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Atlanta Public Schools

Teacher's Curriculum Supplement Web-Edition

Common Core Georgia Performance Standards Mathematics III

Unit 2: Polynomial Functions of Higher Degree

GE Foundation

This document has been made possible by funding from the GE Foundation Developing Futures grant, in partnership with Atlanta Public Schools. It is derived from the Georgia Department of Education Math III Framework and includes contributions from Georgia teachers. It is intended to serve as a companion to the GA DOE Math III Framework Teacher Edition.

Preface

We are pleased to provide this supplement to the Georgia Department of Education's Mathematics III Framework. It has been written in the hope that it will assist teachers in the planning and delivery of the new curriculum, particularly in these first years of implementation. This document should be used with the following considerations.

- The importance of working the tasks used in these lessons cannot be overstated. In planning for the teaching of the Georgia Performance Standards in Mathematics teachers should work the tasks, read the teacher notes provided in the Georgia Department of Education's Mathematics III Framework Teacher Edition, and *then* examine the lessons provided here.

- This guide provides day-by-day lesson plans. While a detailed scope and sequence and established lessons may help in the implementation of a new and more rigorous curriculum, it is hoped that teachers will assess their students informally on an on-going basis and use the results of these assessments to determine (or modify) what happens in the classroom from one day to the next. Planning based on student need is much more effective than following a pre-determined timeline.

- It is important to remember that the Georgia Performance Standards provide a balance of concepts, skills, and problem solving. Although this document is primarily based on the tasks of the Framework, we have attempted to help teachers achieve this all important balance by embedding necessary skills in the lessons and including skills in specific or suggested homework assignments. The teachers and writers who developed these lessons, however, are not in your classrooms. It is incumbent upon the classroom teacher to assess the skill level of students on every topic addressed in the standards and provide the opportunities needed to master those skills.

- In most of the lesson templates, the sections labeled *Differentiated support/enrichment* have been left blank. This is a result of several factors, the most significant of which was time. It is hoped that as teachers use these lessons, they will contribute their own ideas, not only in the areas of differentiation and enrichment, but in other areas as well. Materials and resources abound that can be used to contribute to the teaching of the standards.

On the topic of differentiation, it is critical to reiterate that many of the strategies used in a standards-based mathematics classroom promote differentiation. These strategies include

- the use of rich tasks with multiple points of entry and more than one path to a solution,
- flexible grouping of students,
- multiple representations of mathematical concepts,
- writing in mathematics,
- monitoring of progress through on-going informal and formative assessments, and
- analysis of student work.

We hope that teachers will incorporate these strategies in each and every lesson.

It is hoped that you find this document useful as we strive to raise the mathematics achievement of all students in our district and state. Comments, questions, and suggestions for inclusions related to this document may be emailed to Dr. Dottie Whitlow, Executive Director, Mathematics and Science Department, Atlanta Public Schools, dwhitlow@atlantapublicschools.us.

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Explanation of the Terms and Categories Used in the Lesson Template

Task: This section gives the suggested number of days needed to teach the concepts addressed in a task, the task name, and the problem numbers of the task as listed in the Georgia Department of Education's Mathematics III Framework Teacher Edition (GaDOE TE).

In some cases new tasks or activities have been developed. These activities have been named by the writers.

Standard(s): Key standards addressed in the lesson are listed in this section. Standards listed first, in regular type, are from the Common Core State Standards for Mathematics. Standards in bold type are the corresponding standards from Mathematics III of the Georgia Performance Standards.

New Vocabulary: Vocabulary is listed here the *first* time it is used. It is strongly recommended that teachers, particularly those teaching Math Support, use interactive word walls. Vocabulary listed in this section should be included on the word walls and previewed in Math Support.

Mathematical concepts/skills: Major concepts addressed in the lesson are listed in this section whether they are CCGPS Math III concepts or were addressed in earlier grades or courses.

Prior knowledge: Prior knowledge includes only those topics studied in previous grades or courses. It does not include CCGPS Math III content taught in previous lessons.

Essential Question(s): Essential questions may be daily and/or unit questions.

Suggested materials: This is an attempt to list all materials that will be needed for the lesson, including consumable items, such as graph paper; and tools, such as graphing calculators and compasses. This list does not include those items that should always be present in a standards-based mathematics classroom such as markers, chart paper, and rulers.

Warm-up: A suggested warm-up is included with every lesson. Warm-ups should be brief and should focus student thinking on the concepts that are to be addressed in the lesson.

Opening: Openings should set the stage for the mathematics to be done during the work time. The amount of class time used for an opening will vary from day-to-day but this should not be the longest part of the lesson.

Worktime: The problem numbers have been listed and the work that students are to do during the worktime has been described. A student version of the day's activity follows the lesson template in every case. In order to address all of the standards in CCGPS Math III, some of the problems in some of the original GaDOE tasks have been omitted and less time consuming activities have been substituted for those problems. In many instances, in the student versions of the tasks, the writing of the original tasks has been simplified. In order to preserve all vocabulary, content, and meaning it is important that teachers work the original tasks as well as the student versions included here.

Teachers are expected to both facilitate and provide some direct instruction, when necessary, during the work time. Suggestions related to student misconceptions, difficult concepts, and deeper meaning have been included in this section.

Questioning is extremely important in every part of a standards-based lesson. We included suggestions for questions in some cases but did not focus on providing good questions as extensively as we would have liked. Developing good questions related to a specific lesson should be a focus of collaborative planning time.

Closing: The closing may be the most important part of the lesson. This is where the mathematics is formalized. Even when a lesson must be continued to the next day, teachers should stop, leaving enough time to "close", summarizing and formalizing what students have done to that point. As much as possible students should assist in presenting the mathematics learned in the lesson. The teacher notes are all important in determining what mathematics should be included in the closing.

Homework: In some cases, homework suggestions are provided. Teachers should use their resources, including the textbook, to assign homework that addresses the needs of their students.

Homework should be focused on the skills and concepts presented in class, relatively short (30 to 45 minutes), and include a balance of skills and thought-provoking problems.

Differentiated support/enrichment: On the topic of differentiation, it is critical to reiterate that many of the strategies used in a standards-based mathematics classroom promote differentiation. These strategies include

- the use of rich tasks with multiple points of entry and more than one path to a solution
- flexible grouping of students
- multiple representations of mathematical concepts
- writing in mathematics
- monitoring of progress through on-going informal and formative assessments and
- analysis of student work.

Check for understanding: A check for understanding is a short, focused assessment-a ticket out the door, for example. There are many good resources for these items in Learning Village on-line at www.georgiastandards.org, along with other GaDOE materials related to the standards. The direct link to Learning Village is https://portal.doe.k12.ga.us/Login.aspx .

Resources/materials for Math Support: Again, in some cases, we have provided materials and/or suggestions for Math Support. This section should be personalized to your students, class, and/or school, based on your resources.

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Unit 2: Polynomial Functions of Higher Degree

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Unit 1 Timeline

Task 1: Polynomial Functions: What We Already know	1 day
Task 2: Interval Notation and Power Functions	1 day
Mini-lesson on Division, Remainder and Factor Theorems	1 day
Task 3: Finding Roots When Degree Is Three or Higher	1 day
Task 4: Using the Theorems of Great Mathematicians	2 days

Task Notes

The importance of working the tasks used in these lessons cannot be overstated. In planning for the teaching of the Common Core Georgia Performance Standards in Mathematics, teachers should work the Student Tasks, read any corresponding teacher notes, and *then* examine the lessons provided here.

The tasks provided in this Supplement are based on the content of Unit 2 of the Georgia Department of Education's Mathematics III Framework. We suggest, as always, that teachers use this Supplement along with the GaDOE Teacher Edition which can be found in Learning Village on-line at www.georgiastandards.org. The direct link to Learning Village is https://portal.doe.k12.ga.us/Login.aspx .

Task 1: Polynomial Functions: What We Already Know
The concepts and skills addressed in this task include:

- vocabulary related to polynomials and polynomial functions;
- understanding linear and quadratic functions as polynomial functions;
- determining characteristics of the graphs of linear and quadratic functions ;
- converting a quadratic function from standard to vertex form;
- solving quadratic equations by factoring and by using the quadratic formula;
- relating the zeros of a function to its graph; and
- graphing transformations of $f(x)$, including vertical stretches and shrinks, vertical and horizontal shifts, and reflections across the x- axis.

