson.

Worktime: The problem numbers have been listed and the work that students are to do during the work time have been described. A student version of the day's activity follows the lesson template in every case. In order to address all of the standards in Math I, some of the problems in some of the tasks have been omitted and, in a few instances, substituted less time consuming activities for tasks. In many instances, in the student versions of the tasks, the writing of the original tasks has been simplified. In order to preserve all vocabulary, content, and meaning it is important that teachers work the original tasks as well as the student versions included here.

Teachers are expected to both facilitate and provide some direct instruction, when necessary, during the work time. Some suggestions, related to student misconceptions, difficult concepts, and deeper meaning in this section have been included. However, the teacher notes in the GaDOE Math I Framework are exceptional. In most cases, there is no need to repeat the information provided there. Again, it is imperative that teachers work the tasks and read the teacher notes that are provided in GaDOE support materials.

Questioning is extremely important in every part of a standards-based lesson. We included suggestions for questions in some cases but did not focus on providing good questions as extensively as we would have liked. Developing good questions related to a specific lesson should be a focus of collaborative planning time.

Closing: The closing may be the most important part of the lesson. This is where the mathematics is formalized. Even when a lesson must be continued to the next day, teachers should stop, leaving enough time to "close", summarizing and formalizing what students have done to that point. As much as possible students should assist in presenting the mathematics learned in the lesson. The teacher notes are all important in determining what mathematics should be included in the closing.

Homework: In some cases, additional written homework suggestions are provided or used the homework provided in the GaDOE sample lessons. We hope that you will use your resources, including your textbook, to assign homework related to the lesson that addresses the needs of your students.

Homework should be focused on the skills and concepts presented in class, relatively short (30 to 45 minutes), and include a balance of skills and thought-provoking problems.

Differentiated support/enrichment: On the topic of differentiation, it is critical to reiterate that many of the strategies used in a standards-based mathematics classroom promote differentiation. These strategies include

- the use of rich tasks with multiple points of entry and more than one path to a solution
- flexible grouping of students
- multiple representations of mathematical concepts
- writing in mathematics
- monitoring of progress through on-going informal and formative assessments
- and analysis of student work.

Check for understanding: A check for understanding is a short, focused assessment-a ticket out the door, for example. The Coach Book may be a good resource for these items.

Resources/materials for Math Support: Again, in some cases, we have provided materials and/or suggestions for Math Support. This section should be personalized to your students, class, and/or school, based on your resources.

Table of Contents

Mathematics I
Unit 2

Unit 2 Timeline

Task 1: Tiling	2 days
Task 2: Tiling Pools	2 days
Task 3: I’ve Got Your Number	3 days
Task 4: Paula’s Peaches	5 days
Task 5: Ladder Length	1 day

Task Notes

Task 1: Tiling
Problems #1 - #7 of the GaDOE TE Framework are included in these lessons. We believe that Problem 8 can be omitted without consequence if time is an issue.

Task 2: Tiling Pools
All parts of the original task from GaDOE TE Framework are included in the lesson plans.

Task 3: I've Got Your Number
The problems in this task have been re-ordered to present the special products included in the standards in the following order:

Identity 1: $(x + a)(x + b) = x^2 + (a + b)x + ab$	*Product of Two Binomials*
Identity 2: $(x + y)^2 = x^2 + 2xy + y^2$	*Square of a Sum*
Identity 3: $(x - y)^2 = x^2 - 2xy + y^2$	*Square of a Difference*
Identity 4: $(x + y)(x – y) = x^2 - y^2$	*Product of the Sum and Difference of Two Expressions*
Identity 5: $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$	*Cube of a Sum*
Identity 6: $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$	*Cube of a Difference*

Rearrangement of the identities was done in hopes of streamlining the task. In several cases parts of problems have been deleted or re-written. Of particular note is the fact that multiplication of binomials with terms other than x and a constant has been included in the student task (i.e. $(3x + 2y)(5x – 4y)$).

We strongly suggest the use of algebra tiles and/or area models in the development of the special products included here. Algebra tiles are particularly effective in helping students see that the length $x + a$ is less than x when a is negative. Using the tiles to illustrate specific cases before looking at the general cases in problem 1 is helpful. The process may be reversed for factoring and is illustrated in the lesson plans for Task 4.

Easy examples to illustrate using tiles: $(x + 2)(x +3)$
$(x – 1)(x + 2)$
$(x – 2)(x - 1)$

Task 4: Paula's Peaches
The decision was made by teachers contributing to this supplement to move *Paula's Peaches* to Unit 2 and *Just Jogging* to Unit 5. This decision means that students will address factoring and solving quadratic equations by factoring immediately after multiplication of polynomials. The introduction to rational functions contained in Just Jogging will immediately precede the more in-depth work with rational functions contained in Unit 5 in *The Resistance Task.*

All parts of *Paula's Peaches,* contained in the GaDOE TE, are addressed here. Some problems have been edited to facilitate classroom use and a problem has been added (Problem 8 of the Day 3 Student Task) to more thoroughly address factoring.

Task 5: Ladder Length
All parts of the original task from the GaDOE TE Framework are included in these lesson plans.

Atlanta Public Schools

Teacher's Curriculum Supplement

Mathematics I: Unit 2

Task 1: Tiling

Mathematics I

Task 1: Tiling — Day 1/2

(GA DOE TE #1-4)

Standard(s): **MM1A2. Students will simplify and operate with radical expressions, polynomials, and rational expressions.**

New vocabulary:

Mathematical concepts/topics: patterns, algebraic expressions, equivalent expressions, consecutive integers, consecutive even and consecutive odd integers, square numbers, square roots, sequences

Prior knowledge: patterns, algebraic expressions, equivalent expressions, consecutive integers, consecutive even and consecutive odd integers, square numbers, square roots, sequences

Essential question(s): How can I use algebraic expressions to represent geometric and numerical patterns?

Suggested materials: colored tiles, grid paper,

Warm-up: Post Mario's design. Have pairs or groups of students build Mario's figures using colored tiles or other square objects.

Opening: This task is designed to promote reasoning and discovery and requires little explanation. Read the introduction to problem 1 with students. Make sure students understand the situation by asking questions like: "How many rows does Mario have in his first figure? in the second", etc. Fill out the table in part 1a as a group.

Worktime: Students should do problems 1-4 of the task. As you monitor student progress, be sure that students can explain the number patterns they are discovering in terms of the figures. Encourage different algebraic expressions for the same patterns. Ask students to compare their expressions with the expressions of others. Ask how they are alike and how they are different. Will they give the same results?

Have students present their work on problem 1 before moving on to Latasha's figure. (See teacher notes and comments in the closing section below.)

Have students add the tiles needed to create Latasha's figures. Again answer 2a as a class.

If there is not significant time to finish and present problems 3 and 4 in class, students might work on these problems for homework. Discussion of these problems could serve as a warm-up and opening for day 2 of the task.

Closing: It is important to discuss all parts of each problem. Question 2j has particular significance. (See teacher notes.) Different groups might present different segments of the problems in order to manage time. Choose the groups and their presentations based on how they addressed the questions and to insure that multiple solutions are demonstrated. Any specific insights or discussions that occurred should be shared during presentations. Ask questions like: What made you think to do it that way? Why did you choose this method or representation? Is there another way to explain it? How is your solution different or the same as a previous group? Do they give the same result?

Homework: If not already finished, have students complete problems 3 and 4 of the task.

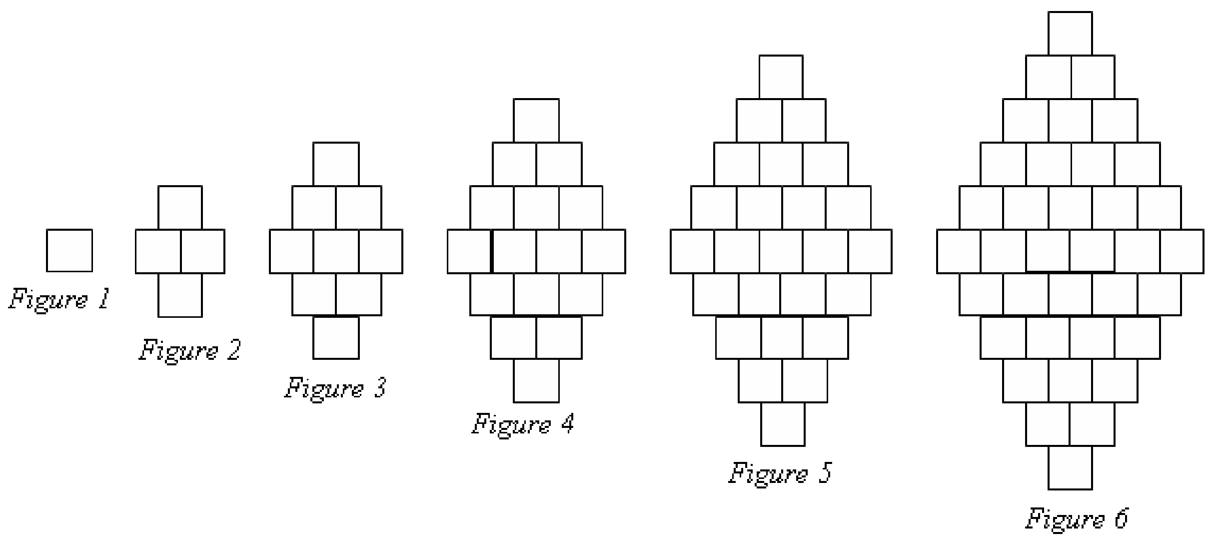
Differentiated support/enrichment:

Resources/materials for Math Support: Vocabulary to be previewed should include integer, even integer, odd integer, consecutive integers, square numbers, square roots. Concepts should include simple patterns, simple algebraic expressions, and the distributive property.

Mathematics I
Tiling
Day 1 Student Task

Latasha and Mario are high school juniors who worked as counselors at a day camp last summer. One of the art projects for the campers involved making designs from colored one-square-inch tiles. As the students worked enthusiastically making their designs, Mario noticed one student making a diamond-shaped design and wondered how big a design, with the same pattern, that could be made if all 5000 tiles available were used. Later in the afternoon, as he and Latasha were putting away materials after the children had left, he mentioned the idea to Latasha. She replied that she saw an interesting design too and wondered if he were talking about the same design. At this point, they stopped cleaning up and got out the tiles to show each other the designs they had in mind.

Mario presented the design that interested him as a sequence of figures as follows:



1. To make sure that you understand the design that was of interest to Mario, answer the following questions.

   a) How many rows of tiles are in each of Mario's figures?

Figure number						
Number of rows						

   b) What pattern do you observe that relates the number of rows to the figure number? Explain in a sentence.

c) Use this pattern to predict the number rows in Figure 12, Figure 47,and Figure 111 if these figures were to be made.

d) Write an algebraic expression for the number of rows in Figure *k*. Explain why your pattern will always give the correct number of rows in Figure *k*. Can your expression be simplified? If so, simplify it.

