ATLANTA PUBLIC SCHOOLS
Mathematics & Science Initiative
Making A Difference

Atlanta Public Schools

Teacher's Curriculum Supplement

Common Core Georgia Performance Standards Mathematics III Unit 1: Matrices

GE Foundation

This document has been made possible by funding from the GE Foundation Developing Futures grant, in partnership with Atlanta Public Schools. It is derived from the Georgia Department of Education Math III Framework and includes contributions from Georgia teachers. It is intended to serve as a companion to the GA DOE Math III Framework Teacher Edition.

Common Core Georgia Performance Standards Mathematics III

This course has been adapted from *Mathematics III* of the Georgia Performance Standards with consideration given to the following documents:

- The Common Core State Standards for Mathematics,
- The Georgia High School Graduation Test Content Descriptions for 2011,
- The Georgia Department of Education Mathematics III Frameworks Teacher Edition, and
- The Georgia Department of Education Mathematics III Curriculum Map.

Care has been taken to assure that those topics tested on the Georgia High School Graduation Test are included in the first semester of this course. Upon completion of *Mathematics I, Mathematics II*, and *CCGPS Mathematics III*, students will have had the opportunity to address the standards specified by Common Core State Standards as *the mathematics that all students should learn in order to be college and career ready*. In an effort to be well aligned with the GPS standards for Mathematics III, a few of the topics included here go beyond the Common Core State Standards benchmark for all students, addressing content recommended for those students wishing to pursue careers and majors in STEM fields.

This course was developed in collaboration with the Georgia Department of Education and may be used in lieu of the GaDOE GPS *Mathematics III*. The prerequisite for the course is successful completion of GPS *Mathematics II*.

Preface

We are pleased to provide this supplement to the Georgia Department of Education's Mathematics III Framework. It has been written in the hope that it will assist teachers in the planning and delivery of the new curriculum, particularly in these first years of implementation. This document should be used with the following considerations.

- The importance of working the tasks used in these lessons cannot be overstated. In planning for the teaching of the Georgia Performance Standards in Mathematics teachers should work the tasks, read the teacher notes provided in the Georgia Department of Education's Mathematics III Framework Teacher Edition, and *then* examine the lessons provided here.

- This guide provides day-by-day lesson plans. While a detailed scope and sequence and established lessons may help in the implementation of a new and more rigorous curriculum, it is hoped that teachers will assess their students informally on an on-going basis and use the results of these assessments to determine (or modify) what happens in the classroom from one day to the next. Planning based on student need is much more effective than following a pre-determined timeline.

- It is important to remember that the Georgia Performance Standards provide a balance of concepts, skills, and problem solving. Although this document is primarily based on the tasks of the Framework, we have attempted to help teachers achieve this all important balance by embedding necessary skills in the lessons and including skills in specific or suggested homework assignments. The teachers and writers who developed these lessons, however, are not in your classrooms. It is incumbent upon the classroom teacher to assess the skill level of students on every topic addressed in the standards and provide the opportunities needed to master those skills.

- In most of the lesson templates, the sections labeled *Differentiated support/enrichment* have been left blank. This is a result of several factors, the most significant of which was time. It is hoped that as teachers use these lessons, they will contribute their own ideas, not only in the areas of differentiation and enrichment, but in other areas as well. Materials and resources abound that can be used to contribute to the teaching of the standards.

On the topic of differentiation, it is critical to reiterate that many of the strategies used in a standards-based mathematics classroom promote differentiation. These strategies include

- the use of rich tasks with multiple points of entry and more than one path to a solution,
- flexible grouping of students,
- multiple representations of mathematical concepts,
- writing in mathematics,
- monitoring of progress through on-going informal and formative assessments, and
- analysis of student work.

We hope that teachers will incorporate these strategies in each and every lesson.

It is hoped that you find this document useful as we strive to raise the mathematics achievement of all students in our district and state. Comments, questions, and suggestions for inclusions related to this document may be emailed to Dr. Dottie Whitlow, Executive Director, Mathematics and Science Department, Atlanta Public Schools, dwhitlow@atlantapublicschools.us.

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Explanation of the Terms and Categories Used in the Lesson Template

Task: This section gives the suggested number of days needed to teach the concepts addressed in a task, the task name, and the problem numbers of the task as listed in the Georgia Department of Education's Mathematics III Framework Teacher Edition (GaDOE TE).

In some cases new tasks or activities have been developed. These activities have been named by the writers.

Standard(s): Key standards addressed in the lesson are listed in this section. Standards listed first, in regular type, are from the Common Core State Standards for Mathematics. Standards in bold type are the corresponding standards from Mathematics III of the Georgia Performance Standards.

New Vocabulary: Vocabulary is listed here the *first* time it is used. It is strongly recommended that teachers, particularly those teaching Math Support, use interactive word walls. Vocabulary listed in this section should be included on the word walls and previewed in Math Support.

Mathematical concepts/skills: Major concepts addressed in the lesson are listed in this section whether they are CCGPS Math III concepts or were addressed in earlier grades or courses.

Prior knowledge: Prior knowledge includes only those topics studied in previous grades or courses. It does not include CCGPS Math III content taught in previous lessons.

Essential Question(s): Essential questions may be daily and/or unit questions.

Suggested materials: This is an attempt to list all materials that will be needed for the lesson, including consumable items, such as graph paper; and tools, such as graphing calculators and compasses. This list does not include those items that should always be present in a standards-based mathematics classroom such as markers, chart paper, and rulers.

Warm-up: A suggested warm-up is included with every lesson. Warm-ups should be brief and should focus student thinking on the concepts that are to be addressed in the lesson.

Opening: Openings should set the stage for the mathematics to be done during the work time. The amount of class time used for an opening will vary from day-to-day but this should not be the longest part of the lesson.

Worktime: The problem numbers have been listed and the work that students are to do during the worktime has been described. A student version of the day's activity follows the lesson template in every case. In order to address all of the standards in CCGPS Math III, some of the problems in some of the original GaDOE tasks have been omitted and less time consuming activities have been substituted for those problems. In many instances, in the student versions of the tasks, the writing of the original tasks has been simplified. In order to preserve all vocabulary, content, and meaning it is important that teachers work the original tasks as well as the student versions included here.

Teachers are expected to both facilitate and provide some direct instruction, when necessary, during the work time. Suggestions related to student misconceptions, difficult concepts, and deeper meanings have been included in this section. However, teacher notes are provided in the GaDOE Math III Framework. In most cases, there is no need to repeat the information provided there. Again, it is imperative that teachers work the tasks and read the teacher notes that are provided in GaDOE support materials.