This task does not occur in the GaDOE Mathematics III Framework. In each unit of the CCGPS Math III course, tasks that re-visit concepts and skills studied in Math I and Math II have been developed for two reasons: to scaffold for Math III concepts, and to provide students with opportunities to prepare for the Georgia High School Graduation Test as they take Math III.

Teachers should use this task to help students review what they already know about vocabulary related to polynomials, linear and quadratic functions, using the quadratic formula to find roots of quadratic equations, and graphing transformations of basic functions.

Task 2: Interval Notation and Power Functions
The concepts and skills addressed in this task include:

- representing intervals using interval notation;
- determining characteristics of the graphs of power functions ($f(x) = ax^n$) including; domain, range, intervals of increase and decrease, end behavior, and symmetry;
- comparing graphs of power functions of odd degree;
- comparing graphs of power functions of even degree;
- determining the effects of the lead coefficient on the graph of $f(x) = ax^n$; and
- graphing transformations of $f(x) = x^n$.

This task does not occur in the GaDOE Mathematics III Framework.

Task 3: Finding Roots When Degree Is Three or Higher
The concepts and skills addressed in this task include:

- finding real and complex roots of higher degree polynomial equations using the Factor Theorem and the Remainder Theorem;
- determining the nature of roots, including real and rational, real and irrational, and imaginary;
- using technology to find and verify roots of higher degree polynomial functions;
- determining the shapes of graphs of polynomial functions near roots of multiplicity *n;*
- investigating and explaining characteristics of polynomial functions, including domain and range, intercepts, zeros, relative and absolute extrema, intervals of increase and decrease, and end behavior.

This task does not occur in the GaDOE Mathematics III Framework.

Task 4: Using the Theorems of Great Mathematicians
The concepts and skills addressed in this task include:

- finding real and complex roots of higher degree polynomials using the Factor Theorem, Remainder Theorem, Rational Root Theorem, Fundamental Theorem of Algebra, Complex Conjugate Theorem, and Conjugate Radical Theorem;
- finding real and complex roots of higher degree polynomials with rational coefficients given one imaginary root or one real and radical root;
- solving polynomial equations analytically, graphically, and using appropriate technology;
- understanding the effects of degree, lead coefficient, and multiplicity of real zeros on the graph of a polynomial function; and
- determining characteristics of polynomial functions, including domain and range, intercepts, zeros, relative and absolute extrema, intervals of increase and decrease, and end behavior.

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Task 1: Polynomial Functions: What We Already Know

CCGPS Mathematics III
Task 1: ***Polynomial Functions: What We Already Know*** **Day 1/1**

CCSS Standard(s):

Algebra
Seeing Structure in Expressions A-SSE
Interpret the structure of expressions
1. Interpret expressions that represent a quantity in terms of its context.*
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*

Write expressions in equivalent forms to solve problems
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*
   c. Factor a quadratic expression to reveal the zeros of the function it defines.

Creating Equations* A-CED
Create equations that describe numbers or relationships
1. Create equations and inequalities in one variable and use them to solve problems.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Represent and solve equations and inequalities graphically
10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Functions
Interpreting Functions F-IF
Analyze functions using different representations
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
   d. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions F-BF
Build new functions from existing functions
3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

GPS Standard(s):
NOTE: Opportunities to revisit concepts learned in previous courses as a means of scaffolding for CCGPS Mathematics III and as a means of preparing students for GHSGT have been provided throughout this course. Concepts and skills addressed in this task were studied in Mathematics I and II. The topics prepare students for work with polynomials of degree 3 or higher and address content descriptors for the GHSGT.

New vocabulary: polynomial function of degree *n*

Mathematical concepts/skills:

- vocabulary related to polynomials and polynomial functions
- understanding linear and quadratic functions as polynomial functions
- determining characteristics of the graphs of linear and quadratic functions
- converting a quadratic function from standard to vertex form
- solving quadratic equations by factoring and by using the quadratic formula
- relating the zeros of a function to its graph
- graphing transformations of $f(x)$, including vertical stretches and shrinks, vertical and horizontal shifts, and reflections across the *x*- axis

Prior knowledge:

- vocabulary related to polynomials and polynomial functions
- understanding linear and quadratic functions as polynomial functions
- determining characteristics of the graphs of linear and quadratic functions
- converting a quadratic function from standard to vertex form
- solving quadratic equations by factoring and by using the quadratic formula
- relating the zeros of a function to its graph
- graphing transformations of $f(x)$, including vertical stretches and shrinks, vertical and horizontal shifts, and reflections across the *x*- axis

Essential question(s): What things do we already know about polynomial functions?

Suggested materials:

- resource materials for investigating vocabulary
- blank Frayer Model templates
- graph paper

Warm-up: Post a Frayer Model template and ask students to use it to define the term *quadratic function*.
(Note: A sample template is provided on page 15.)

Opening: Discuss student responses to the *Warm-up*, carefully explaining the characteristics of the Frayer Model.

Worktime: Students should work in pairs to complete *Items 1 – 8* of the task.
(Note: Teacher notes are provided on page….)

Assign the vocabulary listed in *Item 1* to pairs of students and ask each pair to complete a Frayer Model template on their term.

Once students have had an opportunity to complete the template, allow them to share their work making sure that all students have recorded definitions and examples of the vocabulary terms. (Frayer Model templates may be placed on the Word Wall for student reference.)

Allow time for students to complete *Items 2 – 4*. Have a short discussion of these items and of the formal definition of a polynomial function before allowing students to begin *Items 5 – 8*.

Items 5 – 8 review concepts and skills addressed in Math I and Math II that will be needed as students investigate polynomial functions of degree 3 or higher. Monitor carefully as students work through these problems, asking guiding questions that will help them recall pertinent information. Students should complete these problems without a graphing calculator.

Closing: Allow students to discuss *Items 5 -8*. Questions that help focus on the big ideas might include:

- How did you find the intercepts of the graph?
- What are other names for the x-intercepts of the graph of a function?
- How did you find the vertex?
- Show how you wrote the function in vertex form.
- What is the quadratic formula?
- What kind of roots does the function in *Item 7* have? How do we see these roots on the graph of the function?

Homework: The concepts and skills addressed in this task are review of Math I and Math II content. For this reason, additional practice problems have not been provided as homework. However, as always, teachers should assess student understanding and provide additional practice, if necessary.

Differentiated support/enrichment:

Check for understanding:

Resources/materials for Math Support: Students should preview:

- writing square roots of negative numbers in imaginary form;
- simplifying expressions involving complex numbers;
- finding real and complex solutions of quadratic equations by factoring, taking square roots, and applying the quadratic formula;
- analyzing the nature of roots using the discriminant;
- converting a quadratic function from standard to vertex form; and
- graphing quadratic functions considering domain, range, vertex, axis of symmetry, zeros, intercepts, extrema, intervals of increase and decrease, and rates of change.

CCGPS Mathematics III
Task 1: *Polynomial Functions: What We Already Know*
Student Task

We will begin our study of higher degree polynomial functions by reviewing some of the vocabulary associated with polynomials.

1. Write a definition, in your own words, of each of the following words or phrases. Illustrate your definition with one or more examples.
   a. term
   b. coefficient
   c. monomial
   d. binomial
   e. trinomial
   f. polynomial

2. The **degree of a monomial** is the sum of the exponents on the variables of the monomial. For example, the degree of the term $6x^2y^3$ is 5. Give the degree of each of the following terms.
   a. 7
   b. $6x^2$
   c. m
   d. $5x^2y^4$

3. The **degree of a polynomial** is the degree of the term with the highest degree. Give the degree of each of the following polynomials.
   a. $2x^3 + 5x + 6$
   b. $3x^2y – 7x^3y^2 + 8$
   c. $2x – 4$
   d. $x^2 – 3x^5 + 4x^3$

4. The term of a polynomial with highest degree is referred to as the **leading term** of the polynomial and its coefficient is referred to as the **leading coefficient**. Name the leading coefficient of the polynomial $x^2 – 3x^5 + 4x^3$.