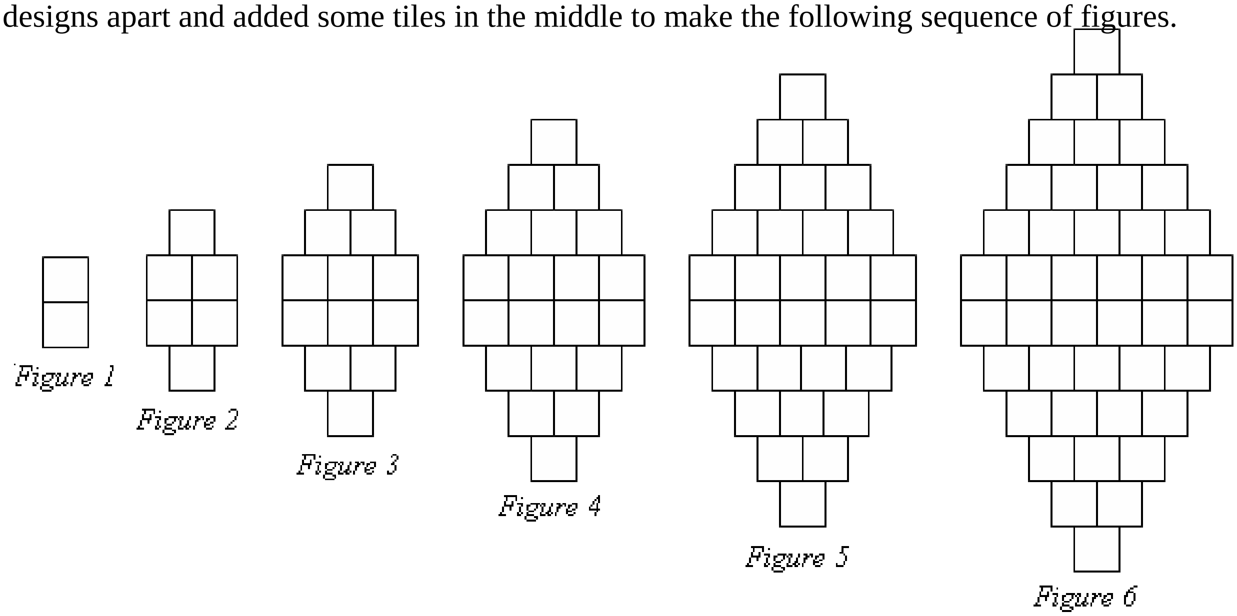
e) What is the total number of tiles in each figure above?

f) What pattern do you observe that relates the total number of tiles to the figure number? Explain in a sentence.

g) Use this pattern to predict the total number of tiles in Figure 12, Figure 47, and Figure 111 if these figures were to be made.

h) Write an algebraic expression for the total number of tiles in Figure *k*. Explain why your pattern will always give the correct total number of tiles in Figure *k*.

When Latasha saw Mario's figures, she realized that the pattern Mario had in mind was very similar to the one that caught her eye, but not quite the same. Latasha pushed each of Mario's designs apart and added some tiles in the middle to make the following sequence of figures.



2. Answer the following questions for Latasha's figures.

a) How many rows of tiles are in each of the figures above?

Figure number						
Number of rows						

b) What pattern do you observe that relates the number of rows to the figure number? Explain in a sentence.

c) Use this pattern to predict the number rows in Figure 12, Figure 47, and Figure 111 if these figures were to be drawn.

d) Write an algebraic expression for the number of rows in Figure *k*. Explain why your pattern will always give the correct number of rows in Figure *k*. Can your expression be simplified? If so, simplify it.

e) What is the total number of tiles in each figure above?

f) What pattern do you observe that relates the total number of tiles to the figure number? Explain in a sentence.

g) Use this pattern to predict the total number of tiles in Figure 12, Figure 47, and Figure 111 if these figures were to be drawn.

h) Write an algebraic expression for the total number of tiles in Figure *k*. Explain why your pattern will always give the correct total number of tiles in Figure *k*.

i) Give a geometric reason why the number of tiles in Figure *k* is always an even number. Look at the algebraic expression you wrote in 2d.

j) Give an algebraic explanation of why this expression always gives an even number. [Hint: If your expression is not a product, use the distributive property to rewrite it as a product.}

3. Mario started the discussion with Latasha wondering whether he could make a version of the diamond pattern that interested him that would use all 5000 tiles that they had in the art supplies. What do you think? Explain your answer. If you can use all 5000 tiles, how many rows will the design have? If a similar design cannot be made, what is the largest design that can be made with the 5000 tiles, that is, how many rows will this design have and how many tiles will be used?

4. What is the largest design in the pattern Latasha liked that can be made with no more than 5000 tiles? How many rows does it have? Does it use all 5000 tiles? Justify your answers.

Mathematics I
Task 1: Tiling **Day 2/2**
(GA DOE TE #5-7)

Standard(s): **MM1A2. Students will simplify and operate with radical expressions, polynomials, and rational expressions.**

New vocabulary: triangular numbers

Mathematical concepts/topics: patterns, algebraic expressions, equality of algebraic expressions, equations, sequences, triangular numbers, the sum of the first *k* positive integers

Prior knowledge: integers, patterns, algebraic expressions, equivalent expressions, equilateral triangles

Essential question(s): How can I use find the sum of the first *k* positive numbers?

Suggested materials: graph paper, isometric dot paper

Warm-up: Have partners compare their work on problems 3 and 4.

Opening: Have students share any remaining parts of problems 3 and 4 from the previous lesson. As an introduction to problem 5, discuss the meaning of the notation M_1, $M_{2, \ldots}$ $L_{1,}$ L_2, etc.

Worktime: Students should do problems 5-7 of the task. Some students may benefit from drawing representations of the triangular numbers on isometric dot paper.

Closing: It is important to discuss all parts of each problem. After students have presented their work, you may want to share the story of Gauss and challenge students to develop the formal proof based on Gauss's idea.

Homework:

Differentiated support/enrichment: This enrichment problem can be done any time after problem 5 has been completed.

Mario said that the relationship between M_k (the number of tiles in his k^{th} figure) and L_k (the number of tiles in Latasha's k^{th} figure) can be represented by the expression $M_k = k/(k + 1)*L_k$. Is he right? Why or why not?

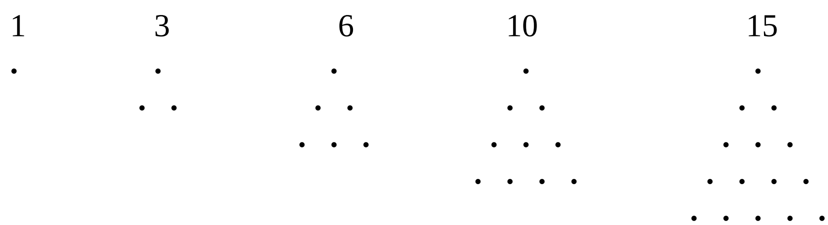
Resources/materials for Math Support: Preview evaluating and simplifying expressions, particularly those containing exponents, parentheses, and division of a binomial by a monomial. Similar expressions can be seen on page 19 of Unit 2 of the GaDOE TE.

Mathematics I
Tiling
Day 2 Student Task

Let M_1, M_2, M_3, M_4, and so forth represent the sequence of numbers that give the total number of tiles in Mario's sequence of figures.
Let L_1, L_2, L_3, L_4 and so forth represent the sequence of numbers that give the total number of tiles in Latasha's sequence of figures.

5. Write an equation that expresses each of the following:
   a) the relationship between L_1 and M_1
   b) the relationship between L_2 and M_2
   c) the relationship between L_3 and M_3
   d) the relationship between L_4 and M_4
   e) the general relationship between L_k and M_k, where k can represent any positive integer.

Triangular numbers are positive integers such that the given number of dots can be arranged in an equilateral triangle. The first few triangular numbers are as follows.



Let T_1, T_2, T_3, T_4, and so forth represent the sequence of triangular numbers.

6. a) Examine the arrangement of dots for T_4. How many dots are in row 1? row 2? row 3? row 4?

   b) Write T_4 as a sum of four positive integers.

   c) Write each of T_2, T_3, and T_5 as a sum of 2, 3, and 5 positive integers, respectively.

   d) Explain what sum you would need to compute to find T_{12}. Compute T_{12} doing the addition yourself.

   e) Explain what sum you would need to compute to find T_{47}. Use technology to find T_{47}.

7. The triangular numbers could also be represented by arrangements of square tiles instead of dots.
   a) Draw the first few figures to represent triangular numbers using square tiles.

   b) Compare these figures to Latasha's diamond-shaped figures. Write a sentence comparing the triangular-number figures and Latasha's figures.

   c) Write an equation using L_k and T_k to express the relationship between the number tiles in the *k*-th one of Latasha's figures and the k^{th} triangular number figure.

   d) Check your equation by comparing the numbers L_1 and T_1, L_2 and T_2, L_3 and T_3, L_4 and T_4, L_5 and T_5, L_{12} and T_{12}, and L_{47} and T_{47}

   e) Write a formula to calculate T_k, the k^{th} triangular number, without summing the first *k* positive integers.

   f) Check your formula by using it to calculate T_1, T_2, T_3, T_4, T_5, T_{12}, and T_{47}.

   g) Give a geometric reason why your formula always gives a whole number.

   h) Give an algebraic reason why the formula must always give a whole number.

   i) What sum does your formula calculate?

Atlanta Public Schools
Teacher's Curriculum Supplement
Mathematics I: Unit 2
Task 2: Tiling Pools

Mathematics I

Task 2: Tiling Pools — Day 1/2

(GA DOE TE #1-9)

Standard(s): MM1A2. Students will simplify and operate with radical expressions, polynomials, and rational expressions.
c. Add, subtract, multiply, and divide polynomials.
g. Use area and volume models for polynomial arithmetic.

New vocabulary:

Mathematical concepts/topics: area, equivalent algebraic expressions, field properties (associative, commutative, and distributive), proof, evaluating algebraic expressions, simplifying algebraic expressions

Prior knowledge: area; area models; associative, commutative, and distributive properties; evaluating and simplifying simple algebraic expressions

Essential question(s): How can I use algebraic expressions to represent real situations and make decisions?

Suggested materials: colored tiles, grid paper

Warm-up: Post the rectangle given in problem 1. Ask students to do the following:

- Write two different but equivalent expressions for the total area of the rectangle.
- Explain how the diagram and expressions illustrate the Distributive property.

Opening: Discuss the warm-up. Give students problem 2 to be sure that all are comfortable with using area models and with the Distributive Property. Answer any questions.

Have students read the introduction to problem 3 silently and then ask a student or students to describe the situation. Question students until all important facts are discussed. (*Read, Write, Talk* would be a good strategy to use here and in many openings.)

Worktime: Students should do problems 3-9 of the task. Make sure that grid paper and/or colored tiles are available to help students visualize the situation.