Questioning is extremely important in every part of a standards-based lesson. We included suggestions for questions in some cases but did not focus on providing good questions as extensively as we would have liked. Developing good questions related to a specific lesson should be a focus of collaborative planning time.

Closing: The closing may be the most important part of the lesson. This is where the mathematics is formalized. Even when a lesson must be continued to the next day, teachers should stop, leaving enough time to "close", summarizing and formalizing what students have done to that point. As much as possible students should assist in presenting the mathematics learned in the lesson. The teacher notes are all important in determining what mathematics should be included in the closing.

Homework: In some cases, homework suggestions are provided. Teachers should use their resources, including the textbook, to assign homework that addresses the needs of their students.

Homework should be focused on the skills and concepts presented in class, relatively short (30 to 45 minutes), and include a balance of skills and thought-provoking problems.

Differentiated support/enrichment: On the topic of differentiation, it is critical to reiterate that many of the strategies used in a standards-based mathematics classroom promote differentiation. These strategies include

- the use of rich tasks with multiple points of entry and more than one path to a solution
- flexible grouping of students
- multiple representations of mathematical concepts
- writing in mathematics
- monitoring of progress through on-going informal and formative assessments and
- analysis of student work.

Check for understanding: A check for understanding is a short, focused assessment-a ticket out the door, for example. There are many good resources for these items which can be found on-line at *Learning Village* at www.georgiastandards.org, along with other GaDOE materials related to the standards.

Resources/materials for Math Support: Again, in some cases, we have provided materials and/or suggestions for Math Support. This section should be personalized to your students, class, and/or school, based on your resources.

Table of Contents

Common Core Georgia Performance Standards Mathematics III
Unit 1: Matrices

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Unit 1 Timeline

Task 1: The Central High School Booster Club	2 days
Task 2: Walk Like a Mathematician	2 days
Task 3: Writing and Solving Matrix Equations	1 day
Task 4: An Okefenokee Food Web	1 day
Task 5: Building Skateboards	1 day

Task Notes

The importance of working the tasks used in these lessons cannot be overstated. In planning for the teaching of the Common Core Georgia Performance Standards in Mathematics, teachers should work the Student Tasks, read any corresponding teacher notes provided in the Georgia Department of Education's Mathematics III Framework Teacher Edition, and *then* examine the lessons provided here.

The tasks provided in this Supplement are based on the content of Unit 1 of the Georgia Department of Education's Mathematics III Framework. We suggest, as always, that teachers use this Supplement along with the GaDOE Teacher which can be found on-line at www.georgiastandards.org.

Task 1: The Central High School Booster Club
The concepts and skills addressed in this task include:

- using units as a way to understand problems and to guide solutions of multi-step problems
- using matrices to represent and manipulate data
- using matrices to solve real-world problems
- operating with matrices

All parts of the original GaDOE task have been used in these lessons. Some items have been rewritten and problems have been added to provide scaffolding for the multiplication of matrices.

Task 2: Walk Like a Mathematician
The concepts and skills addressed in this task include:

- determining properties for matrix addition and multiplication, including the distributive property of multiplication over addition
- finding inverses of invertible 2 x 2 matrices using pencil and paper
- finding inverses of invertible matrices with dimensions larger than 2 x 2 using technology
- finding determinants of 2 x 2 matrices using pencil and paper
- finding determinants of matrices with dimensions larger than 2 x 2 using technology
- using determinants of matrices to solve real-world problems

All parts of the original GaDOE task have been used in these lessons. Some items have been rewritten.

It is important that students complete the Day 1 Homework assignment developed for this task and that these items be discussed thoroughly in class. From this assignment, students should discover that the distributive property of multiplication over addition holds for matrices.

Task 3: Writing and Solving Matrix Equations
The concepts and skills addressed in this task include:

- representing a system of linear equations as a matrix equation
- solving matrix equations using inverse matrices
- representing and solving realistic problems using systems of linear equations

The original GaDOE task containing the items in this lesson is entitled *Candy? What Candy? Do We Get to Eat It?*. All items of the original GaDOE task are included for purposes of this supplement.

Task 4: An Okefenokee Food Web
The concepts and skills addressed in this task include:

- using vertex-edge graphs to represent realistic situations
- using adjacency matrices to represent information given in vertex-edge graphs
- using vertex-edge graphs and associated matrices to answer questions related to realistic situations

Task 5: Building Skateboards
The concepts and skills addressed in this task include:

- graphing solution sets of systems of linear inequalities in two variables
- using a linear programming model to represent and solve problems

All parts of the original GaDOE task have been used in this lesson. Some items have been rewritten.

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Task 1: The Central High School Booster Club

CCGPS Mathematics III
Task 1: ***Central High School Booster Club*** **Day 1/2**
(GaDOE TE Problems 1 - 4)

CCSS Standard(s):
Number and Quantity
Quantities* N -Q
Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.

Vector Quantities and Matrices N-VM
Perform operations on matrices and use matrices in applications.

6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
8. (+) Add, subtract, and multiply matrices of appropriate dimensions.

GPS Standard(s):
MM3A4. Students will perform basic operations with matrices.

a. Add, subtract, multiply, and invert matrices, when possible, choosing appropriate methods, including technology.

New vocabulary: matrix, matrices, element, dimensions, row matrix, column matrix, square matrix, quantity, scalar

Mathematical concepts/skills:

- use matrices to represent and manipulate data
- add and subtract matrices
- multiply a matrix by a scalar
- identify row, column, and square matrices
- determine whether two matrices are equal

Prior knowledge:

- basic arithmetic operations over the set of real numbers
- representing data in arrays

Essential question(s): How are matrices used to solve real-world problems?

Suggested materials:

Warm-up: Post the following:

Read the paragraph below and use a table to organize the information related to wholesale and materials costs for the booster club items.

> *In order to raise money for the school, the Central High School Booster Club offered spirit items prepared by members for sale at the school store and at games. They sold stuffed teddy bears dressed in school colors, tote bags and tee shirts with specially sewn and decorated school insignias. The teddy bears, tote bags, and tee shirts were purchased from wholesale suppliers and decorations were cut, sewn, painted, and attached to the items at three different stations by booster club parents. The wholesale cost for each teddy bear was $4.00, each tote bag was $3.50 and each tee shirt was $3.25. Materials for the decorations cost $1.25 for the bears, $0.90 for the tote bags and $1.05 for the tee shirts.*

Opening: Allow several students to share the tables they developed in order to organize the data. Be sure to choose students who organized the information differently (i.e. students who represented the costs using rows and items using columns (2 x 3) versus those who used rows to represent items and columns to represent costs (3 x 2)).