A **polynomial function of degree *n*** is any function written in the form

$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$, where $a_n \neq 0$, n is a nonnegative integer, and $a_n, a_{n-1}, \ldots a_0$ are real numbers.

You already know a great deal about **polynomial functions**.

5. Consider the polynomial function $f(x) = mx + b$, where m and b represent constants and $m \neq 0$.
   a. What is the degree of this polynomial function?

   b. What name have you used to refer to this function in the past?

   c. Discuss what you already know about the function $f(x) = mx + b$. Use at least one example with specific values for m and b to illustrate your discussion.

6. Consider the polynomial function $f(x) = 2x^2 + 5x - 3$.
   a. What is the degree of this polynomial function?

   b. What name have you used to refer to this function in the past?

   c. Find the intercepts and vertex of this function and then sketch its graph. Show your work.

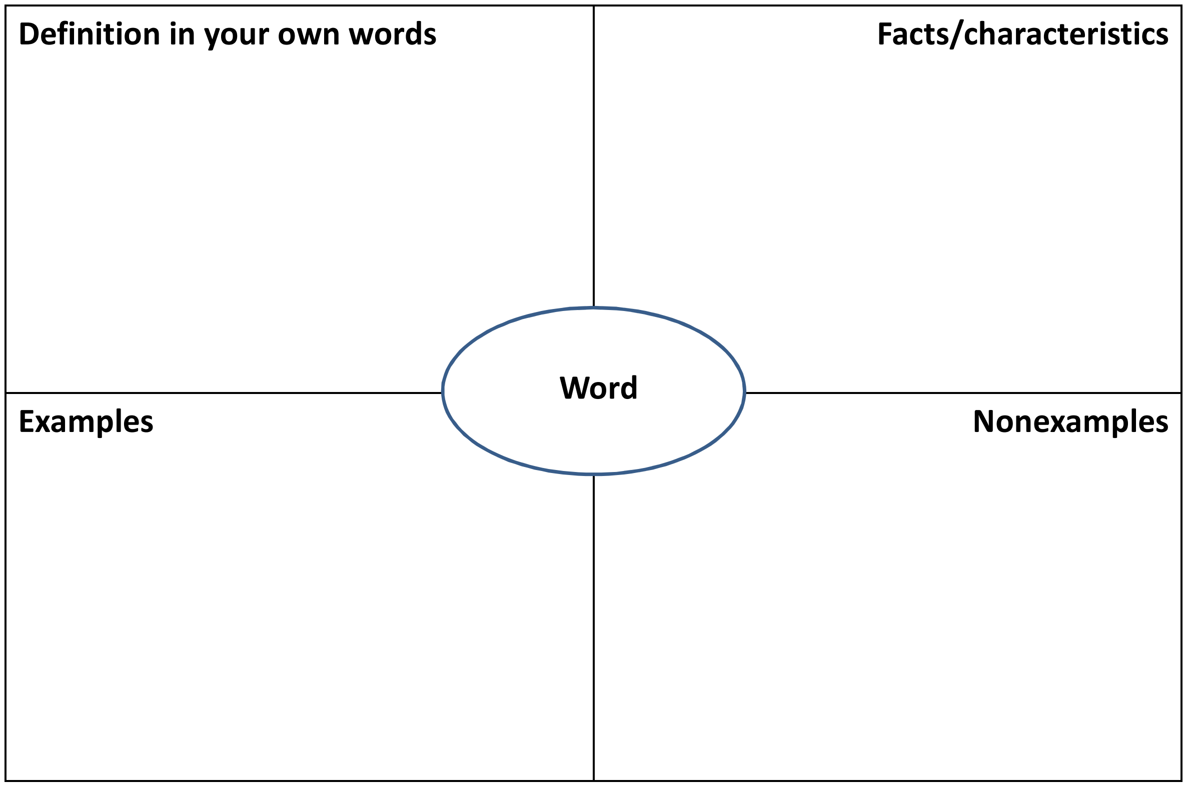
7. Consider the function $g(x) = 2x^2 - 4x + 5$.
   a. Write the function in vertex form.

   b. Find the zeros of the function and sketch its graph.

   c. Discuss how the roots of the function relate to its graph.

8. Graph the polynomial function $h(x) = -\frac{1}{2}(x-3)^3 + 5$ using transformations of the graph of $f(x) = x^3$. Describe each transformation in words.

Frayer Model



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Task 2: Interval Notation and Power Functions

CCGPS Mathematics III
Task 2: ***Interval Notation and Power Functions*** **Day 1/1**

CCSS Standard(s):

Functions
Interpreting Functions F-IF
Analyze functions using different representations

1. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
    c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

Building Functions F-BF
Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

GPS Standard(s):

MM3A1. Students will analyze graphs of polynomial functions of higher degree.

a. Graph simple polynomial functions as translations of the function $f(x) = ax^n$.
b. Understand the effects of the following on the graph of a polynomial function: degree, lead coefficient, and multiplicity of real zeros.
c. Determine whether a polynomial function has symmetry and whether it is even, odd, or neither.
d. Investigate and explain characteristics of polynomial functions, including domain and range, intercepts, zeros, relative and absolute extrema, intervals of increase and decrease, and end behavior.

New vocabulary: interval notation, open interval, closed interval, half-open interval, strictly increasing, strictly decreasing, opposite end behavior, like end behavior

Mathematical concepts/skills:

- represent intervals using interval notation
- determine characteristics of the graphs of power functions, $f(x) = ax^n$, including domain, range, intervals of increase and decrease, end behavior, and symmetry
- compare graphs of power functions of odd degree
- compare graphs of power functions of even degree
- determine the effect of the lead coefficient on the graph of $f(x) = ax^n$
- graph transformations of $f(x) = x^n$

Prior knowledge:

- using set-builder notation to represent sets of real numbers in a given interval
- graphing functions in the form $f(x) = ax^n$, where n = 1, 2, or 3
- graphing transformations of $f(x)$, including vertical stretches and shrinks, reflections across the x-axis, and horizontal and vertical shifts

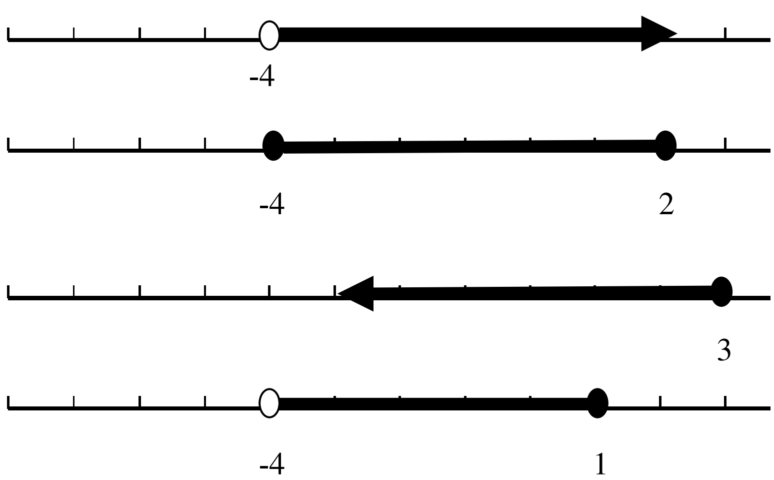
Essential question(s): How can I use interval notation to represent sets of real numbers? How do graphs of power functions compare?

Suggested materials:

- colored pencils
- patty paper
- graph paper
- graphing calculators

Warm-up: Post the following:

Use set-builder notation to represent the interval indicated on each of the number lines shown below.



Opening: Allow students to share their responses to the *Warm-up*. Discuss the information on interval notation on page one of the student task.

Worktime: Students should work in groups of three to complete *Items 1 – 4* of the task. *Item 5* should be completed individually as an assessment of the day's activities.

We suggest that in working *Item 2,* teachers have each student in the group focus on *one* of the graphs of the three functions given. This may be done by having each student complete one table and then students drawing all three graphs on his/her paper. Or, students may each draw one graph on patty paper and compare the three graphs by overlaying the three pieces of patty paper on one coordinate plane.

Discuss *Item 2* before allowing students to begin *Item 3*. (See teacher notes.) Once students understand how graphs of odd-powered functions compare, they should be able to make similar comparisons of even-powered functions by examining the graphs using graphing calculators.