In problem 4, students are asked to "find a different but equivalent formula that can be used to calculate *n*." Several different approaches to this problem are described in the teacher notes including the use of geometric reasoning, numerical patterns, and the algebra of lines. During the work time, encourage and look for different approaches and representations that can be shared during the closing.

Closing: Allow students to share their work. Problem 4 provides an excellent opportunity to share multiple approaches and representations as well as examine equivalent expressions. (See teacher notes.)

Homework:

Differentiated support/enrichment: This enrichment problem can be used any time after problem 9 of the task.

> *Write an equation for finding the number N of border tiles needed to put a border that is k tiles wide around a pool that is L feet long and W feet wide. Explain, with diagrams, how you found your expression.*

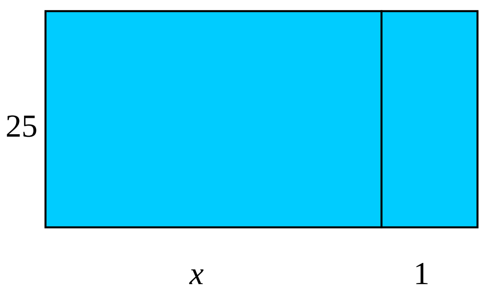
Resources/materials for Math Support: Students may need practice evaluating and simplifying *algebraic* expressions. Provide expressions for which evaluating and/or simplifying requires use of the distributive, associative, and commutative properties; collection of like terms; the division of a polynomial by a constant; and addition of algebraic expressions with unlike, numeric denominators. Expressions should be similar to those found throughout the teacher notes for this task.

Mathematics I
Tiling Pools
Day 1 Student Task

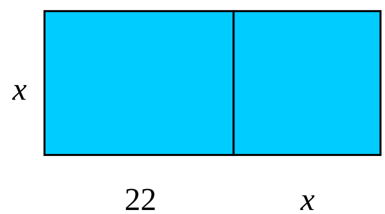
In this task, you will continue to explore how different ways of reasoning about a situation can lead to algebraic expressions that are different but equivalent to each other. We will use swimming pools as the context throughout this task.

In the figures below there are diagrams of swimming pools that have been divided into two sections. Swimming pools are often divided so that different sections are used for different purposes such as swimming laps, diving, area for small children, etc.
(a) For each pool, write two different but equivalent expressions for the total area.
(b) Explain how these diagrams and expressions illustrate the Distributive Property.

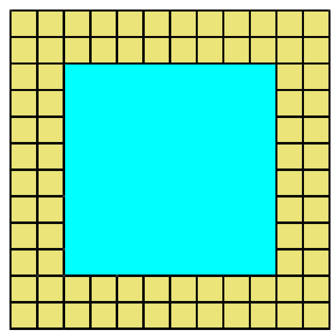
1.



2.



In-ground pools are usually surrounded by a waterproof surface such as concrete. Many homeowners have tile borders installed around the outside edges of their pools to make their pool area more attractive. Superior Pools specializes in custom pools for residential customers and often gets orders for square pools of different sizes. The diagram at the right shows a pool that is 8 feet on each side and is surrounded by two rows of square tiles. Superior Pools uses square tiles that are one foot on each side for all of its tile borders.



The manager at Superior Pools is responsible for telling the installation crew how many border tiles they need for each job and needs an equation for calculating the number of tiles needed for a square pool depending on the size of the pool. Let N represent the total number of tiles needed when the length of a side of the square pool is s feet and the border is two tiles wide.

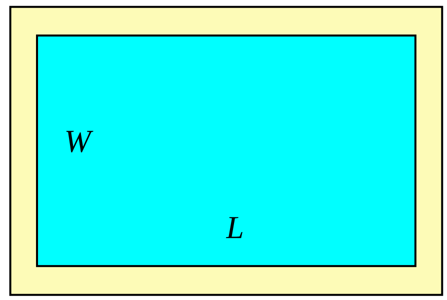
3. Write a formula in terms of the variable s that can be used to calculate N.

4. Write a different but equivalent formula that can be used to calculate N.

5. Give a geometric explanation of why the two different expressions in your formulas for the number of border tiles are equivalent expressions. Include diagrams.

6. Use the Commutative, Associative, and/or Distributive properties to show that your expressions for the number of border tiles are equivalent.

Some customers who have pools installed by Superior Pools want larger pools and choose a rectangular shape that is not a square. Many of these customers also choose to have tile borders that are 2 tiles wide.



7. How many 1-foot square border tiles are needed to put a two-tile-wide border around a pool that is 12 feet wide and 30 feet long?

8. Write an equation for finding the number N of border tiles needed to put a two-tile-wide border around a pool that is L feet long and W feet wide. Explain, with diagrams, how you found your expression.

9. Explain why the area A of the tile border (in square feet) is the same number as the number of tiles that are needed for the border. Write an equation for finding the area A of the tile border using an expression that is different from but equivalent to the expression used in the equation for N given in answering question 8. Use algebraic properties to show that your expressions for A and N are equivalent.

Mathematics I
Task 2: Tiling Pools   Day 2/2
(GA DOE TE #10-13)

Standard(s): MM1A2. Students will simplify and operate with radical expressions, polynomials, and rational expressions.
c. Add, subtract, multiply, and divide polynomials.
g. Use area and volume models for polynomial arithmetic.

New vocabulary:

Mathematical concepts/topics: area, equivalent algebraic expressions, field properties (associative, commutative, and distributive), proof, counterexample, evaluating algebraic expressions, simplifying algebraic expressions

Prior knowledge: area; area models; associative, commutative, and distributive properties; counterexample; evaluating and simplifying simple algebraic expressions

Essential question(s): How can I use algebraic expressions to represent real situations and make decisions?

Suggested materials: colored tiles, grid paper

Warm-up: Ask students to evaluate the following expression when $x = 3$.

$$2(x-5)^2 + 4\left(\frac{x}{6}+\frac{x}{4}\right)$$

Discuss the warm-up but do not spend more than 5 or 6 minutes on this problem. Students struggling with the arithmetic here can be helped one-on-one as you monitor the work time. The Support teacher should also be alerted to students needing extra help with evaluating simple expressions.

Opening: Have students read the introduction to problem 10 silently and then ask a student or students to describe the situation. Question students until all important facts have been discussed.

Worktime: Students should do problems 10-13 of the task. Make sure that grid paper and colored tiles are available to help students visualize the situation.

Problem 12 ask students to “write a different but equivalent expression for the number of border tiles N” and to explain why the expression is equivalent to the expression in problem 11. In the teacher notes, there are 4 different approaches to this problem utilizing area, lines, tables, and graphs as well as algebraic manipulation. Ask guiding questions to encourage these different approaches and representations so that they can be shared during the work time.

Closing: Allow students to share their work. Problem 12 provides an excellent opportunity to share multiple approaches and representations as well as examine equivalent expressions. (See teacher notes.)

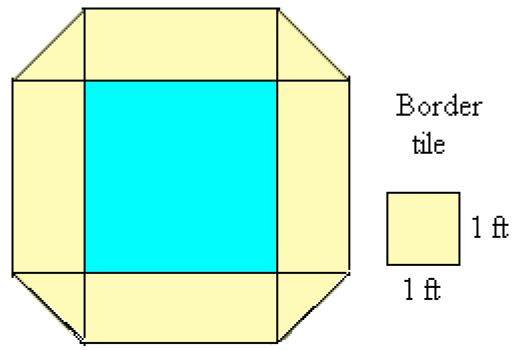
Homework:

Differentiated support/enrichment:

Resources/materials for Math Support: Vocabulary to be previewed for this lesson should include the term *counterexample*. Students may continue to need practice evaluating and simplifying *algebraic* expressions. Provide expressions for which evaluating and/or simpliying requires use of the distributive, associative, and commutative properties; collection of like terms; the division of a polynomial by a constant; and addition of algebraic expressions with unlike, *numeric* denominators. Expressions should be similar to those found throughout the teacher notes for this task.

Mathematics I
Tiling Pools
Day 2 Student Task

A company that sells hot tubs creates a tile border for its products by placing 1-foot-square tiles along the edges of the tub and triangular tiles at the corners as shown. The triangular tiles are made by cutting the square tiles in half along a diagonal.



10. Suppose a hot tub has sides of length 6 feet. How many square tiles are needed for the border?

11. Write an equation for the number of square tiles N needed to create such a border on a hot tub that has sides that are s feet long.

12. Write a different but equivalent expression for the number of border tiles N. Explain why this expression is equivalent to the one given in your answer to question 11.

13. Below are three expressions that some students wrote for the number of tiles needed for the border of a square hot tub with sides s feet long.

(i) $4\left(\frac{s+s+1}{2}\right)$ (ii) $4\left(\frac{s}{2}+\frac{s}{4}\right)+2$ (iii) $(s+2)^2-4\left(\frac{1}{2}\right)-s^2$

(a) Use each expression to find the number of border tiles if $s = 0$.

(b) Do you think that the expressions are equivalent? Explain.

(c) Use each expression to find the number of tiles if $s = 10$. Does this result agree with your answer to part (b)? Explain.

(d) What can you say about testing specific values as a method for determining whether two different expressions are equivalent?

(e) Use algebraic properties to show the equivalence of those expressions in 11, 12, and 13 which are equivalent.

* Adapted from the "Equivalent Expressions" section of *Say It With Symbols: Making Sense of Symbols* in the *Connected Mathematics 2* series.

Atlanta Public Schools

Teacher's Curriculum Supplement

Mathematics I: Unit 2

Task 3: I've Got Your Number

Mathematics I
Task 3: I've Got Your Number — Day 1/3

(GaDOE TE #1, and #2)

Standard(s): MM1A2. Students will simplify and operate with radical expressions, polynomials, and rational expressions.
c. Add, subtract, multiply, and divide polynomials.
g. Use area and volume models for polynomial arithmetic.

New vocabulary: algebraic identities, binomial

Mathematical concepts/topics: multiplying binomials, area, equivalent algebraic expressions, field properties (associative, commutative, and distributive), proof, simplifying algebraic expressions, absolute value, factors and multiples of real numbers

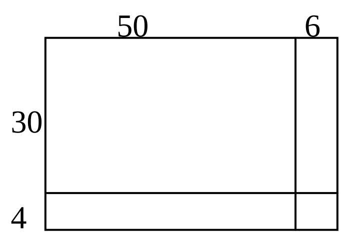
Prior knowledge: area; area models; associative, commutative, and distributive properties; simplifying simple algebraic expressions, absolute value, factors and multiples of real numbers

Essential question(s): How can I use algebraic identities to multiply binomials quickly?

Suggested materials: algebra tiles, grid paper

Warm-up: Ask students to multiply (34)(56) by:

- writing as (30 + 4)(50 + 6) and using the distributive property;
- using an array.



Opening: Discuss the warm-up with students making sure that all understand the use of the distributive property and the area model (array) to multiply (34)(56).