Students should realize that it is appropriate to organize the data as a 2 x 3 or as a 3 x 2 array. Emphasis should be placed on the importance of labeling rows and columns and paying attention to those labels.

Use student tables to introduce the vocabulary and notation included in the task prior to *Item 1*. Discussion should focus on the following terms and symbols: *arrays, rows, columns, matrix, matrices, dimensions, elements, quantity, [m x n], and* a_{mn}.

Worktime: Students should work in pairs to complete *Items 1 - 4* of the task.

(Note: The homework assignment following this task contains items that foster deeper understanding of the current content and link to prior and future learning as well. If time allows, students would be well served to complete as much of this assignment as possible in class. All items in the homework should be discussed at some point.)

When students have had ample time to complete *Item 1*, have a whole class discussion of this item. Be sure students understand the terminology and notation related to matrices. It is *not* necessary for all students to write Matrices *B*, *C*, *D*, and *E* using the same rows and columns. What is important is that they pay attention to labeling their rows and columns and that they understand what quantities are represented by individual elements of each matrix.

Closing: Allow students to discuss problems 2 – 4. Questions that help focus on the big ideas might include:

- What must be true in order for two matrices to be equal?
- What must be true about two matrices in order to add or subtract them?
- Did any of you have to change the dimensions of Matrix *E* that you wrote in *Problem 1e* to find Matrix *P* in *Problem 3*? Why was this necessary?

Homework: See attached.

Differentiated support/enrichment:

Check for understanding: *Find the values of x and y that make the following equation a true statement. Explain how you arrived at your solution.*

$$\begin{bmatrix} x & -7 & -5 \\ 4 & y & 2 \end{bmatrix} = \begin{bmatrix} 6 & 2 & 1 \\ 0 & x & 3 \end{bmatrix} - \begin{bmatrix} x & 9 & 6 \\ -4 & 7 & 1 \end{bmatrix}$$

Resources/materials for Math Support: Students should preview:

- organizing information in tables
- arrays
- difference between rows and columns
- computing with signed numbers

CCGPS Mathematics III
Task 1: *The Central High School Booster Club*
Day 1 Student Task

In order to raise money for the school, the Central High School Booster Club offered spirit items prepared by members for sale at the school store and at games. They sold stuffed teddy bears dressed in school colors, tote bags and tee shirts with specially sewn and decorated school insignias. The teddy bears, tote bags, and tee shirts were purchased from wholesale suppliers and decorations were cut, sewn, painted, and attached to the items at three different stations by booster club parents. The wholesale cost for each teddy bear was $4.00, each tote bag was $3.50 and each tee shirt was $3.25. Materials for the decorations cost $1.25 for the bears, $0.90 for the tote bags and $1.05 for the tee shirts. Parents estimated the time necessary to complete a bear was 15 minutes for station 1, 20 minutes for station 2, and 5 minutes for station 3. A tote bag required 10 minutes for station 1, 15 minutes for station 2, and 10 minutes for station 3. Tee shirts were made using computer generated transfer designs for each sport which took 5 minutes for station1, 6 minutes for station 2, and 20 minutes for station 3.

The booster club parents made spirit items at three different work meetings and produced 30 bears, 30 tote bags, and 45 tee shirts at the first session. Fifteen bears, 25 tote bags, and 30 tee shirts were made during the second meeting; and, 30 bears, 35 tote bags and 75 tee shirts were made at the third session. They sold the bears for $12.00 each, the tote bags for $10.00 each and the tee shirts for $10.00 each. In the first month of school, 10 bears, 15 tote bags, and 50 tee shirts were sold at the bookstore. During the same time period, Booster Club members sold 50 bears, 20 tote bags, and 100 tee shirts at the games.

The following is a **matrix**, a rectangular array of values, showing the wholesale cost of each item as well as the cost of decorations. "*Wholesale*" and "*decorations*" are labels for the matrix **rows** and "*bears*", "*totes*", and "*shirts*" are labels for the matrix **columns**. The **dimensions** of this matrix called ***A*** are 2 rows and 3 columns and matrix ***A*** is referred to as a [**2 x 3] matrix**. Each number in the matrix is called an **element** of the matrix.

Cost per Item

$$A = \begin{array}{r} \\ \textit{wholesale} \\ \textit{decorations} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{bears} & \textit{totes} & \textit{shirts} \end{array} \\ \begin{bmatrix} 4.00 & 3.50 & 3.25 \\ 1.25 & .90 & 1.05 \end{bmatrix} \end{array}$$

It is sometimes convenient to write **matrices** (plural of matrix) in a simplified format without labels for the rows and columns. Matrix *A* can be written as an array.

$$A = \begin{bmatrix} 4.00 & 3.50 & 3.25 \\ 1.25 & .90 & 1.05 \end{bmatrix} \text{ where the elements can be identified as } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}.$$

The notation $\boldsymbol{a_{mn}}$ refers to the element in the m^{th} row and the n^{th} column of the matrix. For example, the element $\boldsymbol{a_{13}}$ is the value in the first row and the third column. For matrix *A*, $a_{13} = 3.25$, which is the wholesale cost of a tee-shirt.

1. Write and label matrices for the information given on the Central High School Booster Club's spirit project.

   a. Let Matrix *B* contain the information given on the time necessary to complete each station for each item.

   b. The elements in Matrix *B* are **quantities**, numbers with units. Describe the units represented by these elements. What does the element b_{32} tell you?

   c. Write a Matrix *C* to show the numbers of bears, totes, and shirts produced at each of the three meetings. Choose and label a specific element of your matrix. What does the element tell you?

   d. Write a Matrix *D* that contains the information on items sold at the bookstore and at the game.

   e. Let Matrix *E* show the sales prices of the three items.

2. Matrices are called **square matrices** when the number of rows is equal to the number of columns. A matrix with only one row or only one column is called a **row matrix** or a **column matrix**. Are any of the matrices from *Item 1* square matrices, row matrices, or column matrices? If so, identify them.

Since matrices are arrays containing sets of discrete data with dimensions, they have a particular set of rules, or algebra, governing operations such as addition, subtraction, and multiplication.

Two matrices are **equal** if and only if they have the same dimensions and their corresponding elements are equal.

In order to **add two matrices**, the matrices must have the same dimensions. And, if the matrices have row and column labels, these labels must also match. Consider the following problem.

Several local companies wish to donate spirit items which can be sold along with the items made by the Booster Club to raise money for Central High School. J J's Sporting Goods store donates 100 caps and 100 pennants in September and 125 caps and 75 pennants in October. Friendly Fred's Food store donates 105 caps and 125 pennants in September and 110 caps and 100 pennants in October. How many items are available each month from both sources?