Discuss *Items 3* and *4* before allowing students to begin working individually on *Item 5*.

Closing: Allow students to share their responses to *Item 5*. It will be useful to record graphs of even- and odd-powered functions, along with their characteristics, on an Anchor Chart. This information will be referenced throughout the remainder of the unit.

Homework: See attached.

Differentiated support/enrichment:

Check for understanding:

Resources/materials for Math Support: Students should preview:

- using set-builder notation to represent sets of real numbers on a given interval;
- graphing functions in the form $f(x) = ax^n$, where $n = 1, 2,$ or 3*; and*
- graphing transformations of $f(x)$, including vertical stretches and shrinks, reflections across the x-axis, and horizontal and vertical shifts.

CCGPS Mathematics III
Task 2: *Interval Notation and Power Functions*
Student Task

In discussing higher order polynomial functions we will use a new notation for intervals. In previous grades and courses you used the symbols <, >, ≤, and ≥ to represent intervals. For example, $\{x | -1 \leq x < 1\}$ indicates that x is the set of all real numbers greater than or equal to –1 and less than 1.

In Math III we will use **interval notation** to represent an interval on the number line or on an axis of the coordinate plane. Consider the following examples.

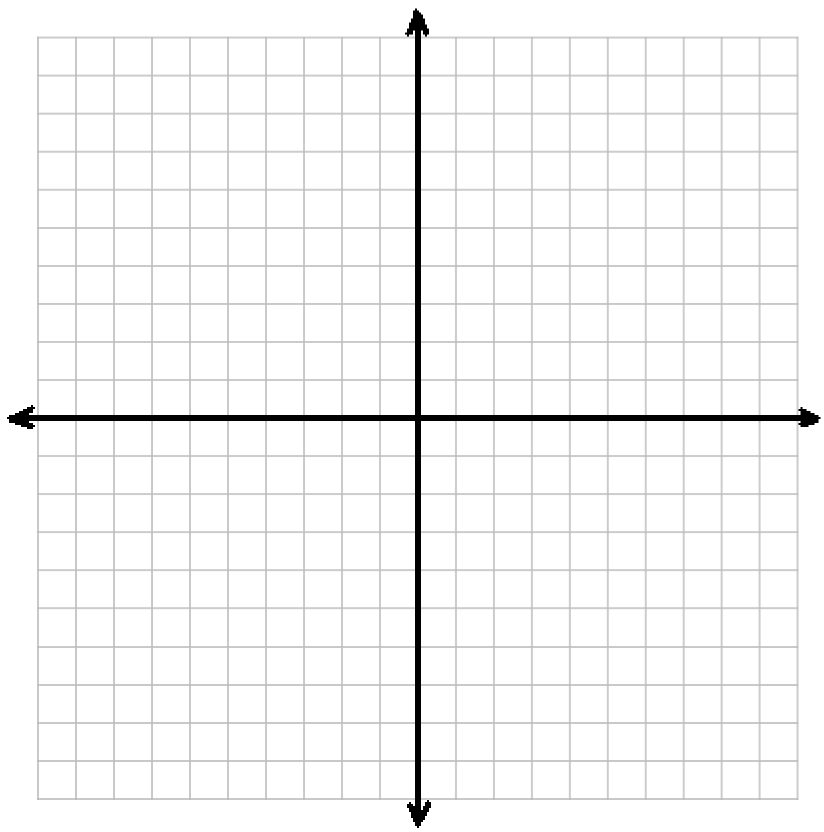
- The interval $-1 < x < 1$ is represented in interval notation as *(–1, 1)*. The *parentheses* indicate that –1 and 1 are *not included* in the interval. We say that the interval is **open** at –1 and 1.
- The interval $-1 \leq x < 1$ is represented in interval notation as [–1, 1). The *bracket* on the left indicates that –1 is *included* in the interval. The parenthesis on the right indicates that 1 is *not included* in the interval. We say that this interval is **half-open**.
- The interval $-1 \leq x \leq 1$ is represented in interval notation as [–1, 1]. The *brackets* indicate that both –1 and 1 are *included* in the interval and we say that this interval is **closed** at –1 and at 1.
- The notation $\{x | x \geq 5\}$ indicates that x is the set of all real numbers greater than or equal to 5. This interval is represented in interval notation as [5, ∞). A parenthesis is always used to enclose an infinity symbol.
- The set of all real numbers is represented in interval notation as (-∞, ∞).

1. Represent each of the following intervals using interval notation:
   a. $-3 < x \leq 5$
   b. $x < -4$
   c. $a \geq -8$

Now that we are familiar with interval notation, we will begin our study of higher degree polynomial functions by examining the graphs of some simple polynomial functions. Functions in the form $f(x) = ax^n$ are called **power functions**.

2. Graph the power functions $f_1(x) = x$, $f_2(x) = x^3$, and $f_3(x) = x^5$ on the coordinate plane provided below using the given values for x and a scale of ½ on both the x-and the y-axes.

x	y
-3	
-2	
-1	
-3/4	
-1/2	
-1/4	
0	
1/4	
1/2	
3/4	
1	
2	
3	



a. How do the graphs of these three functions compare:
   i. on the interval (–1, 1)?

   ii. when $x = 1$ and $x = -1$?

   iii. when x is greater than 1? Less than 1?

b. Describe the end behavior of these functions.

c. On what intervals are the functions:
   i. increasing?

   ii. decreasing?

d. How would you describe the symmetry of these functions? How does this relate to what you have already learned about the symmetry of functions?

3. Use your graphing calculator to graph the power functions $g_1(x) = x^2$, $g_2(x) = x^4$, and $g_3(x) = x^6$ all on the same set of axes.
   a. How do the graphs of these three functions compare:
      i. on the interval (–1, 1)?

      ii. when $x = 1$ and $x = -1$?

      iii. when x is greater than 1? Less than 1?

   b. Describe the end behavior of these functions.

   c. On what intervals are the functions:
      i. increasing?
      ii. decreasing?

   d. How would you describe the symmetry of these functions? How does this relate to what you have already learned about the symmetry of functions?

4. Power functions tell us a lot about the graphs of higher degree polynomial functions. Consider the function $h(x) = ax^n$. Use what you learned about transformations in Math I and Math II to show how a, the leading coefficient, would affect the graph of $h(x)$ when $a \neq 1$ by writing and graphing at least two power functions. In your examples include combinations of the following characteristics, addressing each characteristic at least once.

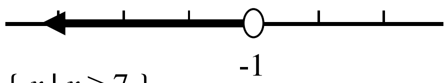
   - a power function of odd degree,
   - a power function of even degree,
   - a positive value for *a,*
   - a negative value for *a,*
   - an absolute value of *a* greater than 1, and
   - an absolute value of *a* greater than 1.

5. Summarize what you have learned to this point about graphs of functions in the form $f(x) = ax^n$ when n is odd and when n is even. Include in your discussion the shape of the graph, end behavior, symmetry, and the effect of the value of a (the leading coefficient).

CCGPS Mathematics III
Task 2: *Interval Notation and Power Functions*
Homework

In problems 1 – 3, use interval notation to represent each of the indicated sets of numbers.

1. $\{ x \mid -4 < x < 5 \}$

2. 

3. $\{ x \mid x \geq 7 \}$

4. Consider the function $g(x) = -2(x-3)^3 + 4$.

    a. Graph $g(x)$ as a transformation of $f(x) = x^3$.

    b. Describe each transformation in words.

    c. On what intervals is the function increasing? On what intervals is it decreasing?

    d. What kind of symmetry does $g(x)$ have? Explain your thinking.

    e. Describe the end behavior of $g(x)$.

5. Consider the function $h(x) = \frac{-1}{2}(x+2)^4 - 1$.

    a. Graph $h(x)$ as a transformation of $f(x) = x^4$.

    b. Describe each transformation in words.

    c. On what intervals is the function increasing? On what intervals is it decreasing?

    d. What kind of symmetry does $h(x)$ have? Explain your thinking.

    e. Describe the end behavior of $h(x)$.

Statement Concerning Mini-Lesson on Division of Polynomials and Related Theorems

Before beginning *Task 3* of this unit, teachers should conduct a mini-lesson on both long division and synthetic division. Due to the procedural nature of this content, a task was not developed to address these topics.