Allow students to work in pairs to investigate the number "trick" and the related questions given at the beginning of the student task. Once they have had time to examine the "trick" and discuss whether they think the algebraic expressions representing Answers A and B are equivalent, discuss the problem.

Extend the opening discussion to include use of the distributive property to multiply $(x + 2)(x + 3)$, Identity 1, and the area model representing $(x + 2)(x + 3) = x^2 + 5x + 6$. All of this material is contained in the student task prior to problem 1.

Make sure all students understand the derivation of Identity 1. Leave Identity 1, $(x + a)(x + b) = x^2 + (a + b)x + ab$, posted so students may refer to it throughout the task.

Worktime: Students should do problems 1 and 2 of the task. The area models in problem 1 may be difficult for some students. Algebra tiles are an excellent vehicle for helping students see lengths $x + a$ when a is a negative number. Make sure that grid paper and/or algebra tiles are available to help visualize the situations. You may want to give students a couple of products with specific values for a and b to illustrate using area models where a and/or b are negative before beginning problem 1. Examples might be $(x + 2)(x - 3)$ and $(x – 4)(x - 2)$.

Closing: Allow students to share their work. Although problems in this task have been re-ordered and simplified, the GaDOE TE teacher notes address all problems included in the student task.

Homework: Practice using Identity 1 to multiply binomials.

Differentiated support/enrichment: For enrichment, have students multiply binomials with terms other than x and constant terms. For example, $(3x + 8)(2x - 7)$ and $(2x – y)(3x + 8y)$.

Resources/materials for Math Support: Make sure students are able to use arrays and the distributive property to multiply two digit numbers such as (45)(37). Provide opportunities to model products of binomials in which a and/or b are negative using algebra tiles. Practice in operating with signed numbers, operating with monomials, and using the rules of exponents is also important (e.g. $x \cdot x = x^2$, $3x + 4x = 7x$, $(4y^2)^3$).

Mathematics I
I've Got Your Number
Day 1 Student Task

Trick or No Trick?

1. Choose a number. _______________
2. Add 2 to your original number. _________
3. Add 3 to your original number. __________
4. Multiply the numbers you got in steps 2 and 3. ____________

Label the number you got in step 4 as Answer A. ____________

6. Square your original number. _______________
7. Multiply your original number by 5. ______________
8. Add 6, the number you got in step 6, and the number you got in step 7. _____________

Label the number you got in step 8 as Answer B. _______________

Are the numbers you have for Answers A and B the same?

Do you think this will always happen no matter what you choose as your original number?
To help answer this question, do the "trick" again. This time use x to represent your original number.
Are the two algebraic expressions representing Answer A and Answer B equivalent? Why or why not?

☺ ☺

In this task, you will learn six identities related to products of binomials. These products are very common in algebra, so common as a matter of fact, that they are referred to as *special products*. Knowing these identities will help you solve algebraic problems with greater ease much like knowing the multiplication facts helps you compute more quickly and easily.

In the scenario above, *Trick or No Trick*, the algebraic expression for Answer A should be:

$$(x + 2)(x + 3)$$

To multiply the number $x + 2$ by the number $x + 3$, we can use the distributive property several times to write a different but equivalent expression.

First treat $(x + 2)$ as a single number but think of $x + 3$ as the sum of the numbers x and 3, and apply the distributive property to obtain:

$$(x + 2)\cdot x \ + \ (x + 2)\cdot 3$$

Now, change your point of view and think of $x + 2$ as the sum of the numbers x and 2, and apply the distributive property to each of the expressions containing $x + 2$ as a factor to obtain:

$$x \cdot x \ + 2\cdot x \ + \ x\cdot 3 \ + \ 2\cdot 3$$

Using our agreements about algebra notation, rewrite as:

$$x^2 \ + 2x \ + \ 3x \ + \ 6$$

Add the like terms $2x$ and $3x$:

$$2x \ + \ 3x = (2 + 3)\,x = 5x$$

The final expression equivalent to $(x + 2)(x + 3)$ is:

$$x^2 + 5x + 6$$

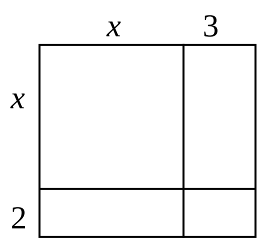
The expression $x^2 + 5x + 6$ is Answer B. We have shown that $(x + 2)(x + 3) = x^2 + 5x + 6$ no matter what number you choose for x. Notice that 5 is the sum of 2 and 3 and 6 is the product of 2 and 3.

The identity $(x + 2)(x + 3) = x^2 + 5x + 6$ is an example of the first special product you will study in this task.

$$(x + a)(x + b) = x^2 + (a + b)x + ab \qquad \text{(Identity 1)}$$

For the remainder of this task, we'll refer to the above identity as Identity 1. Note that we used $a = 2$ and $b = 3$ in our example, but a could represent any real number and so could b.

Area models are an excellent way to illustrate identities representing the product of two binomials. Use the area model below to show why $(x + 2)(x + 3) = x^2 + 5x + 6$.



We stated that Identity 1 will always be true for any real number a and b. That means that a and b could be positive or negative.

1. (a) Each of the diagrams below illustrates Special Product 1. Match each diagram with one of the following cases for a and b.

Case 1: a positive, b positive   Figure _____.

Case 2: a positive, b negative Figure _____ .

Case 3: a negative, b positive Figure ______.

Case 4: a negative, b negative Figure _____.

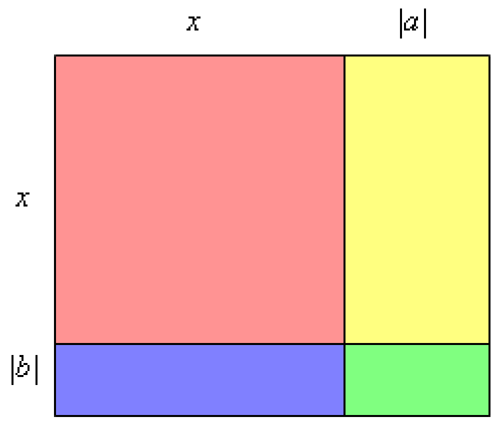


Figure A

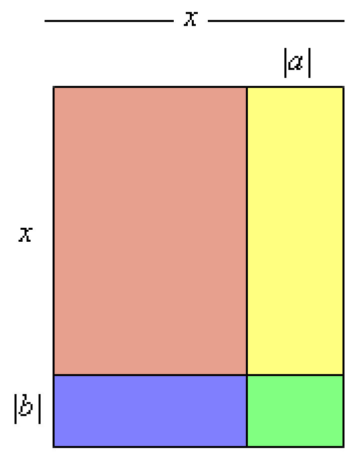


Figure B

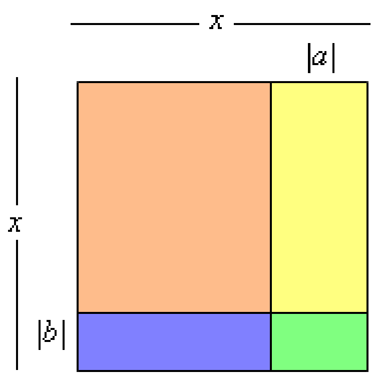


Figure C

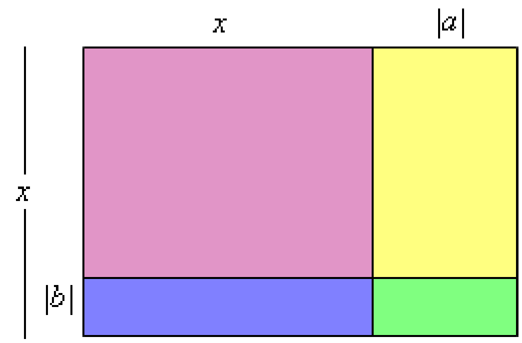


Figure D

(b) Thinking of the case it represents, for each diagram above, find the rectangle whose area is $(x+a)(x+b)$ and use a pencil to put diagonal stripes on this rectangle. Then explain how the diagram illustrates the pattern. Note that when *a* is a negative number, $a = -|a|$, and when *a* is a positive number, then $a = |a|$; and similarly for *b*. Tell two of your explanations to another student and let that student explain the other two to you.

(c) Use Identity 1 to multiply $(x - 3)(x + 4)$. Draw your own diagram, similar to the appropriate Case (1, 2, 3, or 4 above) to illustrate the product.

2. Identity 1 can be used to give an alternate way to multiply two digit numbers that have the same digit in the ten's place. For example, (68)(73) can be thought of as (70 - 2)(70 + 3). Using the distributive property, we get:

$(70 – 2)(70 + 3) = (70 \cdot 70) + (70 \cdot 3) – (2 \cdot 70) – (2 \cdot 3) = 4900 + 210 - 140 – 6 = 4964$

Using Identity 1, with $x = 70$, $a = - 2$, and $b = 3$, we get:

$(70 – 2)(70 + 3) = 70^2 + (- 2 + 3)70 + - 2 \cdot 3 = 4900 + 70 – 6 = 4964$

.

Use Identity 1 to calculate each of the following products.

(a) (52)(57)

(b) (35)(44)

(c) $(x + 7)(x + 8)$

(d) $(x – 4)(x – 5)$

(e) $(x – 2)(x + 8)$

Mathematics I

Task 3: I've Got Your Number — Day 2/3

(GaDOE TE #4, #5, #7, #9, and #10)

Standard(s): MM1A2. Students will simplify and operate with radical expressions, polynomials, and rational expressions.
c. Add, subtract, multiply, and divide polynomials.
g. Use area and volume models for polynomial arithmetic.

New vocabulary: square of a sum, square of a difference

Mathematical concepts/topics: multiplication of binomials, including the square of a sum, the square of a difference, and the product of the sum and the difference of two numbers; area; simplifying algebraic expressions; equivalent algebraic expressions; algebraic identities; field properties (associative, commutative, and distributive); proof

Prior knowledge: area; area models; associative, commutative, and distributive properties; evaluating and simplifying simple algebraic expressions

Essential question(s): How can I use algebraic identities to multiply binomials quickly and easily?

Suggested materials: algebra tiles, grid paper

Warm-up: Have students review Identity 1 by comparing homework. Tell them to be prepared to ask any questions that they might have about homework problems. Spot check student work as they are comparing with partners. Address only those problems that students have not resolved. It is important that teachers not use valuable time going over work that students already have and understand.

Post the work below so that students may begin simplifying the given expressions as they finish comparing homework.

Write each of the following expressions in simplest form:

$(5x)^2$ $\qquad$ $2x + 3xy + 5x - xy$ $\qquad$ $(4x^2)^3$

Opening: Discuss simple form of the expressions above. Address any issues related to homework. Be sure to review Identity 1: $(x + a)(x + b) = x^2 + (a + b)x + ab$.

Worktime: Students should do problems 3-10 of the Student Task. Many students will need to use algebra tiles to visualize the area model used in problem 6 to illustrate $(x - y)^2$. Problem 8 of the student task has been added to the GaDOE task in order to give students more practice in manipulating a variety of algebraic expressions.