To add two matrices, add corresponding elements.

$$\text{Let } J = \begin{matrix} & \textit{Sept} & \textit{Oct} \\ \textit{caps} & 100 & 125 \\ \textit{pennants} & 100 & 75 \end{matrix} \quad \text{and} \quad F = \begin{matrix} & \textit{Sept} & \textit{Oct} \\ \textit{caps} & 105 & 110 \\ \textit{pennants} & 125 & 100 \end{matrix}$$

$$\text{then } J + F = \begin{matrix} & \textit{Sept} & \textit{Oct} \\ \textit{caps} & 100+105 & 125+110 \\ \textit{pennants} & 100+125 & 75+100 \end{matrix} \quad \text{and} \quad J + F = \begin{matrix} & \textit{Sept} & \textit{Oct} \\ \textit{caps} & 205 & 235 \\ \textit{pennants} & 225 & 175 \end{matrix}$$

Subtraction is handled like addition by subtracting corresponding elements.

3. Construct a matrix *G* with dimensions [1 x 3] corresponding to production cost per item. Use this new Matrix *G* and Matrix *E* from *Item 1* to find Matrix *P*, the profit the Booster Club can expect from the sale of each bear, tote bag, and tee shirt.

Another type of matrix operation is known as scalar multiplication. A **scalar** is a single number such as 3 and matrix **scalar multiplication** is done by multiplying each element in a matrix by the same scalar.

$$\text{Let } M = \begin{bmatrix} -2 & 0 & 5 \\ 1 & -3 & 4 \end{bmatrix}, \text{ then } 3M = \begin{bmatrix} -6 & 0 & 15 \\ 3 & -9 & 12 \end{bmatrix}.$$

4. Use scalar multiplication to change Matrix *B* (*Item 1*) from minutes required per item to hours required per item.

CCGPS Mathematics III
Task 1: ***The Central High School Booster Club***
Day 1 Homework

State the dimensions of each matrix. Identify any row, column, or square matrix.

1. $\begin{bmatrix} 3 \\ 2 \\ 5 \\ 1 \end{bmatrix}$ 2. $\begin{bmatrix} 3 & -1 \\ 2 & 4 \\ 5 & 9 \end{bmatrix}$ 3. $\begin{bmatrix} 1 & 7 \\ 2 & 8 \end{bmatrix}$ 4. $\begin{bmatrix} -1 & 3 & -4 \\ 2 & -5 & 7 \end{bmatrix}$

Use matrices *A, B*, and *C* to perform the indicated operations, if possible. If the operation is not possible, say why.

$A = \begin{bmatrix} 1 & 3 & 5 & 9 \\ 2 & 4 & 7 & 8 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 3 & -4 & 9 \\ 2 & -5 & 7 & -3 \end{bmatrix}$ $C = \begin{bmatrix} 3 & -1 \\ 2 & 4 \\ 5 & 9 \\ 1 & 6 \end{bmatrix}$

5. $A + B$ 6. $B - C$ 7. $4C$ 8. $2A + 3B$

9. In the equation below, *X* represents a matrix. Use what you know about solving equations and what you have learned about matrices to find the Matrix *X* that makes the equation a true statement. How can you check your work to make sure that you are correct?

$$3X + 2\begin{bmatrix} 1 & 5 & 0 \\ 2 & -1 & -3 \end{bmatrix} = \begin{bmatrix} 5 & 13 & 9 \\ -2 & 4 & 12 \end{bmatrix}$$

10. Find the value of *x* and *y* in the equation below.

$$x[2 \quad 1] + y[5 \quad -4] = [-4 \quad 11]$$

CCGPS Mathematics III
Task 1: ***Central High School Booster Club*** **Day 2/2**
(GaDOE TE Problems 5 - 6)

CCSS Standard(s):
Number and Quantity
Quantities* N -Q
Reason quantitatively and use units to solve problems.
1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.

Vector Quantities and Matrices N-VM
Perform operations on matrices and use matrices in applications.
6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
8. (+) Add, subtract, and multiply matrices of appropriate dimensions.

GPS Standard(s):
MM3A4. Students will perform basic operations with matrices.
a. Add, subtract, multiply, and invert matrices, when possible, choosing appropriate methods, including technology.

New vocabulary: transpose of a matrix

Mathematical concepts/skills:
- use matrices to represent and manipulate data
- multiply matrices
- transpose matrices

Prior knowledge:
- basic arithmetic operations over the set of real numbers
- representing data in arrays

Essential question(s): How are matrices used to solve real-world problems?

Suggested materials:

Warm-up: If students did not complete and discuss the Day 1 homework assignment during the previous class period, ask them to compare homework responses with a partner and be prepared to share solutions and/or ask any questions they still have related to the work

Opening: After a discussion of the Day 1 homework assignment, ask students to do the following:

*Read the paragraph below silently. Represent the **sales price** of each item in a [1 x 3] matrix N. Represent the **number of each item sold at the bookstore** and the **number of each item sold at extracurricular activities** in a [3 x 2] matrix S.*

At the end of the school year, the Booster Club members decided to reduce the prices of all unsold items and offer them for sale in the school store and at extracurricular events. Bears went on sale for $10, totes for $5, and shirts for $9. As a result of the clearance sale, 30 bears, 70 totes, and 20 shirts were sold in the book store. Twelve bears, 22 totes, and 10 shirts were sold at extracurricular activities.

After making sure that all students have the correct matrices, ask that they work alone for five or so minutes to investigate how they might use these two matrices to determine the amount of revenue produced by selling items at the bookstore and the amount of revenue produced by selling items at extracurricular activities.

Choose a student who has realized that you must multiply the elements in the row of Matrix *N* by the elements in a column of Matrix *S* and then add those products to determine revenue. Use this work to introduce multiplication of matrices. Students should answer *Items 5* and *6* during this discussion and before moving on to the worktime.

Worktime: Students should work in pairs to complete *Items 7- 10* of the task.

When students have had ample time to complete *Item 7*, have a whole class discussion of these problems to be sure that all students understand the mechanics of matrix multiplication.

As you monitor student progress, look for students who discovered that rows and columns need to be interchanged to obtain the correct response in *Item 8*. Allow these students to share their work and then discuss transposing matrices.

Closing: Allow students to discuss problems 9 and 10.

Choose students who did and who did not have to transpose their matrices to obtain the correct response in *Item 9*.

Questions that help focus on the big ideas might include:

- Why did you have to transpose Matrix … to obtain the correct result?
- What does element a_{12} of your matrix tell us?
- Did you transpose a matrix in *Problem 10*? Which one? Why was that necessary?

Homework: See attached.