It is also worthwhile to introduce the Factor and Remainder Theorems as the division algorithms are taught. A statement of the theorems, along with a short student assignment, is provided on page 25 of this Supplement.

Student Assignment

The Factor and Remainder Theorems

We know that if r is a root of the quadratic function $f(x) = ax^2 + bx + c$, then $(x - r)$ is a factor of the quadratic polynomial $ax^2 + bx + c$. This is true for all polynomials and is stated in a very important theorem called the **Factor Theorem**.

Factor Theorem: A number r is a solution of the polynomial equation $f(x) = 0$ if and only if $(x - r)$ is a factor of $f(x)$.

Notice that the theorem is written in if-and-only-if form. What does this tell us?

Saying that $(x - r)$ is a factor of $f(x)$ means that when $f(x)$ is divided by $(x - r)$, the remainder is zero. This is a special case of another very important theorem called the **Remainder Theorem**.

Remainder Theorem: When the polynomial $f(x)$ is divided by $(x - r)$, the remainder is $f(r)$.

1. Consider the polynomial $P(x) = x^4 - 3x^3 + 7x^2 - 60x - 125$.
   a. Determine whether $(x - 5)$ is a factor of $P(x)$ by using the Factor Theorem.

   b. Determine whether $(x - 5)$ is a factor of $P(x)$ by first using the Remainder Theorem and then considering the Factor Theorem.

2. Use synthetic division and the Factor Theorem to prove that $(x + 4)$ is a factor of $P(x) = x^3 + 4x^2 - x - 4$.

3. Find a value for k such that $P(x) = x^3 - 3x^2 + k x + 3$ has $(x - 3)$ as a factor.

4. If $P(x) = x^4 - 3x^3 + kx^2 + x - 5$, find k such that when $P(x)$ is divided by $(x + 3)$ the remainder is - 8.

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Task 3: Finding Roots When Degree Is Three or Higher

CCGPS Mathematics III
Task 3: *Finding Roots When Degree Is Three or Higher* **Day 1/1**

CCSS Standard(s):

Algebra
Seeing Structure in Expressions A-SSE
Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*
    c. Factor a quadratic expression to reveal the zeros of the function it defines.

Arithmetic with Polynomials and Rational Expressions A-APR
Understand the relationship between zeros and factors of polynomials

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Functions
Interpreting Functions F-IF
Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
    d. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

GPS Standard(s):

MM3A1. Students will analyze graphs of polynomial functions of higher degree.

a. Graph simple polynomial functions as translations of the function $f(x) = ax^n$.
b. Understand the effects of the following on the graph of a polynomial function: degree, lead coefficient, and multiplicity of real zeros.
c. Determine whether a polynomial function has symmetry and whether it is even, odd, or neither.
d. Investigate and explain characteristics of polynomial functions, including domain and range, intercepts, zeros, relative and absolute extrema, intervals of increase and decrease, and end behavior.

MM3A3. Students will solve a variety of equations and inequalities.

a. Find real and complex roots of higher degree polynomial equations using the factor theorem, remainder theorem, rational root theorem, and fundamental theorem of algebra.
b. Solve polynomial…equations analytically, graphically, and using appropriate technology.

New vocabulary: distinct roots, multiplicity of roots, relative (or local) extrema, relative (or local) maximum values, relative (or local) minimum values

Mathematical concepts/skills:

- finding real and complex roots of higher degree polynomial equations using the Factor Theorem and the Remainder Theorem
- determining the nature of roots, including real and rational, real and irrational, and imaginary
- using technology to find and verify roots of higher degree polynomial functions
- determining the shapes of graphs of polynomial functions near roots of multiplicity *n*
- investigating and explaining characteristics of polynomial functions, including domain and range, intercepts, zeros, relative and absolute extrema, intervals of increase and decrease, and end behavior

Prior knowledge:

- finding roots of quadratic functions
- determining the nature of roots, including real, imaginary, rational, and irrational

Essential question(s): What tools can I use to investigate polynomial functions of degree 3 or higher?

Suggested materials:

- graph paper
- graphing calculators

Warm-up: Have students work individually to complete *Item 1* of the task.

Opening: Allow students to share responses to *Item 1*. Discuss whether roots are real and rational, real and irrational, or imaginary. Remind students that imaginary roots of polynomials with real coefficients occur in conjugate pairs. Irrational roots of polynomial functions with rational coefficients also occur in conjugate pairs. (In this course, we are only concerned with polynomial functions with rational coefficients.)

Worktime: Students should work in pairs to complete *Items 2 - 8* of the task. *Items 9 - 11* can be assigned as homework if time does not permit that they be completed in class.

After students have had time to complete *Item 4* have a whole class discussion of *Items 2 - 4* to be sure that all students understand the concepts addressed. (See teacher notes.) Encourage students to use the *CALCULATE* menu and then the *zero* function, of their graphing calculators to verify or find zeros of functions.

Discuss the information included in the task on roots of multiplicity *n* and shapes of graphs near those roots before allowing students to begin *Item 5*.

Monitor work carefully as students complete *Items 5 – 7*. When ample time has been given to complete these problems, have a class discussion to share responses and address any questions that students may have.

Note that in *Item 6*, students are asked to sketch a graph that could represent *any* polynomial function with the given characteristics. In *Item 7*, they are asked to find a *specific* polynomial function that meets the given conditions.

Questions for discussion might include:

- How many roots does a sixth degree polynomial have? Explain.
- How many roots does an nth degree polynomial have? Explain.
- What is meant by *distinct* roots?
- Explain how we write a polynomial function of degree n as a product of linear factors.
- How do imaginary roots affect the graph of a polynomial function?

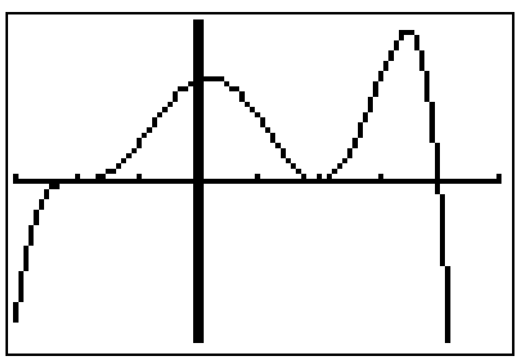
Before allowing students to begin *Item 8*, discuss extrema. Make sure students understand if a turning point occurs at the point (x_1, y_1), then y_1 is the extreme value of the function and it occurs at $x = x_1$.

Closing: Allow students to share responses to *Item 8.*

Homework: Students should complete *Items 9 - 11* of the task if not completed in class.

Differentiated support/enrichment:

Check for understanding: List everything you know about this polynomial function. (The scale on the x-axis is 1.)



Resources/materials for Math Support: Students should preview:

- solving quadratic equations by factoring and using the quadratic formula
- determining the nature of roots of quadratic equations-including real and rational, real and irrational, or imaginary

CCGPS Mathematics III
Task 3: *Finding Roots When Degree Is Three or Higher*
Student Task

Finding Roots When Degree Is 3 or Higher

In the first task of this unit, you reviewed what you already know about linear, quadratic, and cubic functions - in other words, polynomial functions of degrees 1, 2, and 3, respectively. In the second task you found, by examining power functions, that the graphs of odd powered functions have a kind of *S* shape. Graphs of even powered functions are shaped somewhat like a *U*.

In this task we will focus on finding the roots of polynomial functions. Knowing the real roots of functions is a great help not only in drawing the graphs of the functions but also in examining real-world situations that can be modeled by the functions. You know that you can always find the roots of a linear or quadratic function even when, in the case of many quadratic functions, those roots are imaginary numbers. For higher degree polynomial functions (degree 3 and higher) it is often very difficult to find the roots and equally difficult to draw their graphs without the help of technology. In this task we will investigate some of the simpler techniques used to examine roots and rely on technology to help us address the more challenging issues.

We will begin by reviewing what we know about finding roots of quadratic functions.