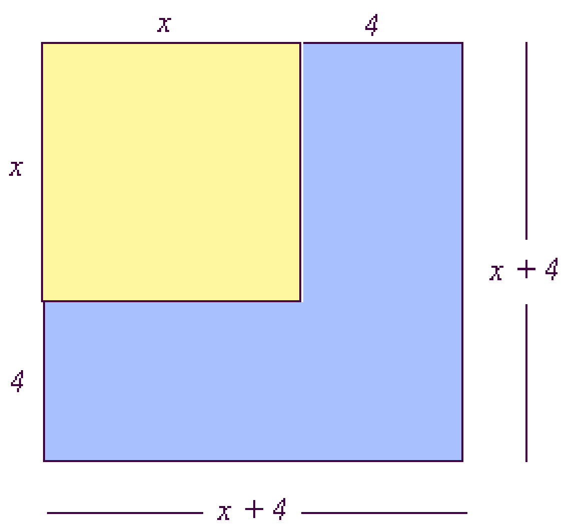
Closing: Allow students to share their work. Although problems in this task have been revised, the GaDOE TE notes address most problems and all of the concepts included in the student task.

Homework: Practice using Identities 2, 3, and 4 to multiply binomials.

Differentiated support/enrichment:

Resources/materials for Math Support: Continue to use arrays, area models, and the distributive property to multiply two digit numbers and binomials that contain algebraic expressions. Provide opportunities to model products of binomials in which *a* and/or *b* are negative using algebra tiles. Practice in operating with monomials and using the rules of exponents is also important.

3. Let's consider a special case of Identity 1. What happens when a and b in Identity 1 are the same number? Start by considering the square below created by adding 4 to the length of each side of a square with side length x.



(a) What is the area of the square with side length x?

(b) The square with side length $x + 4$ has greater area than the original square. Use Identity 1 to calculate its total area. When you use Identity 1, what are a, b, $a + b$?

(c) How much greater is the area of the square with side length $x + 4$? Use the figure to show this additional area. Where is the square with area 16 square units?

(d) How would your answers to parts (b) and (c) change if the larger square had been created to have side length $x + y$, that is, if both a and b are the same number y?

(e) Draw a figure to illustrate the area of a square with side length $x + y$ assuming that x and y are positive numbers. Use your figure to help you find the product $(x + y)(x + y)$ or $(x + y)^2$. Label this product Identity 2.

$(x + y)^2 =$ ______________________________

(Identity 2 is the *Square of a Sum*)

4. Identity 2 gives a rule for squaring a sum. Use it to calculate each of the following by making convenient choices for x and y.

(a) 302^2

(b) 2.1^2

5. Use Identity 2 to find each of the following products:

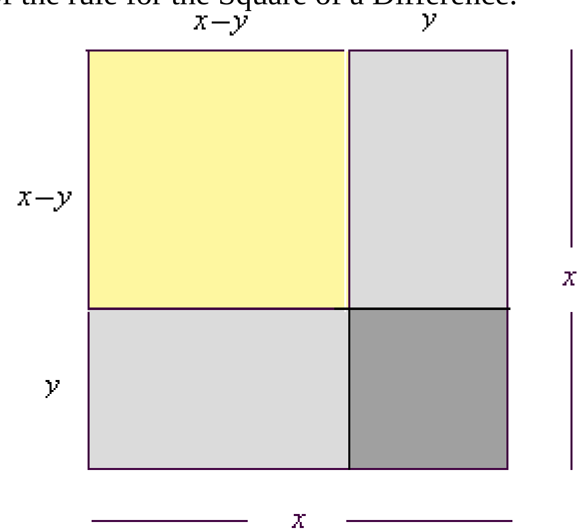
(a) $(x + 3)^2$

(b) $(4x + 5y)^2$

6. (a) Let's consider another special case of Identity 1. Let y represent any positive number. Go back to Identity 1 and substitute $-y$ for a and for b to get a rule for squaring a difference.

$(x - y)^2 =$ ______________________ (Identity 3 is the *Square of a Difference*)

(b) In the diagram below find the square with side x, the square with side y, and two different rectangles with area xy. Now, use the diagram to give a geometric explanation of the rule for the Square of a Difference.



7. By making convenient choices for x and y, use the *Square of a Difference* identity to find the following squares.

(a) $99^2 =$

(b) $17^2 =$

8. Use Identity 3 to find the squares of the following differences.

(a) $(x - 6)^2$

(b) $(3x - 5y)^2$

9. Now let's consider what happens in Identity 1 if a and b are opposite real numbers. Use Identity 1 to calculate each of the following. Substitute other variables or expressions for x as necessary.

(a) Calculate $(x + 8)(x - 8)$. Remember that $x - 8$ can also be expressed as $x + (-8)$.

(b) Calculate $(x - 6)(x + 6)$.

(c) Calculate $(3z + 12)(3z - 12)$.

(f) Substitute y for a and $-y$ for b in Identity 1 to find a pattern for the product $(x + y)(x - y)$.

$(x + y)(x - y) =$ ____________________ (Identity 4)

10. Make appropriate choices for x and y to use Identity 4 to calculate each of the following.

(a) (101)(99)

(b) (22)(18)

(f) $\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)$

Mathematics I

Task 3: I've Got Your Number — Day 3/3

(GaDOE TE #11, #6, and #8)

Standard(s): MM1A2. Students will simplify and operate with radical expressions, polynomials, and rational expressions.
c. Add, subtract, multiply, and divide polynomials.
g. Use area and volume models for polynomial arithmetic.

New vocabulary: cube of a sum, cube of a difference

Mathematical concepts/topics: multiplication of binomials, including the cube of a sum, and the cube of a difference; zero product property; area; volume; equivalent algebraic expressions; algebraic identities; field properties (associative, commutative, and distributive); proof; simplifying algebraic expressions

Prior knowledge: area; volume of right rectangular prisms; area and volume models; associative, commutative, and distributive properties; evaluating and simplifying simple algebraic expressions

Essential question(s): How can I use algebraic identities to multiply binomials quickly and easily?

Suggested materials: models of cubes, algebra tiles, grid paper

Warm-up: Have students review Identities 2, 3, and 4 by comparing homework. Tell them to be prepared to ask any questions they have about homework problems. Spot check student work as they are comparing with partners.

When students have finished comparing homework, they should answer the following question in their notebook.

How do you find the volume of a cube? State two different formulas for finding the volume of a cube and explain why they would both give the same result.

Opening: Answer any questions related to homework and then discuss the warm-up. Students should give the formulas $V = l \cdot w \cdot h$ and $V = B \cdot h$ and be able to explain both formulas.

Worktime: Students should do problems 11-13 of the Student Task. Problem 11 is important for a variety of reasons. It foreshadows use of the zero product property which will be used in the next task. It also helps develop a conceptual understanding of why $(x + y)^2$ can never be $x^2 + y^2$and similarly$(x – y)^2$ can never be $x^2 + y^2$ similarly $(x – y)^2$ can never be $x^2 – y^2$. Treated properly, Problem 11 may help students avoid some very common misconceptions.

Closing: Allow students to share their work. Discuss problem 11 thoroughly. (See GaDOE TE notes.)

Homework: Practice using Identities 1-6 to multiply binomials.

Differentiated support/enrichment:

Resources/materials for Math Support: Topics to be reviewed/previewed: finding the volume of a rectangular prism in general, and a cube in particular; multiplication by 0.

Mathematics I
I've Got Your Number
Day 3 Student Task

11. You have now studied 4 special products:

$$(x + a)(x + b) = x^2 + (a + b)\,x + ab$$
$$(x + y)^2 = x^2 + 2xy + y^2$$
$$(x - y)^2 = x^2 - 2xy + y^2$$
$$(x + y)(x - y) = x^2 - y^2$$

(a) In Question 9, you computed several products of the form $(x + y)(x - y)$ verifying that the product is always of the form $x^2 - y^2$. Thus, if we choose values for x and y so that $x = y$, then the product $(x + y)(x - y)$ will equal 0. Explain why this is true.

(b) Is there any other way to choose numbers to substitute for x and y so that the product $(x + y)(x - y)$ will equal 0? Explain.

(c) In general, if the product of two numbers is zero, what must be true about one of them?

(d) Consider Identity 2 for the Square of a Sum: $(x+y)^2 = x^2 + 2xy + y^2$. Is there a way to choose numbers to substitute for x and y so that the product xy equals 0?

(e) Is it ever possible that $(x + y)^2$ could equal $x^2 + y^2$? Explain your answer.

(f) Could $(x - y)^2$ ever equal $x^2 + y^2$? Could $(x - y)^2$ ever equal $x^2 - y^2$? Explain.

12. We can extend the strategies used to find squares of binomials to find cubes of binomials.

(a) What is the volume of a cube with side length 4?

(b) What is the volume of a cube with side length x?

(c) Now determine the volume of a cube with side length $x + 4$. First, use the rule for squaring a sum to find the area of the base of the cube.

Then use the distributive property several times to multiply the area of the base by the height, $x + 4$. Simplify your answer.

$(x + 4)^3 =$ ________________________

(d) Repeat parts (b) and (c) for a cube with side length $x + y$. Write your result as a rule for the *Cube of a Sum*.

$(x + y)^3 =$ ______________________________

(Identity 5 – the *Cube of a Sum*)

(e) Making convenient choices for x and y, use Pattern 3 to find the following cubes.

$23^3 =$

$101^3 =$

Use the rule for cubing a sum to cube 2 = 1 + 1. Do you get the same number as (2)(2)(2)?

(f) Use the cube of the sum pattern to simplify the following expressions.

$(t + 5)^3 =$

$(w + 2)^3 =$

$(3x + 4)^3 =$

13. (a) Find a rule for the cube of a difference.

$(x – y)^3 =$ __________________

(Identity 6 – *The Cube of a Difference*)

(b) Check your rule for the Cube of a Difference by using it to calculate the cube of 1 using 1 = 2 – 1 and the cube of 2 using 2 = 5 – 3.

$1^3 =$

$2^3 =$

(c) Use the cube of a difference to simplify the following expressions.

$(t - 5)^3 =$

$(w - 7)^3 =$

$(2x - y)^3 =$

Atlanta Public Schools
Teacher's Curriculum Supplement
Mathematics I: Unit 2
Task 4: Paula's Peaches

Mathematics I
Task 4: Paula's Peaches Day 1/5
(GaDOE TE #1, and #2)

Standard(s): MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.
a. Represent functions using function notation.
d. Investigate and explain the characteristics of a function: domain, range, zeros, intercepts, intervals of increase and decrease, maximum and minimum values, and end behavior.
e. Relate to a given context the characteristics of a function, and use graphs and tables to investigate its behavior.
g. Explore rates of change, comparing constant rates of change (i.e., slope) versus variable rates of change. Compare rates of change of linear, quadratic, square root, and other function families.