Differentiated support/enrichment: It is often difficult for students to see which elements are multiplied when multiplying two matrices. One way to help student visualize this is to cut strips from transparencies and use the strips to "slide" a row of the matrix one the left on top of columns of the matrix on the right. As this is done, write out the equation that shows the sums of these products.

Check for understanding: *The Prom committee is trying to raise funds for this year's prom. They are selling t-shirts, washing cars, and selling raffle tickets for $10, $5, and $2, respectively. By the end of the campaign, 72 shirts were sold, 14 cars were washed, and 50 raffle tickets were sold. Which of the following products represents how to find the total revenue?*

a. $\begin{bmatrix} 10 & 5 & 2 \end{bmatrix} \begin{bmatrix} 72 & 14 & 50 \end{bmatrix}$ b. $\begin{bmatrix} 10 \\ 5 \\ 2 \end{bmatrix} \begin{bmatrix} 72 \\ 14 \\ 50 \end{bmatrix}$ c. $\begin{bmatrix} 10 & 5 & 2 \end{bmatrix} \begin{bmatrix} 72 \\ 14 \\ 50 \end{bmatrix}$

d. none of the above

Resources/materials for Math Support: Students should preview:

- organizing information in tables
- arrays
- difference between rows and columns
- computing with signed numbers

CCGPS Mathematics III
Task 1: ***The Central High School Booster Club***
Day 2 Student Task

Matrices can also be multiplied together. Since each matrix represents an array of data, rules for multiplying them depends on the position of each element. Consider the following example.

At the end of the school year, the Booster Club members decided to reduce the prices of all unsold items and offer them for sale in the school store and at extracurricular events. Bears went on sale for $10, totes for $5, and shirts for $9. As a result of the clearance sale, 30 bears, 70 totes, and 20 shirts were sold in the book store. Twelve bears, 22 totes, and 10 shirts were sold at extracurricular activities.

Matrix *N* shows the clearance price of each item. Matrix *S* shows the number of each item sold during the clearance sale at the bookstore and at extracurricular activities.

$$N = \begin{array}{c} \quad \textit{bears} \;\; \textit{totes} \;\; \textit{shirts} \\ \$\begin{bmatrix} 10 & 5 & 9 \end{bmatrix} \end{array} \qquad S = \begin{array}{c} \\ \textit{bears} \\ \textit{totes} \\ \textit{shirts} \end{array} \begin{array}{c} \textit{store} \;\; \textit{activities} \\ \begin{bmatrix} 30 & 12 \\ 70 & 22 \\ 20 & 10 \end{bmatrix} \end{array}$$

Suppose the club members would like to know how much revenue was produced by selling clearance items in the bookstore and the amount of revenue produced by selling items at extracurricular activities.

By definition, we multiply two matrices by multiplying the elements in the rows of the first matrix by the elements in the columns of the second. **Products of the elements in a row and a column are added to obtain one element of the product matrix.** In our example, we multiply the elements in the row of Matrix *N* by the elements in the first column of Matrix *S* (as shown below) to obtain 830, the first element in the matrix *N* * *S*.

$$10(30) + 5(70) + 9(20) = 830$$

5. Write the equation above attaching appropriate units to each number in the sentence. What quantity is represented by 830?

Next we multiply the elements in the row of Matrix *N* by the elements in the second column of Matrix *S* to obtain 320, the second element in the matrix *N* * *S (as shown below)*.

$$10(12) + 5(22) + 9(10) = 320$$

6. Write this equation attaching appropriate units to each number in the sentence. What quantity is represented by 320?

Matrix $N * S = [830 \quad 320]$.

The first element of $N * S$ tells us that revenue from selling items in the bookstore was $830 and the revenue from selling items at extracurricular activities was $320.

Notice that in multiplying $N * S$, we multiplied a matrix with dimensions 1 x 3 times a matrix with dimensions 3 x 2 to obtain a matrix with dimensions 1 x 2.

.   [1 x 3] * [3 x 2] = [1 x 2]

[revenue x item] * [item x location] = [revenue x location]

This procedure illustrates the multiplication of two matrices. In order to multiply two matrices, the number of columns of the matrix on the left **must** equal the number of rows of the matrix on the right. The resulting product matrix will have the same number of rows as the matrix on the left and the same number of columns as the matrix on the right.

If the dimensions of two matrices are not appropriately matched, it is not possible to multiply them.

7. Given the following matrices, find their products, if possible.

$$L = \begin{bmatrix} 1 & 3 \\ -5 & 4 \end{bmatrix} \qquad M = \begin{bmatrix} -1 & 2 & 7 & -1 \\ 5 & 4 & 3 & 2 \end{bmatrix} \qquad N = \begin{bmatrix} 3 & 0 \\ -2 & 1 \\ 5 & 5 \\ -1 & 2 \\ 6 & 3 \end{bmatrix} \qquad T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

   a. *LM*
   b. *LN*
   c. *LT*
   d. *TL*
   e. *MN*

There is another very important factor to consider when multiplying matrices. In all real applications the dimensions of matrices have labels. In other words, the elements of each row and column represent quantities, not just numbers. If matrices have labels, one should take care before multiplying to make sure that the labels corresponding to numerical dimensions are matched as are the dimensions.

8. Suppose the booster club had only bears and shirts to sell on one particular weekend. Matrix *P* shows the prices of each item. Matrix *W* shows the number of each item sold on Friday and the number of each item sold on Saturday. Suppose the club wants to know how much money they made on Friday and how much money they made on Saturday.

$$P = \begin{matrix} & \textit{bears} & \textit{shirts} \\ \textit{dollars} & [10 & 9] \end{matrix} \qquad W = \begin{matrix} & \textit{bears} & \textit{shirts} \\ \textit{Friday} & 12 & 15 \\ \textit{Saturday} & 20 & 30 \end{matrix}$$

a. Does the Matrix $P * W$ provide the information the club needs? Why or why not?

b. Is there any way that you could rewrite Matrices P and/or W in order to multiply and obtain the needed information? Explain your thinking.

The process of interchanging the rows and columns of a matrix is referred to as **transposing** the matrix. For example, if Matrix M is given to be the 3 x 2 matrix

$M = \begin{bmatrix} 1 & 2 \\ 4 & -3 \\ 8 & 9 \end{bmatrix}$, the **transpose** of Matrix M is labeled as M^T and is the 2 x 3 matrix

$$M^T = \begin{bmatrix} 1 & 4 & 8 \\ 2 & -3 & 9 \end{bmatrix}.$$

In applying matrices to obtain a specific result, it is often possible (or necessary) to transpose the matrices before multiplying them.

9. Using Matrix D from *Item 1* and Matrix P from *Item 3* of this task, calculate the matrix that gives the amount of profit the booster club made at the bookstore and at the games. (You may, or may not, need to transpose your matrices).