When the roots of a quadratic function are rational, we can often find them quickly by factoring. Whether the roots are rational, irrational or imaginary, we can *always* find them by using the quadratic formula:

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Remember that $b^2 - 4ac$, the discriminant, gives us the ability to determine the nature of the roots.

$$b^2 - 4ac \begin{cases} > 0, & 2\ real\ roots \\ = 0, & 1\ real\ root \\ < 0, & 2\ imaginary\ roots \end{cases}$$

1. Find the roots of the following quadratic functions. State whether the roots are real and rational, real and irrational, or imaginary.

    a. $f(x) = x^2 - 5x - 14$
    b. $f(x) = 4x^2 - 2x + 9$
    c. $f(x) = 3x^2 + 4x - 8$

When a polynomial is of higher degree, we can use a lot of what we already know to help us find the zeros of the function. Suppose we want to find the roots of $f(x) = x^3 + 2x^2 - 5x - 6$. By using technology to inspect the graph, we discover that one of the real roots of the function is 2. Then, by the Factor Theorem, we know that $(x - 2)$ is a factor of the polynomial $x^3 + 2x^2 - 5x - 6$. We can now use long division to find the remaining quadratic factor. Once we obtain a quadratic factor, we can find the remaining roots by using the quadratic formula whether the roots are real or imaginary.

Using long division or the shortcut of synthetic division, we find that $x^3 + 2x^2 - 5x - 6$ factors as $(x - 2)(x^2 + 4x + 3)$. Factoring again we obtain $(x - 2)(x + 3)(x + 1)$.
Thus $f(x) = (x - 2)(x + 3)(x + 1)$ and the roots of the function are – 3, - 1, and 2.

2. Explain how you could use your graphing calculator to verify that the roots of

   $f(x) = (x - 2)(x + 3)(x + 1)$ are – 3, - 1, and 2.

3. Four is a real root of the function $g(x) = x^3 - 2x^2 - 11x + 12$.

   a. Use division and factoring to find the remaining roots.

   b. Use your graphing calculator to verify your results.

4. The function $h(x) = x^4 - 9x^3 + 17x^2 + 33x - 90$ factors as $h(x) = (x - 3)^2 (x + 2)(x - 5)$.

   a. What are the roots of this function?

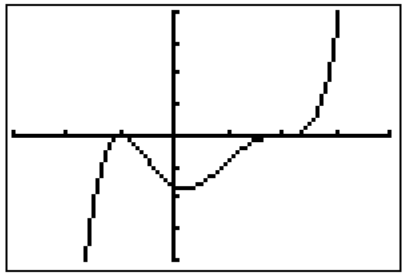
   b. Graph the function on your graphing calculator and copy the graph onto graph paper.

   c. What do you notice about the shape of the function near $x = 3$? What do you notice about the factor corresponding to a root of 3?

When the factor corresponding to a given root occurs n times, we say the root has a **multiplicity** of n. For example, in *Item 4,* the factor $(x - 3)$ is squared. Therefore we say that the root 3 has a *multiplicity of 2*. Notice that the graph of $h(x)$ near $x = 3$ is shaped much like a parabola, the shape of a quadratic function. A similar situation occurs for any given multiplicity.

For any function $f(x)$:

- If the multiplicity of a root r is 1, the graph of $f(x)$ will cross the x-axis appearing to be somewhat linear near $x = r$.
- If the multiplicity of a root r is odd and greater than 1, the graph of $f(x)$ will flatten out and cross the x-axis as it passes through $x = r$. The higher the multiplicity of r, the flatter the graph will become near $x = r$.
- If the multiplicity of a root r is even, the graph of $f(x)$ will be tangent to, but will not cross, the x-axis at $x = r$. The higher the multiplicity of r, the flatter the graph will be near $x = r$.



5. On the graph shown above, the scale on the *x*-axis is 1. Choose the function below that best fits the graph. Explain your choice. Include in your explanation the degree of the polynomial, the lead coefficient, end behavior, and zeros.
    a. $f(x) = (x - 1)^2 (x + 2)$
    b. $f(x) = -(x + 1)^2 (x + 2)$
    c. $f(x) = (x + 1)^2 (x - 2)^3$
    d. $f(x) = -(x - 1)^2 (x + 2)^3$

6. Sketch your own graphs that indicate the shape of a polynomial function with the given characteristics.
    a. A polynomial function has degree 4; a negative leading coefficient; and real zeros 3 with multiplicity 2, - 2 with a multiplicity of 1, and - 3 with a multiplicity of 1.

    b. A polynomial function has degree 5; a positive leading coefficient; real zeros 3 with multiplicity 2, - 1 with a multiplicity of 1, and 2 imaginary zeros.

From your work to this point, you have probably realized that a polynomial function of degree *n* has *n* roots if we count each multiplicity as a root. We say that a polynomial of degree *n* has at most *n distinct* (or different) roots. Real roots appear as points of intersection of the function and the *x*-axis. Imaginary roots occur in pairs and are not visible on a graph. We will talk briefly about these roots in the next task.

Since a polynomial function of degree *n* can have *n* roots (given that we count multiplicities), it can also have *n* linear factors and can be written in the form $f(x) = a(x - r_1)(x - r_2)\ldots\ldots(x - r_n)$ where *a* is the leading coefficient and $r_1, r_2, \ldots r_n$ are zeros of the polynomial.

7. A third degree polynomial function has roots 2, -1, and 3 and a *y*-intercept at 12. Find the function.

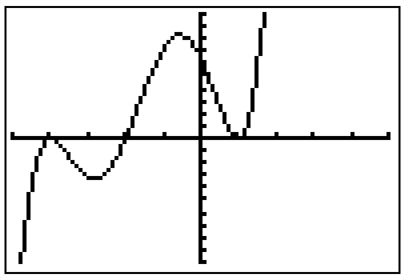
A turning point is a point where the graph of a function changes from increasing to decreasing or vice versa. These points are lower (or higher) than nearby points and so they are referred to as **relative** (or **local) extrema**. A point that is lower (has a smaller value) than nearby points is called a **relative (or local) minimum value** of the function. A point that is higher (has a larger value) than nearby points is called a **relative maximum** value of the function. **A polynomial function of degree *n* can have at most *n - 1* relative extrema.**

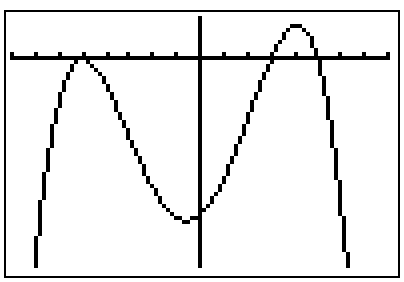
For each of the graphs in *Items 8 – 11*:

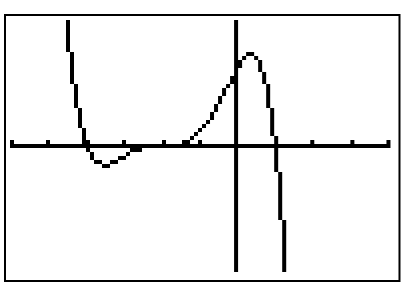
- the *x*-scale is 1 unit,
- the polynomial function represented has only real roots, and
- the leading coefficient of each polynomial function is 1 or – 1.

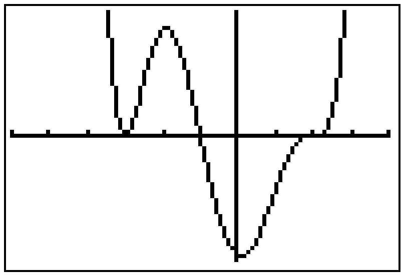
For each graph:

a. Name the zeros of the polynomial function. State the multiplicity of each zero. (Assume the lowest possible multiplicity.)
b. Write a function of lowest possible degree in factored form that will produce the graph.
c. Give the degree of the polynomial function.
d. Give the number of relative extrema for each graph.

8. 

9. 

10. 

11. 

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Task 4: Using the Theorems of Great Mathematicians

CCGPS Mathematics III
Task 4: ***Using the Theorems of Great Mathematicians*** **Day 1/2**

CCSS Standard(s):

Algebra
Seeing Structure in Expressions A-SSE
Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*
    c. Factor a quadratic expression to reveal the zeros of the function it defines.