New vocabulary: average yield

Mathematical concepts/topics: modeling using linear and quadratic functions; using equations, tables, and graphs to investigate functions and the real situations modeled by those functions; writing linear and quadratic equations, function notation, domain, constant versus variable rates of change

Prior knowledge: linear functions

Essential question(s): How can I use algebra to model real situations and solve problems related to those situations?

Suggested materials: graph paper

Warm-up: Give students the Day 1 Student Task for Paula's Peaches. Ask them to read the opening scenario silently, making notes of important information in the margins.

Opening: Discuss the context of the task by asking students what they feel are the important facts in Paula's situation. Some may be unfamiliar with the term average *yield*.

Worktime: Students should complete problems 1 and 2 of the student task. At this point, students should have mastered concepts and skills related to linear functions. Having them work alone on problem 1 for a period of time would provide an excellent opportunity to check for individual student understanding of these concepts and skills.

Closing: Allow students to share their work. Using multiple representations (equations, tables, and graphs) will strengthen student understanding of the mathematics and of the situation addressed in this task. Make sure that all representations are presented. The unit of measure for the rate of change calculated in part 2d is complicated but extremely realistic. Discuss the unit of measure thoroughly. (See teacher notes.)

Homework:

Differentiated support/enrichment:

Resources/materials for Math Support: Make sure students understand the term *average yield*. Provide practice in reading, examining, and writing equations for situations similar to the "Peaches" scenario. Practice in simplifying linear and quadratic expressions is also important.

Mathematics I
Paula's Peaches
Day 1 Student Task

Paula is a peach grower in central Georgia and wants to expand her peach orchard. In her current orchard, there are 30 trees per acre and the average yield per tree is 600 peaches. Data from the local agricultural experiment station indicates that if Paula plants more than 30 trees per acre, once the trees are in production, the average yield of 600 peaches per tree will decrease by 12 peaches for each tree over 30. She needs to decide how many trees to plant in the new section of the orchard. Throughout this task assume that, for all peach growers in this area, the average yield is 600 peaches per tree when 30 trees per acre are planted and that this yield will decrease by 12 peaches per tree for each additional tree per acre.

1. Paula believes that algebra can help her determine the best plan for the new section of orchard and begins by developing a mathematical model of the relationship between the number of trees per acre and the average yield in **peaches per tree.**

   a. Is this relationship linear or nonlinear? Explain your reasoning.

   b. If Paula plants 6 more trees per acre, what will be the average yield in peaches per tree?

   c. What is the yield in peaches per tree if she plants 42 trees per acre?

   d. Let T be the function for which the input x is the number of trees planted on each acre and $T(x)$ is the average yield in peaches per tree. Write a formula for $T(x)$ in terms of x and express it in simplest form. Explain how you know that your formula is correct.

   e. Draw a graph of the function T. Given that the information from the agricultural experiment station applies only to increasing the number of trees per acre, what is an appropriate domain for the function T?

2. Since her income from peaches depends on the total number of peaches she produces, Paula realized that she needed to take a next step and consider the total number of peaches that she can produce **per acre**.

   a. With the current 30 trees per acre, what is the yield in total peaches per acre?

   b. If Paula plants 36 trees per acre, what will be the yield in total peaches per acre?

   c. What is the yield in total peaches per acre if she plants 42 trees per acre?

   d. Find the average rate of change of peaches per acre with respect to number of trees per acre when the number of trees per acre increases from 30 to 36. Write a sentence to explain what this number means.

   e. Find the average rate of change of peaches per acre with respect to the number of trees per acre when the number of trees per acre increases from 36 to 42. Write a sentence to explain the meaning of this number.

   f. Is the relationship between number of trees per acre and yield in peaches per acre linear? Explain your reasoning.

   g. Let Y be the function that expresses this relationship, that is, the function for which the input x is the number of trees planted on each acre and the output $Y(x)$ is the total yield in peaches per acre. Write a formula for $Y(x)$ in terms of x and express your answer in expanded form.

   h. Calculate $Y(30)$, $Y(36)$, and $Y(42)$. What is the meaning of these values? How are they related to your answers to parts 2a through 2c?

   i. What is the relationship between the domain for the function T and the domain for the function Y? Explain.

Mathematics I

Task 4: Paula's Peaches Day 2/5

(GaDOE TE #3, and #4)

Standard(s): MM1A2. Students will simplify and operate with radical expressions, polynomials, and rational expressions.

c. Add, subtract, multiply, and divide polynomials.

e. Factor expressions by greatest common factor, grouping, trial and error, and special products limited to the formulas below.

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$(x-y)^2 = x^2 - 2xy + y^2$$

$$(x+y)(x-y) = x^2 - y^2$$

$$(x+a)(x+b) = x^2 + (a+b)x + ab$$

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

MM1A3. Students will solve simple equations.

a. Solve quadratic equations in the form $ax^2 + bx + c = 0$ where $a = 1$, by using factorization and finding square roots where applicable.

New vocabulary: quadratic expression, quadratic equation, factor, zero product property

Mathematical concepts/topics: factoring quadratic expressions, multiplicative property of equality, addition property of equality, equivalent equations, solving quadratic equations by factoring

Prior knowledge: solving multi-step linear equations in one variable, solving linear equations in two variables

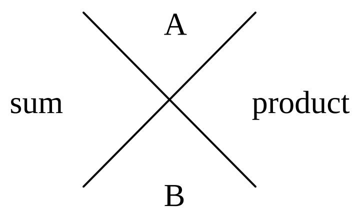
Essential question(s): How can I use algebra to model real situations and solve problems related to those situations?

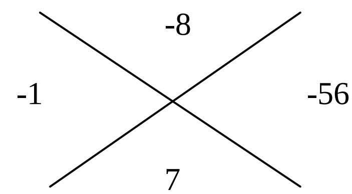
Suggested materials: grid paper, algebra tiles

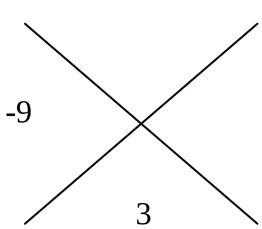
Warm-up: Post the directions and the puzzles below. Many students will be familiar with these puzzles as they were used in 7th grade tasks to address signed numbers.

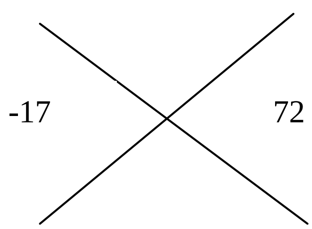
X GAMES

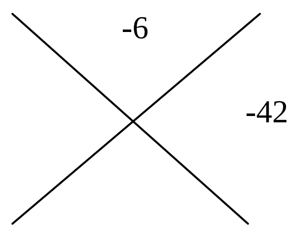
Each of the puzzles below has a space for two different integers, their sum, and their product. You are given the position of each value and a sample puzzle. Your job is to find the missing numbers. For each space you fill, write a number sentence that results in the missing value.









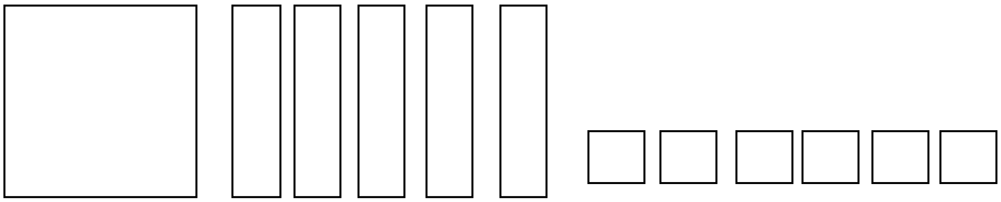


Opening: Have students share their solutions to the puzzles above, including the number sentences that resulted in their solutions. Ask students to solve the linear equation below. Discuss their solutions. This discussion should include the use of the addition and multiplication properties of equality in solving an equation. Students have spent extensive time in 8th grade solving multi-step equations. It is important to link to this prior knowledge before beginning this lesson.

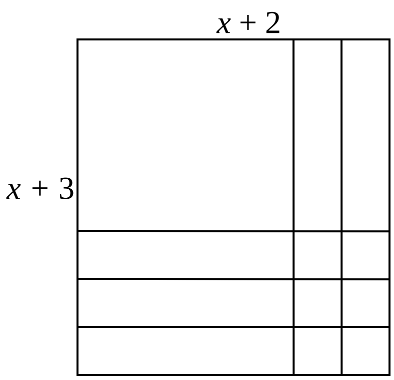
$$2(4x + 5) - 4 = 3x - 11$$

Worktime: Students should complete problems 3 and 4 of the student task. It is a very good idea to have the six special products learned in the previous task posted in the room so that students may refer to them quickly. This is an example of using ***anchor charts*** to reinforce newly learned material. Having reference materials in the form of concept books, student-made note pages, or anchor charts, not only reinforces concepts and skills, but also helps students take more responsibility for their own learning.

Algebra tiles are particularly useful in helping students see the factoring of a trinomial. For example, to illustrate factoring $x^2 + 5x + 6$, begin by laying out the pieces as such:



Have students arrange their pieces to form a rectangle and represent the area of the rectangle as the product of the side lengths.



$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

Closing: Allow students to share their work. (See teacher notes.)
Homework: Practice in solving quadratic equations in the form $x^2 + bx + c = 0$ by factoring.

Differentiated support/enrichment:

Resources/materials for Math Support: Students will need practice in solving equations, including: writing equivalent equations, factoring, and using the zero product property. Students may benefit from more practice with games like those in the warm-up and from continuing to use manipulatives (as shown above and in the previous task) and/or area models to reinforce multiplication of binomials and factoring of trinomials.

3. Paula wants to know whether there is a different number of trees per acre that will give the same yield per acre as the yield when she plants 30 trees per acre.

   a. Write an equation that expresses the requirement that *x* trees per acre yields the same total number of peaches per acre as planting 30 trees per acre.

   b. Use the algebraic rules for creating equivalent equations to obtain an equivalent equation with an expression in *x* on one side of the equation and 0 on the other.

   c. Multiply this equation by an appropriate rational number so that the new equation is of the form $x^2 + bx + c = 0$. Explain why this new equation has the same solution set as the equations from parts a and b.

   d. When the equation is in the form $x^2 + bx + c = 0$, what are the values of *b* and *c*?

   e. Find integers *m* and *n* such that $m \cdot n = c$ and $m + n = b$.

   f. Using the values of *m* and *n* found in part 3e, form the algebraic expression $(x+m)(x+n)$ and use Identity 1 from to simplify it.

   g. Combining parts 3d through 3f, rewrite the equation from part c in the form $(x+m)(x+n) = 0$.

   h. This equation expresses the idea that the product of two numbers, $x + m$ and $x + n$, is equal to 0. We know from our discussion in the previous task that, when the product of two numbers is 0, one of the numbers has to be 0. This property is called the ***Zero Factor Property***. For these particular values of *m* and *n*, what value of *x* makes $x + m = 0$ and what value of *x* makes $x + n = 0$?

   i. Verify that the answers to part 3h are solutions to the equation written in part 3a. It is appropriate to use a calculator for the arithmetic.

   j. Write a sentence to explain the meaning of your solutions to the equation in relation to planting peach trees.