10. A company produces posters for the Atlanta Braves. Matrix T below gives the time in hours of each type labor required to produce posters of three different players: Chipper Jones, Brian McCann, and Gregor Blanco. Matrix P gives the total number of orders for posters of these players received in May and June.

    How many hours of each type labor will be needed to fill the orders for May and June? Show how you know.

$$T = \begin{matrix} & \textit{print} & \textit{package} & \textit{ship} \\ \textit{Jones} & .04 & .06 & .02 \\ \textit{McCann} & .05 & .07 & .03 \\ \textit{Blanco} & .03 & .05 & .02 \end{matrix} \qquad P = \begin{matrix} & \textit{May} & \textit{June} \\ \textit{Jones} & 23000 & 45000 \\ \textit{McCann} & 42000 & 37000 \\ \textit{Blanco} & 18000 & 46000 \end{matrix}$$

CCGPS Mathematics III
Task 1: *The Central High School Booster Club*
Day 2 Homework

Use the matrices given below to answer questions 1 – 4.

$A = \begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 3 \\ -2 & -5 \end{bmatrix}$

1. Find AB.

2. Find BA.

3. What do you notice about AB and BA? Explain.

4. Find B^2.

5. If $E = \begin{bmatrix} 10 & 5 & 2 \end{bmatrix}$, give an example of a matrix F such that EF and FE exists. Find EF and FE using your example.

6. Solve the equation below for x and y.

$$\begin{bmatrix} x & 1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2y \end{bmatrix} = \begin{bmatrix} 7 & 7 \\ 9 & 10 \end{bmatrix}$$

7. A stockbroker sold a customer 200 shares of stock A, 400 shares of stock B, 600 shares of stock C, and 250 shares of stock D. The price per share of A, B, C, and D, was \$80, \$120, \$200, and \$300, respectively.

    a. Let Matrix N represent the number of shares of each stock the customer bought. (Be sure to label carefully.)

    b. Let Matrix P represent the price per share of each stock.

    c. Use matrices N and P to calculate the matrix that gives the total cost of the stocks that the customer bought.

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Task 2: Walk Like a Mathematician

CCGPS Mathematics III
Task 2: ***Walk Like a Mathematician*** **Day 1/2**
(GaDOE TE Problems 1 - 5)

CCSS Standard(s):

Vector Quantities and Matrices N-VM
Perform operations on matrices and use matrices in applications.
8. (+) Add, subtract, and multiply matrices of appropriate dimensions.
9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

GPS Standard(s):
MM3A4. Students will perform basic operations with matrices.
a. Add, subtract, multiply, and invert matrices, when possible, choosing appropriate methods, including technology.
c. Examine the properties of matrices, contrasting them with properties of real numbers.

New vocabulary: additive identity matrix, multiplicative identity matrix

Mathematical concepts/skills:
- determine properties of matrix addition and multiplication

Prior knowledge:
- properties of real numbers
- counterexample
- proof

Essential question(s): What properties, similar to the properties of real numbers, hold for matrix addition and matrix multiplication? How can I use technology to operate with matrices?

Suggested materials: graphing calculators

Warm-up: Allow students to work in pairs to complete the table below.

Write an equation to illustrate each of the following properties of real numbers:

	Addition	*Multiplication*
Commutative property		
Associative property		
Identity property		
Inverse property		

Opening: After students have had time to complete the table above, allow them to share the statements they have written to illustrate each of the properties listed. As students share their equations, ask them to state the properties in words as well as by using mathematical symbols. Some students will use constants to illustrate the property (i.e., $2 + 3 = 3 + 2$). Others will use variables. Both are fine but be sure ultimately to generalize the properties using variables to show that they are true for all real numbers.

Worktime: Students should work in pairs to complete *Items 1 – 5* of the task. After a "closing" on this material, we suggest that students be introduced to the matrix features of the graphing calculator. Operations at this time should be limited to addition, subtraction, multiplication, and finding the transposes of matrices. Students will use technology to find inverses and determinants of higher order matrices after completing *Items 6 -8* of this task.

Students should justify their work in *Items 1 – 3*. For example, in *Item 1* justification should include the idea that to add matrices, we add corresponding elements. If the elements of the matrices are real numbers, since addition of real numbers is commutative, addition of matrices will be commutative. In *Item 2* students need only to find a counterexample to show that matrix multiplication is not commutative.

After students have had ample time to complete *Item 3*, have a whole class discussion of the properties investigated to this point.

Be sure to allow students struggle time with *Item 5*. Many students may begin by thinking that a matrix for which every element is 1 will serve as an identity matrix. Allowing students to find an identity matrix by trial and error will reinforce what they have learned to this point about matrix multiplication.

Closing: Allow students to discuss *Items 4* and *5*. After a complete discussion of the properties of matrix addition and multiplication introduce students to the matrix features of the graphing calculator.

Homework: See attached.

Differentiated support/enrichment:

Check for understanding:

Consider matrices A, B, and C.

1. *If A + B exists, what can you say about B + A? Explain.*
2. *If AB exists, what can you say about BA? Explain.*

Resources/materials for Math Support: Students should preview properties of real numbers and the fundamental ideas of proof, including counterexample. They may also need additional practice on multiplication of matrices.

CCGPS Mathematics III
Walk Like a Mathematician
Day 1 Student Task

Matrices allow us to perform many useful mathematical tasks which ordinarily require a large number of computations. Some types of problems which can be done efficiently with matrices include solving systems of equations, finding the areas of triangles given the coordinates of the vertices, finding equations for graphs given sets of ordered pairs, and determining information contained in vertex edge graphs. In order to address these types of problems, it is necessary to understand more about matrix operations and properties. It is also useful to learn to use technology to perform some matrix operations.

Matrix operations have many of the same properties as real numbers. There are more restrictions on matrices than on real numbers, however, because of the rules governing matrix addition, subtraction, and multiplication. Some of the real number properties which are more useful when considering matrix properties are listed below.