Arithmetic with Polynomials and Rational Expressions A-APR
Understand the relationship between zeros and factors of polynomials

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Functions
Interpreting Functions F-IF
Analyze functions using different representations

a. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
    c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

GPS Standard(s):

MM3A1. Students will analyze graphs of polynomial functions of higher degree.

b. Understand the effects of the following on the graph of a polynomial function: degree, lead coefficient, and multiplicity of real zeros.
d. Investigate and explain characteristics of polynomial functions, including domain and range, intercepts, zeros, relative and absolute extrema, intervals of increase and decrease, and end behavior.

MM3A3. Students will solve a variety of equations and inequalities.

a. Find real and complex roots of higher degree polynomial equations using the factor theorem, remainder theorem, rational root theorem, and fundamental theorem of algebra.
b. Solve polynomial…equations analytically, graphically, and using appropriate technology.

New vocabulary: Integral Root Theorem, Rational Root Theorem

Mathematical concepts/skills:

- finding real and complex roots of higher degree polynomial equations using the Factor Theorem, Remainder Theorem, Rational Root Theorem, and Fundamental Theorem of Algebra.
- solving polynomial equations analytically, graphically, and using appropriate technology
- understanding the effects of degree, lead coefficient, and multiplicity of real zeros on the graph of a polynomial function
- determining characteristics of polynomial functions, including domain and range, intercepts, zeros, relative and absolute extrema, intervals of increase and decrease, and end behavior

Prior knowledge:

- finding roots of linear and quadratic functions
- determining the nature of roots, including real, imaginary, rational, and irrational
- determining characteristics of functions, including domain and range, intercepts, zeros, relative and absolute extrema, intervals of increase and decrease, and end behavior

Essential question(s): How can I find the roots of higher degree polynomial equations?

Suggested materials:

- graph paper
- graphing calculators

Warm-up: Students should work with a partner to compare responses to *Items 9 – 11* of the previous task, if these items have not been addressed in a whole class discussion. Ask students to be prepared to share responses and to ask any remaining questions they have related to the items.

Opening: After a discussion of the items described in the *Warm-up,* ask students to read the first three paragraphs of the task and complete *Item 1*. Have a whole-class discussion of the Integral Root Theorem and responses to *Item 1* before allowing students to begin *Item 2*.

Worktime: Students should work in pairs to complete *Items 2 – 8* of the task.

Give students an opportunity to complete *Items 2 – 4* before having a whole-class discussion of the Rational Root Theorem and responses to these items. Questions that might be asked, in addition to those included in the task, are:

- How did you decide which possible integral roots were actual roots of $g(x)$? Did anyone use a different method for finding the roots?
- State the Rational Root Theorem in your own words.
- How does the Rational Root Theorem differ from the Integral Root Theorem?
- Once we know the Rational Root Theorem, do we still need the Integral Root Theorem? Why of why not?

Students should complete *Items 5 – 8* of the task. Questions in *Item 8* will require students to use the *maximum/minimum* feature under the *CALCULATE* menu of the graphing calculator. It may be necessary to demonstrate these features if they have not been used previously.

Closing: Allow students to share responses to *Items 5 – 8*.

In examining the graph of each function, it is important to discuss the characteristics studied in this and previous tasks. Students are asked, in the task, to answer the questions below only as they relate to the graph in *Item 8.* However, you may want to ask these questions verbally of each graph as students present during the closing.

a. Does the graph of the function indicate that the roots you found algebraically are correct? How so?
b. Does this function have any relative extrema? If so, how many?
c. Give each local maximum/minimum value of the function and state where it occurs.
d. On what intervals is this function increasing? On what intervals is it decreasing?
e. Describe the end behavior of the function.
f. Explain the effects of the degree and the lead coefficient on the graph of the function.

Homework: (See attached.)

Differentiated support/enrichment:

Check for understanding:

Resources/materials for Math Support: Students should preview:

- solving quadratic equations by factoring and using the quadratic formula
- determining the nature of roots of quadratic equations-including real and rational, real and irrational, or imaginary
- determining characteristics of functions, including domain and range, intercepts, zeros, relative and absolute extrema, intervals of increase and decrease, and end behavior

CCGPS Mathematics III
Task 4: ***Using the Theorems of Great Mathematicians***
Day 1 Student Task

In this task we will look more closely at finding roots of higher degree polynomial functions. We will be aided by theorems written by great mathematicians, including Carl Friedrich Gauss. Gauss was the first mathematician to demonstrate that every polynomial function can be written as a product of linear and quadratic factors. Just think, if we can write a function as a product of linear and quadratic factors, we can find all of its zeros!

You have already been introduced to two very important theorems – the Remainder Theorem and the Factor Theorem. We will now consider a third theorem called the **Integral Root Theorem**.

The Integral Root Theorem: If an integer r is a root of a polynomial function with integral coefficients and a leading coefficient of 1, than r is a factor of the constant term of the polynomial.

1. Consider the cubic function $f(x) = x^3 + 2x^2 - x - 2$.
    a. Does the function $f(x)$ fit the conditions of the Integral Root Theorem? Explain your thinking.

    b. Based on this theorem, what are the possible integral roots of $f(x)$?

    c. Are any of these possibilities actual roots of $f(x)$? Show how you know.

    d. Find all roots of $f(x)$.

2. Consider the cubic function $g(x) = x^4 + 7x^3 + 12x^2 - 4x - 16$.
    a. Does the function $g(x)$ fit the conditions of the Integral Root Theorem? Explain your thinking.

    b. Based on this theorem, what are the possible integral roots of $f(x)$?

    c. Are any of these possibilities actual roots of $f(x)$? Show how you know.

    d. Find all roots of $g(x)$.

The **Rational Root Theorem** is an extension of the Integral Root Theorem. It allows us to find possible roots of polynomials with integral coefficients but with a leading coefficient other than 1.

Rational Root Theorem: If the nonzero rational number p/q, in lowest terms, is a root of a polynomial function in standard form that has integral coefficients, then p must be a factor of the constant term of the polynomial function and q must be a factor of the leading term.

Let's consider the function $h(x) = 2x^3 - 3x^2 - 2x + 3$. The factors of the constant term 3 are {± 3, ± 1}. The factors of the leading term are {± 1, ± 2}. According to our theorem, any possible rational roots must be a factor of 3 divided by a factor of 2. So possible rational roots are $\left\{\pm 3, \frac{\pm 3}{2}, \pm 1, \frac{\pm 1}{2}\right\}$. This gives us eight possible rational roots to test. (Glad we know how to use synthetic division!)

3. Consider $h(x) = 2x^3 - 3x^2 - 2x + 3$ discussed above.

   a. How many rational roots of *h(x)* would you need to find by using synthetic division or by evaluating *h* at possible roots? Explain.

   b. Find all roots of *h(x).* Show your work.

4. Consider the polynomial $f(x) = x^3 - 5x^2 - 4x + 20$.

   a. Identify *p* (all the factors of 20).

   b. Identify *q* (all the factors of the lead coefficient, 1).

   c. Identify all possible combinations of $\frac{p}{q}$.

   d. Are any of the values for $\frac{p}{q}$ in *part c* roots of *f(x)*? Show how you know.

   e. Find all roots of *f(x).* Show your work.

For each of the functions in *Items 5 - 7*, find the roots and write the function in factored form. Verify your work by graphing the function on your graphing calculator and copying the graph onto your paper.

5. $f(x) = x^3 + 2x^2 - 5x - 6$

   Possible rational roots:

   Work for finding roots:

   Complete factorization: ______________________________________

   Graph:

6. $f(x) = -2x^3 + 5x^2 + 8x - 3$

   Possible rational roots:

   Work for finding roots:

   Complete factorization: ________________________________

   Graph:

7. $f(x) = x^4 + 2x^3 - 9x^2 - 2x + 8$

   Possible rational roots:

   Work for finding roots:

   Complete factorization: ________________________________

   Graph:

For the function *f(x)* below, find the roots and write the function in factored form. Verify your work by graphing the function on your graphing calculator and copying the graph onto your paper. Answer questions *a - f* related to the graph. (You may need to use the *CALCULATE* menu of your graphing calculator to help you answer some of these questions.)

8. $f(x) = x^5 - 4x^4 - 15x^3 + 14x^2 + 28x - 24$

   Possible rational roots:

   Work for finding roots:

   Complete factorization: ________________________________

   Graph:

   a. Does the graph of the function indicate that the roots you found algebraically are correct? How so?
   b. Does this function have any relative extrema? If so, how many?
   c. Give each local maximum/minimum value of the function and state where it occurs.
   d. On what intervals is this function increasing? On what intervals is it decreasing?
   e. Describe the end behavior of the function.
   f. Explain the effects of the degree and the lead coefficient on the graph of the function.