4. Paula saw another peach grower, Sam, from a neighboring county at a farm equipment auction and began talking to him about the possibilities for the new section of her orchard. Sam was surprised to learn about the agricultural research and said that it probably explained the drop in yield for a orchard near him. This peach farm has more than 30 trees per acre and is getting an average total yield of 14,400 peaches per acre.

   a. Write an equation that expresses the situation that x trees per acre results in a total yield per acre of 14,400 peaches per acre.

   b. Use the algebraic rules for creating equivalent equations to obtain an equivalent equation with an expression in x on one side of the equation and 0 on the other.

   c. Multiply this equation by an appropriate rational number so that the new equation is of the form $x^2 + bx + c = 0$. Explain why this new equation has the same solution set as the equations from parts a and b.

   d. When the equation is in the form $x^2 + bx + c = 0$, what is value of b and what is the value of c?

   e. Find integers m and n such that $m \cdot n = c$ and $m + n = b$.

   f. Using the values of m and n found in part 4e, form the algebraic expression $(x+m)(x+n)$ and use Identity 1 of your *Special Products* to simplify $(x+m)(x+n)$.

   g. Combining parts 4d through 4f, rewrite the equation from part d in the form $(x+m)(x+n) = 0$.

   h. This equation expresses the idea that the product of two numbers, $x + m$ and $x + n$, is equal to 0. We know when the product of two numbers is 0, one of the numbers has to be 0. What value of x makes $x + m = 0$? What value of x makes $x + n = 0$?

   i. Verify that the answers to part 4h are solutions to the equation written in part 4a. It is appropriate to use a calculator for the arithmetic.

   j. Which of the solutions verified in part 4i is (are) in the domain of the function Y? How many peach trees per acre are planted at the peach orchard getting 14400 peaches per acre?

The steps in items 3 and 4 outline a method of solving equations of the form $x^2 + bx + c = 0$. These equations are called ***quadratic equations*** and an expression of the form $x^2 + bx + c$ is called a ***quadratic expression***. In general, quadratic expressions may have any nonzero coefficient on the x^2 term, but in Mathematics I we focus on quadratic expressions with coefficient 1 on the x^2 term. An important part of this method for solving quadratic equations is the process of rewriting an expression of the form $x^2 + bx + c$ in the form $(x + m)(x + n)$. The rewriting step is an application of Identity 1 of your *Special Products*. The identity tells us that the product of the numbers *m* and *n* must equal *c* and that the sum of *m* and *n* must equal *b*. In Mathematics I, we will apply Identity 1 in this way only when the values of *b*, *c*, *m*, and *n* are integers.

Mathematics I

Task 4: Paula's Peaches   Day 3/5

(GaDOE TE #5 - #7, CS 8)

Standard(s): MM1A2. Students will simplify and operate with radical expressions, polynomials, and rational expressions.
e. Factor expressions by greatest common factor, grouping, trial and error, and special products limited to the formulas below.

$$(x+y)^2 = x^2 + 2xy + y^2$$
$$(x-y)^2 = x^2 - 2xy + y^2$$
$$(x+y)(x-y) = x^2 - y^2$$
$$(x+a)(x+b) = x^2 + (a+b)x + ab$$
$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$
$$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

g. Use area and volume models for polynomial arithmetic.

New vocabulary: greatest common factor, factoring by grouping, trinomial, polynomial

Mathematical concepts/topics: factoring using special products, factoring out the greatest common factor, factoring by grouping

Prior knowledge: distributive property, rules of exponents

Essential question(s): How can I write a polynomial as a product?

Suggested materials: grid paper, algebra tiles

Warm-up: Post the following:

(i) $6x^2 - 12x$

(ii) $2x(x-1) + 4(x-1)$

For each of the expressions above, answer the following:

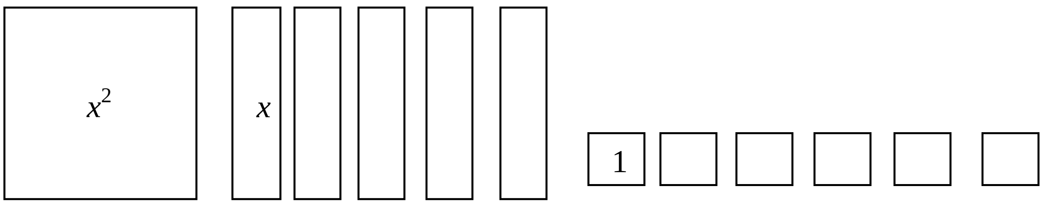
- What is the largest factor common to both terms of the expression?
- Use the distributive property to write the expression as the product of its largest common factor and a binomial.

Opening: Discuss the warm-up. Students have had extensive work with the distributive property and the greatest common factors of numerical expressions. They have not had a lot of practice with common factors of algebraic expressions.

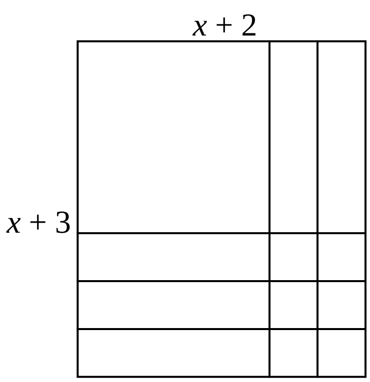
Worktime: Students should complete problems 5 - 8 of the student task. Problem 8 has been added to the task to give students more practice in factoring out the greatest common factor and in factoring by grouping. We again recommend that the special products learned in the previous task be posted so that students may refer to them quickly.

We also suggest that the use of algebra tiles (explained in the last lesson and again below) be used in problems 5 and 6.

To illustrate factoring $x^2 + 5x + 6$, begin by laying out the pieces as such:



Have students arrange their pieces to form a rectangle and represent the area of the rectangle as the product of the side lengths.



$x^2 + 5x + 6 = (x + 2)(x +$

Closing: Allow students to share their work. (See teacher notes.)
Homework: Practice factoring quadratic polynomials.

Differentiated support/enrichment:

Resources/materials for Math Support: Students will need practice in solving equations, including: writing equivalent equations, factoring, and using the zero product property. Students may benefit from more practice with games like those in the warm-up and from continuing to use manipulatives (as shown above and in the previous task) and/or area models to reinforce multiplication of binomials and factoring of trinomials.

Mathematics I
Paula's Peaches
Day 3 Student Task

5. Since the whole expression $(x+m)(x+n)$ is a product, we call the expressions $x+m$ and $x+n$ the **factors** of this product. For the following expressions in the form x^2+bx+c, rewrite the expression as a product of factors of the form $x + m$ and $x + n$. Verify each answer by creating a rectangle with sides of length $x+m$ and $x+n$, respectively, and showing geometrically that the area of the rectangle is x^2+bx+c.

a. x^2+3x+2

b. x^2+6x+5

c. x^2+5x+6

d. $x^2+7x+12$

6. In item 5, the values of *b* and *c* were positive. Now use Identity 1 in reverse to factor each of the following quadratic expressions of the form x^2+bx+c where *c* is positive but *b* is negative. Verify each answer by multiplying the factored form to obtain the original expression.

a. x^2-8x+7

b. $x^2-9x+18$

c. x^2-4x+4

d. $x^2-8x+15$

7. Use Identity 1 in reverse to factor each of the following quadratic expressions of the form $x^2 + bx + c$ where c is negative. Verify each answer by multiplying the factored form to obtain the original expression.

a. $x^2 + 6x - 7$

b. $x^2 - 6x - 7$

c. $x^2 + x - 42$

d. $x^2 - x - 42$

8. Use the distributive property and the six special products you have learned to factor each of the following polynomials.

a. $x^3 - 9x$

b. $5x^2 - 40x + 80$

c. $2x^4 - 8x^3 - 42x^2$

d. $x(2x - 1) - 3(2x - 1)$

e. $x^3 - 3x^2 - x + 3$

Mathematics I

Task 4: Paula's Peaches   Day 4/5

(GaDOE TE #8 - #12)

Standard(s): MM1A2. Students will simplify and operate with radical expressions, polynomials, and rational expressions.

e. Factor expressions by greatest common factor, grouping, trial and error, and special products limited to the formulas below.

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$(x-y)^2 = x^2 - 2xy + y^2$$

$$(x+y)(x-y) = x^2 - y^2$$

$$(x+a)(x+b) = x^2 + (a+b)x + ab$$

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

MM1A3. Students will solve simple equations.

a. Solve quadratic equations in the form $ax^2 + bx + c = 0$ where $a = 1$, by using factorization and finding square roots where applicable.

New vocabulary: greatest common factor, factoring by grouping

Mathematical concepts/topics: factoring quadratic expressions, multiplicative property of equality, addition property of equality, equivalent equations, solving quadratic equations by factoring

Prior knowledge: solving multi-step linear equations in one variable, solving linear equations in two variables, greatest common factor (GCF)

Essential question(s): How can I use algebra to model real situations and solve problems related to those situations?

Suggested materials: calculators

Warm-up: Have students review factoring quadratic expressions by comparing homework. Tell them to be prepared to ask any questions that they might have about homework problems. Spot check student work as they are comparing with partners. Address only those problems that students have not resolved.

Opening: Address any homework problems that students have not resolved.

Worktime: Students should complete problems 9 – 13 of the student task. In problem 9, it is particularly important that students verify their work by checking solutions in the original equations.

Problems 11 – 13 require that students apply the mathematical concepts they have learned. Make sure they understand how to use the function they have developed to answer questions related to the real situation.

Closing: Allow students to share their work. (See teacher notes.)

Homework: Homework at this point should provide practice in reading a scenario, writing an equation that models the situation described, and solving equations that answer questions related to the situation.

Differentiated support/enrichment:

Resources/materials for Math Support: Students will continue to need practice in solving equations, including: writing equivalent equations, factoring, and using the zero product property.