Let a, b, and c be real numbers

	ADDITION PROPERTIES	MULTIPLICATION PROPERTIES
COMMUTATIVE	a + b = b + a	ab = ba
ASSOCIATIVE	(a + b) + c = a + (b + c)	(ab)c = a(bc)
IDENTITY	There exists a unique real number zero, 0, such that a + 0 = 0 + a = a	There exists a unique real number one, 1, such that a * 1 = 1 * a = a
INVERSE	For each real number a, there is a unique real number - a such that a + (-a) = (-a) + a = 0	For each nonzero real number a, there is a unique real number $\frac{1}{a}$ such that $a(\frac{1}{a}) = (\frac{1}{a})a = 1$

The following is a set of matrices without row and column labels. Use these matrices to help you answer questions 1 –5 below.

$$D = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \quad E = \begin{bmatrix} 2 & 4 & 1 \\ 0 & -1 & 3 \\ 1 & 0 & 2 \end{bmatrix} \quad F = \begin{bmatrix} 5 \\ -2 \end{bmatrix} \quad G = \begin{bmatrix} 0 & -1 \\ 1/3 & 2/3 \end{bmatrix} \quad H = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$J = \begin{bmatrix} 4 & 2 \\ -1 & 3 \end{bmatrix}$ $K = \begin{bmatrix} 3 & 4 & -1 \\ 0 & -2 & 3 \\ 5 & 1 & 2 \end{bmatrix}$ $L = \begin{bmatrix} 5 & -2 \\ -1 & 0 \end{bmatrix}$

1. Is matrix addition commutative? Justify your response.
2. Is matrix multiplication commutative? Justify your response.
3. Are matrix addition and matrix multiplication associative? Show how you know.
4. Could there be an **identity matrix** for matrix addition similar to the additive identity for the set of real numbers? Explain your thinking.
5. Could there be an **identity matrix** for matrix multiplication similar to the multiplicative identity for the set of real numbers? Explain your thinking.

CCGPS Mathematics III
Walk Like a Mathematician
Day 1 Homework

Use the matrices given below to answer questions 1 – 4.

$A = \begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix}$ $C = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$ $D = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$

1. Find $A(C + D)$.
2. Find $AC + AD$.
3. What do you notice about $A(C + D)$ and $AC + AD$? Explain.
4. What property of matrices do *Problems 1 – 3* illustrate?
5. What must be true about any matrix that can be raised to a power? Explain your thinking.

CCGPS Mathematics III
Task 2: *Walk Like a Mathematician* **Day 2/2**
(GaDOE TE Problems 6 - 8)

CCSS Standard(s):

Vector Quantities and Matrices N-VM
Perform operations on matrices and use matrices in applications.
6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

GPS Standard(s):
MM3A4. Students will perform basic operations with matrices.
a. Add, subtract, multiply, and invert matrices, when possible, choosing appropriate methods, including technology.
b. Find the inverses of two-by-two matrices using pencil and paper, and find inverses of larger matrices using technology.
c. Examine the properties of matrices, contrasting them with properties of real numbers.

New vocabulary: multiplicative inverse of a matrix, determinant

Mathematical concepts/skills:
- find the inverses of invertible 2 x 2 matrices using pencil and paper
- find inverses of invertible matrices with dimensions larger than 2 x 2 using technology
- find determinants of 2 x 2 matrices using pencil and paper
- find determinants of matrices with dimensions larger than 2 x 2 using technology
- use determinants of matrices to solve real-world problems

Prior knowledge:
- properties of real numbers
- properties of inverse functions (i.e., $f(f^{-1}(x)) = f^{-1}(f(x)) = x$)
- formula for finding the area of a triangle

Essential question(s): How can I find the inverse of a matrix? How can I use matrices to solve real-world problems?

Suggested materials: graphing calculators

Warm-up: Ask students to compare homework from the previous lesson with a partner and be prepared to share their responses.

Opening: Discuss the homework assignment from the previous lesson. From this assignment, students should have discovered that the distributive property of multiplication over addition holds for matrices. They should have also discovered that only square matrices can be raised to a power.

Ask students what they know about inverse *functions.* Through discussion and questioning address the following concepts.

- Inverse functions "undo each other". In other words, the composition of two inverse functions, in either order, yields the identity function.
- Only one-to-one functions have inverses that are also functions.
- The domains and ranges of inverse functions are interchanged.
- Inverse functions can be found by interchanging *x* and *y* and solving for *y*.
- Graphs of inverse functions are reflections of each other across the line $y = x$.

Use this discussion to introduce multiplicative inverses of matrices. Include the conditions that must be met in order for a matrix to have a multiplicative inverse, the notation used for the inverse of a matrix, and the fact that the product of matrices that are multiplicative inverses yields the identity matrix. Ask how these ideas compare to those of inverse functions.

Worktime: Students should work in pairs to complete *Items 6 -8* of the task.

After students have had time to complete *Item 7*, have a whole class discussion to be sure that all students understand the procedure for finding the inverse of a matrix and for verifying that the matrices are truly inverses. The matrices below may be used for additional practice in finding the inverse of a 2 x 2 matrix. Matrix *A* has an inverse. Matrix *B* does not have an inverse.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & -4 \\ -6 & 12 \end{bmatrix}$$

Allow students to complete *Item 8*. Allow time for students to find inverses and determinants of higher order matrices using graphing calculators. Students should always verify that two matrices are inverses by showing that their product is the identity matrix. For matrices larger than 2 x 2 finding inverses and verifying that the matrices are inverses may be done with technology.

Closing: Allow students to discuss *Item 8.*

Homework: See attached.

Differentiated support/enrichment: In discussing the formula for finding the inverse of a 2 x 2 matrix, it is important for students to understand that the variables *a*, *b*, *c*, and *d* actually refer to the *positions* those variables hold in the 2 x 2 matrix.

Check for understanding: *Explain the process for finding the inverse of a 2 x 2 matrix.*

Resources/materials for Math Support: Students should preview properties of inverse functions, and the formula for finding the area of a triangle. They may also need additional practice using technology to operate with matrices.

CCGPS Mathematics III
Walk Like a Mathematician
Day 2 Student Task

$D = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$ $G = \begin{bmatrix} 0 & -1 \\ 1/3 & 2/3 \end{bmatrix}$ $J = \begin{bmatrix} 4 & 2 \\ -1 & 3 \end{bmatrix}$

Consider matrices *D* and *G* above. *D* and *G* are **inverse matrices**.

> In order for a matrix to have an inverse, it must satisfy two conditions.
> 1. The matrix must be a square matrix.
> 2. No row of the matrix can be a multiple of any other row.

Both *D* and *G* are 2 x 2 matrices; and, the rows in *D* are not multiples of each other. The same is true of *G*.

The notation normally used for a matrix and its inverse is *D* and D^{-1} or *G* and G^{-1}.

The product of two inverse matrices should be the identity matrix, ***I***.

6. Find *D* * *G* and *G* * *D*.

If a 2 x 2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has a multiplicative inverse, the inverse can be found using the following formula:

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

The expression $ad - bc$ is referred to as the **determinant** of the 2 x 2 matrix. It is often represented as ***det A*** and denoted with vertical bars. For example,

$$\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

If the determinant of Matrix *A* is equal to 0, then *A* does not have an inverse.