CCGPS Mathematics III
Task 4: ***Using the Theorems of Great Mathematicians***
Day 1 Homework

For the functions below, find the roots and write the functions in factored form. Verify your work by graphing the functions on your graphing calculator and copying the graph onto your paper. Answer the questions related to the graph. (You may need to use the *CALCULATE* menu of your graphing calculator to help you answer some of these questions.)

1. $f(x) = x^6 - x^4$

    Possible rational roots:

    Work for finding roots:

    Complete factorization: ______________________________________

    Graph:

    a. Does the graph of the function indicate that the roots you found algebraically are correct? How so?

    b. Is the function an even function, an odd function, or neither. Explain your thinking.

    c. Does this function have any relative extrema? If so, how many?

    d. Give each local maximum/minimum value of the function and state where it occurs.

    e. On what intervals is this function increasing? On what intervals is it decreasing?

    f. Describe the end behavior of the function.

    g. Explain the effects of the degree and the lead coefficient on the graph of the function.

2. $f(x) = x^4 - 6x^3 + 10x^2 + 2x - 15$

Possible rational roots:

Work for finding roots:

Complete factorization: ______________________________________

Graph:

a. Does the graph of the function indicate that the roots you found algebraically are correct? How so?

b. Is the function an even function, an odd function, or neither. Explain your thinking.

c. Does this function have any relative extrema? If so, how many?

d. Give each local maximum/minimum value of the function and state where it occurs.

e. On what intervals is this function increasing? On what intervals is it decreasing?

f. Describe the end behavior of the function.

g. Explain the effects of the degree and the lead coefficient on the graph of the function.

CCGPS Mathematics III
Task 4: ***Using the Theorems of Great Mathematicians*** **Day 2/2**

CCSS Standard(s):

Algebra
Seeing Structure in Expressions A-SSE
Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*
    c. Factor a quadratic expression to reveal the zeros of the function it defines.

Arithmetic with Polynomials and Rational Expressions A-APR
Understand the relationship between zeros and factors of polynomials

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Functions
Interpreting Functions F-IF
Analyze functions using different representations

a. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
    c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

GPS Standard(s):

MM3A1. Students will analyze graphs of polynomial functions of higher degree.

b. Understand the effects of the following on the graph of a polynomial function: degree, lead coefficient, and multiplicity of real zeros.
d. Investigate and explain characteristics of polynomial functions, including domain and range, intercepts, zeros, relative and absolute extrema, intervals of increase and decrease, and end behavior.

MM3A3. Students will solve a variety of equations and inequalities.

a. Find real and complex roots of higher degree polynomial equations using the factor theorem, remainder theorem, rational root theorem, and fundamental theorem of algebra.
b. Solve polynomial…equations analytically, graphically, and using appropriate technology.

New vocabulary: Corollary, Complex Conjugate Theorem, Conjugate Radical Theorem

Mathematical concepts/skills:

- finding real and imaginary roots of higher degree polynomials with rational coefficients given one imaginary root or one real and radical root

- finding real and imaginary roots of higher degree polynomials using the Factor Theorem, Remainder Theorem, Rational Root Theorem, Fundamental Theorem of Algebra, Complex Conjugate Theorem, and Conjugate Radical Theorem.
- solving polynomial equations analytically, graphically, and using appropriate technology
- understanding the effects of degree, lead coefficient, and multiplicity of real zeros on the graph of a polynomial function
- determining characteristics of polynomial functions, including domain and range, intercepts, zeros, relative and absolute extrema, intervals of increase and decrease, and end behavior

Prior knowledge:

- finding roots of linear and quadratic functions
- determining the nature of roots, including real and rational, real and irrational, or imaginary
- determining characteristics of functions, including domain and range, intercepts, zeros, relative and absolute extrema, intervals of increase and decrease, and end behavior

Essential question(s): How can I find the roots of higher degree polynomial equations?

Suggested materials:

- graph paper
- graphing calculators

Warm-up: Post the following:

Perform each of the following operations. Write the result in simplest form.

1. $(2 + 3i)(2 - 3i)$
2. $(1 + 2\sqrt{3})(1 - 2\sqrt{3})$
3. $[x - (1 + 4i)][x - (1 - 4i)]$

Opening: Discuss the *Warm-up*.

After discussing *Problems 1 – 3* above have students read silently and then discuss each of the four theorems presented in this part of task. As stated, these theorems contain concepts that students have been using to find roots of both quadratic and higher degree polynomial functions.

Worktime: Students should work in pairs to complete *Items 9 – 12* of the task.

After students have had an opportunity to complete *Item 9*, have a whole-class discussion to make sure students understand all four theorems.

Closing: Allow students to share responses to *Items 10 – 12*.

Homework:

Differentiated support/enrichment:

Check for understanding:

Resources/materials for Math Support: Students should preview:

- solving quadratic equations by factoring and using the quadratic formula
- determining the nature of roots of quadratic equations-including real and rational, real and irrational, or imaginary
- determining characteristics of functions, including domain and range, intercepts, zeros, relative and absolute extrema, intervals of increase and decrease, and end behavior

CCGPS Mathematics III
Task 4: ***Using the Theorems of Great Mathematicians***
Day 2 Student Task

The following theorems are simply formal statements of ideas that we have learned to this point.

(Note: In reading and working with these theorems, remember that complex numbers are numbers written in the form $a + bi$, where a and b are real numbers. If b is zero, then $a + 0i$ is a real number. In other words, the set of complex numbers includes both the real and the imaginary numbers.)

A Corollary of the Fundamental Theorem of Algebra:
If $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$, $a_n \neq 0$, is a polynomial of positive degree with complex coefficients, then $f(x)$ has n linear factors.

Corollary: If $f(x)$ is a polynomial function with complex coefficients and degree n, then f has exactly n zeros.
(Note: This corollary does not state that f has n *distinct* zeros. In this case, we are counting each multiplicity as a zero.)

Complex Conjugate Theorem: If a complex number $a + bi$ is a zero of a polynomial function with real coefficients, then its conjugate, $a – bi$, is also a zero of the polynomial.
(Note: We already know this from our work with the Quadratic Formula, right?)

Conjugate Radical Theorem: If $a+\sqrt{b}$ is a zero of a polynomial function with rational coefficients, where a and b are rational, but $\sqrt{b}$ is irrational, then the conjugate number $a-\sqrt{b}$ is also a root of the polynomial.

9. Let's use the function $h(x) = x^4 – 4x^3 + 5x^2 – 16x + 4$ to make sure we understand the four theorems stated above.
   a. Considering each multiplicity, how many roots does $h(x)$ have?

   b. If $2i$ is one root, what other number is a root of $h(x)$? Write the factors that correspond to these two roots and multiply them to obtain a quadratic polynomial.

   c. If $2-\sqrt{3}$ is a root, what other number is a root of $h(x)$? Write the factors that correspond to these two roots and multiply them to obtain a quadratic polynomial.

   d. Multiply the two quadratic polynomials obtained in *parts b* and *c*. How does this polynomial compare to $h(x)$?

10. Consider $k(x) = x^3 - 5x^2 + 8x - 6$.
    a. Find and classify the roots of $k(x)$. Roots should be classified as real and rational, real and irrational, or imaginary.

    b. Write $k(x)$ in factored form.

    c. Verify your work by graphing the function on your graphing calculator and copying the graph onto your paper.

11. Consider $f(x) = x^4 - 2x^2 - 3$.
    a. Find all roots of $f(x)$, if $x = \sqrt{3}$ is one root. Show your work

    b. Classify each root of $f(x)$.

    c. Write $f(x)$ in factored form.

    d. Verify your work by graphing $f(x)$ on your graphing calculator and copying the graph onto your paper.

12. Consider $f(x) = x^4 - x^3 + 3x^2 - 9x - 54$.
    a. Find and classify all roots of $f(x)$. Show your work

    b. Write $f(x)$ in factored form.

    c. Verify your work by graphing $f(x)$ on your graphing calculator and copying the graph onto your paper.