9. In items 3 and 4, we used factoring as part of a process to solve equations that are equivalent to equations of the form $x^2 + bx + c = 0$ where b and c are integers. Look back at the steps you did in items 3 and 4, and describe the process for solving an equation of the form $x^2 + bx + c = 0$. Use this process to solve each of the following equations, that is, to find all of the numbers that satisfy the original equation. Verify your work by checking each solution in the original equation.

   a. $x^2 - 6x + 8 = 0$

   b. $x^2 - 15x + 36 = 0$

   c. $x^2 + 28x + 27 = 0$

   d. $x^2 - 3x - 10 = 0$

   e. $x^2 + 2x - 15 = 0$

   f. $x^2 - 4x - 21 = 0$

   g. $x^2 - 7x = 0$

   h. $x^2 + 13x = 0$

10. The process you used in item 8 works whenever you have an equation in the form $x^2 + bx + c = 0$. There are many equations, like those in items 3 and 4, that look somewhat different from this form but are, in fact, equivalent to an equation in this form. Remember that the ***Addition Property of Equality*** allows us to get an equivalent equation by adding the same expression to both sides of the equation and the ***Multiplicative Property of Equality*** allows us to get an equivalent equation by multiplying both sides of the equation by the same number as long as the number we use is not 0. For each equation below, find an equivalent equation in the form $x^2 + bx + c = 0$.

    a. $6x^2 + 12x - 48 = 0$

    b. $x^2 - 8x = 9$

    c. $3x^2 = 21x - 30$

    d. $4x^2 + 24 = 20x$

    e. $x(x - 11) + 30 = 0$

    f. $\frac{1}{2}x(x + 8) = 10$

    g. $(x + 1)(x + 5) + 3 = 0$

    h. $(2x + 3)(x + 4) = x + 24$

    i. $5x(x + 3) = 200$

11. Now we return to the peach growers in central Georgia. How many peach trees per acre would result in only 8400 peaches per acre?

12. If there are no peach trees on a property, then the yield is zero peaches per acre. Write an equation to express the idea that the yield is zero peaches per acre with x trees planted per acre, where x is number greater than 30. Is there a solution to this equation, that is, is there a number of trees per acre that is more than 30 and yet results in a yield of zero peaches per acre? Explain.

13. At the same auction where Paula heard about the peach grower who was getting a low yield, she talked to the owner of a major farm supply store in the area. Paula began telling the store owner about her plans to expand her orchard, and the store owner responded by telling her about a local grower that gets 19,200 peaches per acre. Is this number of peaches per acre possible? If so, how many trees were planted?

Mathematics I

Task 4: Paula's Peaches **Day 5/5**

(GaDOE TE #1, and #2)

Standard(s): MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.

a. Represent functions using function notation.

d. Investigate and explain the characteristics of a function: domain, range, zeros, intercepts, intervals of increase and decrease, maximum and minimum values, and end behavior.

e. Relate to a given context the characteristics of a function, and use graphs and tables to investigate its behavior.

i. Understand that any equation in x can be interpreted as the equation $f(x) = g(x)$, and interpret the solutions of the equation as the x-value(s) of the intersection point(s) of the graphs of $y = f(x)$ and $y = g(x)$.

MM1A3. Students will solve simple equations.

c. Use a variety of techniques, including technology, tables, and graphs to solve equations resulting from the investigation of $x^2 + bx + c = 0$.

New vocabulary: constant function

Mathematical concepts/topics: modeling using quadratic functions; using equations, tables, and graphs to investigate functions and the real situations modeled by those functions; function notation; domain; solutions of the equation $f(x) = g(x)$ as the x-values of the intersection point(s) of the graphs of $y = f(x)$ and $y = g(x)$.

Prior knowledge:

Essential question(s): How can I use algebra and technology to model real situations and solve problems related to those situations?

Suggested materials: graph paper, graphing calculators

Warm-up: Have students compare homework. Tell them to be prepared to ask any questions that they might have about homework problems. Spot check student work as they are comparing with partners.

Opening: Discuss any homework issues. Ask students to graph the function $f(x) = 7$. Ask what they think the graph of the function looks like and why.

Worktime: Students should complete problems 14 through 16 of the student task. Allow them to graph points they have calculated while working the task to obtain the general shape of the graph, then check their work with the calculator.

The idea that the graph of Y is discrete with domain restricted to integers between 30 and 80, inclusive, while f is continuous with a domain of all real numbers is important. (Please see teacher notes for the correct notation for the domain of Y).

Also important here is the idea that the solution(s) of the equation $f(x) = g(x)$ is the x-value(s) of the intersection point(s) of the graphs of $y = f(x)$ and $y = g(x)$.

Closing: Allow students to share their work. (See teacher notes.)

Homework:

Differentiated support/enrichment:

Resources/materials for Math Support: Make sure students are comfortable using a graphing calculator and understand the concept of a constant function.

Mathematics I
Paula's Peaches
Day 5 Student Task

14. Using graph paper, explore the graph of Y as a function of x.
    a. In part 3 of this task, you were asked if there were a different number of trees per acre, other than 30, that would yield 18,000 peaches per acre. What point on the graph corresponds to the answer to this question?

       In part 4 of this task, you were asked to find the number of trees per acre that would yield 14,400 peaches per acre. What point on the graph corresponds to the answer to this question?

    b. In problems 11, 12, and 13, you were asked how many trees per acre would result in 8400, 0, and 19200 peaches per acre, respectively? What points on the graph correspond to the answers to these questions?

    c. What is the relationship of the graph of the function Y to the graph of the function f, where the formula for $f(x)$ is the same as the formula for $Y(x)$ but the domain for f is all real numbers?

    d. Items 4, 11, and 12 give information about points that are on the graph of f but not on the graph of Y. What points are these?

    e. Graph the functions f and Y on the same axes. How does your graph show that the domain of f is all real numbers? How is the domain of Y shown on your graph?

15. In answering parts a, b, and d of item 14, you gave one geometric interpretation of the solutions of the equations solved in items 3, 4, 11, 12, and 13. We now explore a slightly different viewpoint.
    a. Draw the line $y = 18000$ on the graph drawn for item 14e. This line is the graph of the function with constant value 18000. Where does this line intersect the graph of the function Y? Based on the graph, how many trees per acre give a yield of more than 18000 peaches per acre?

    b. Draw the line $y = 8400$ on your graph. Where does this line intersect the graph of the function Y? Based on the graph, how many trees per acre give a yield of fewer than 8400 peaches per acre?

c. Use a graphing utility and this intersection method to find the number of trees per acre that give a total yield closest to the following numbers of peaches per acre:
(i) 10000 (ii) 15000 (iii) 20000

d. Find the value of the function *Y* for the number of trees given in answering 15c(i) – (iii) above.

16. For each of the equations solved in item 8, do the following.
   a. Use technology to graph a function whose formula is given by the left-hand side of the equation.
   b. Find the points on the graph which correspond to the solutions found in item 8.
   c. How is each of these results an example of the intersection method explored in item 14?

Atlanta Public Schools

Teacher's Curriculum Supplement

Mathematics I: Unit 2

Task 5: Ladder Length

Mathematics I

Task 5: Ladder Length   Day 1/1

(GaDOE TE #1 - #6)

Standard(s): MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.
a. Represent functions using function notation.
e. Relate to a given context the characteristics of a function, and use graphs and tables to investigate its behavior.
MM1A2. Students will simplify and operate with radical expressions, polynomials, and rational expressions.

a. Simplify algebraic and numeric expressions involving square root.
b. Perform operations with square roots.

New vocabulary:

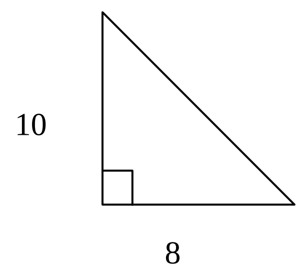
Mathematical concepts/topics: modeling using linear functions; comparing linear to non-linear relationships; using equations, tables, and graphs to investigate functions and the real situations modeled by those functions; simplifying radical expressions; exact versus approximate answers; Pythagorean Theorem; function notation

Prior knowledge: simplifying numerical expressions containing square roots, operating with irrational numbers, Pythagorean Theorem, linear functions

Essential question(s): How can I use algebra to model real situations and solve problems related to those situations?

Suggested materials: graph paper

Warm-up: Post the following:



Use the given right triangle to do the following:

- Find the length of the hypotenuse of the triangle.
- Give your answer as an exact value in simplest form.
- Give your answer as an approximation rounded to the nearest tenth.

Opening: Discuss the warm-up. Students have worked extensively with the Pythagorean Theorem and simplifying square roots of numerical expressions in middle school. These topics are not the focus of the task. However, they are important in preparing students to represent the situations given using the appropriate functions and to simplify the algebraic expressions involved in the functions.

Worktime: Students should complete problems 1 and 6 of the student task.

Closing: Allow students to share their work. (See teacher notes.)

Homework:

Differentiated support/enrichment:

Resources/materials for Math Support: The following topics should be previewed: simplifying numerical expressions containing square roots, operating with irrational numbers, determining exact values versus approximations, finding side lengths of right triangles using the Pythagorean Theorem.

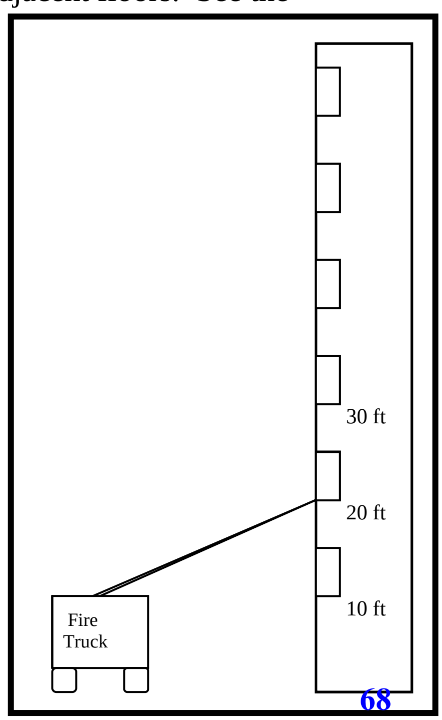
Mathematics I
Ladder Length
Day 1 Student Task

Firefighters are important members of the community who not only fight fires but also participate in a variety of rescue activities. A ladder truck is an important tool that firefighters use to get water to and rescue people from heights above the ground floor. This activity explores relationships between the building floor firefighters need to reach and the length to which they need to extend the ladder mounted on the truck.

One metropolitan fire department has a truck with a 100 ft extension ladder. The ladder is mounted on the top of the truck. When the ladder is in use, the base of the ladder is 10 feet above the ground.

Suppose that a ladder truck is parked so that the base of the ladder is 20 feet from the side of an apartment building. Because there are laundry and storage rooms in the basement of the building, the base of the windows in first floor apartments are 10 feet above the ground, and there is a distance of 10 feet between the base of the windows on adjacent floors. See the diagram.

1. Find the length to which the ladder needs to be extended to reach the base of a window on the second floor. What is the exact answer? Would an approximation be more meaningful in this situation? Make an approximation to the nearest tenth of a foot and check your answer.



2. Find the length to which the ladder needs to be extended to reach the base of a window on the third floor.

3. Find the length to which the ladder needs to be extended to reach the base of a window on the fourth floor.

4. Let n represent the number of the floor that the ladder needs to reach. Write a function where the input is n and the output is the height h, in feet above the ground, of the base of windows on that floor. Does this function express a linear relationship?

5. Write a function where the input is n and the output is the length L of the ladder when it is extended to reach the base of a window on floor n. If possible, simplify this expression. Does this function express a linear relationship?

6. In this situation, what is the highest floor that the ladder could reach?