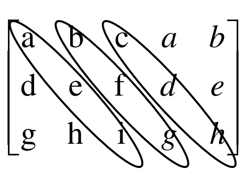
7. Find the inverse of Matrix J listed above. Verify that J and J^{-1} are inverses.

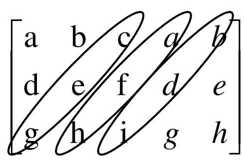
To find the inverse of a 2 x 2 matrix, you used the formula for finding the determinant of a 2 x 2 matrix. Square matrices of order higher than 2 also have determinants, although finding them can be quite complicated. We will show one method for finding the determinant of a 3 x 3 matrix here. Determinants of matrices larger than 2 x 2 are most often found using technology.

To find determinants of 3 x 3 matrices you can use the following procedure.

Given Matrix $B = \begin{bmatrix} \mathrm{a} & \mathrm{b} & \mathrm{c} \\ \mathrm{d} & \mathrm{e} & \mathrm{f} \\ \mathrm{g} & \mathrm{h} & \mathrm{i} \end{bmatrix}$, rewrite the matrix and repeat columns 1 and 2 to get

$\begin{bmatrix} \mathrm{a} & \mathrm{b} & \mathrm{c} & a & b \\ \mathrm{d} & \mathrm{e} & \mathrm{f} & d & e \\ \mathrm{g} & \mathrm{h} & \mathrm{i} & g & h \end{bmatrix}$. Now multiply and combine products according to the following patterns.



and


to give

$\det(B) = aei + bfg + cdh - ceg - afh - bdi$.

8. The determinant of a 3 x 3 matrix can be used to find the area of a triangle. If (x_1, y_1), (x_2, y_2), and (x_3, y_3) are vertices of a triangle, the area of the triangle is

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}.$$

a. Given a triangle with vertices (-1, 0), (1, 3), and (5, 0), find the area using the determinant formula. Verify that the area you found is correct using a *geometric* formula.

b. Suppose you are finding the area of a triangle with vertices (-1, -1), (4, 7), and (9, -6). You find the value of the determinant created to be -105 and so to find the area of the triangle you must multiply by $\frac{-1}{2}$. Your partner works the same problem and gets +105 for the determinant that she created and so she multiplies by $\frac{1}{2}$ to get the area of the triangle. After checking both solutions, you each have done your work correctly. How can you explain this discrepancy?

c. Suppose another triangle with vertices (1, 1), (4, 2), and (7, 3) gives an area of 0. What do you know about the triangle and the points?

d. A gardener is trying to find a triangular area behind his house that encloses 1750 square feet. He has placed the first two fence posts at (0, 50) and at (40, 0). The final fence post is on the property line at $y = 100$. Find the point where the gardener can place the final fence post.

CCGPS Mathematics III
Walk Like a Mathematician
Day 2 Homework

$$A = \begin{bmatrix} 5 & -2 \\ 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} \frac{1}{3} & -\frac{a}{4} \\ 1 & \frac{-3a}{4} \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 4 & 1 \\ 0 & -1 & 3 \\ 1 & 0 & 2 \end{bmatrix}$$

Use matrices *A*, *B*, and *C* to perform each of the operations in *Problems 1 - 4*, if possible. If the operation cannot be performed, explain why.

1. Find A^{-1}. Verify that *A* and A^{-1} are multiplicative inverses.
2. Find B^{-1}. Verify that *B* and B^{-1} are multiplicative inverses.
3. Use technology to find C^{-1}. Verify that *C* and C^{-1} are multiplicative inverses.
4. Find the determinant of *C*.

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Task 3: Writing and Solving Matrix Equations

CCGPS Mathematics III
Task 3: ***Writing and Solving Matrix Equations*** **Day 1/1**
(GaDOE TE Problems 1- 4)

CCSS Standard(s):
Reasoning with Equations and Inequalities A-REI
Solve systems of equations
8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.
9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

GPS Standard(s):
MM3A5. Students will use matrices to formulate and solve problems.
a. Represent a system of linear equations as a matrix equation.
b. Solve matrix equations using inverse matrices.
c. Represent and solve realistic problems using systems of linear equations.

New vocabulary: matrix equation, coefficient matrix, variable matrix

Mathematical concepts/skills:
- write systems of equations as single matrix equations
- solve matrix equations using inverse matrices
- given three points, find the equation of a parabola by writing and solving a matrix equation
- solve real-world problems by writing and solving a matrix equation

Prior knowledge:
- solving systems of two linear equations in two unknowns
- standard form of an equation of a parabola

Essential question(s): How can I solve real-world problems by writing and solving matrix equations?

Suggested materials:
- graphing calculators
- small brown paper bags (one for each group of 3 students)
- at least 3 different kinds of miniature chocolate candy bars (3 bars per student)

Warm-up: Post the following:

Write the following matrix equation as a system of two linear equations in two unknowns. Be prepared to explain your work.

$$\begin{bmatrix} 1 & 2 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 10 \end{bmatrix}$$

Opening: Students should be able to use what they have learned about matrix multiplication and the definition of equality for matrices to write the equation given in the warm up as two linear equations in two unknowns. Allow students to share their work, explaining the steps taken.

After the system has been presented, ask students to solve the system using any method they choose. In previous grades students learned to solve systems by substitution, elimination of a variable, and by graphing. Monitor student work to find students who recall these methods. If necessary, ask guiding questions that remind students of these procedures. Share all three methods. Ask students what it means to find the *solution* of a system of linear equations in two unknowns. Both an algebraic response (an ordered pair that makes both equations true) and a geometric response (the point where the two lines intersect) should be discussed. Have students verify their solutions.

Explain to students that in today's lesson they will begin with a *system* of equations, write the system as a single matrix equation, and then solve the equation using inverse matrices. Discuss the terms coefficient matrix, variable matrix, and answer matrix.

Worktime: Students should work in groups of three to complete *Items 1 - 6* of the task. *Items 4a* and *4b* can be assigned as homework if time does not permit that they be completed in class.

After students have had time to complete *Item 1* have a whole class discussion to be sure that all students understand *why* multiplying each side of the matrix equation by the inverse of the coefficient matrix isolates the variable matrix.

Monitor work carefully as students complete *Items 2 – 4*. When ample time has been given to complete these problems, have a class discussion to address any questions that students may have.

Have students read the material related to finding an equation of a line and allow them to begin *Item 5*. Ask guided questions, if necessary, to remind students of the standard form of an equation of a parabola. Students will know that two points determine a line but many may not know that three noncolinear points determine a specific parabola.

Closing: Before beginning *Item 6*, allow students to discuss *Item 5.* It may be difficult for some students to grasp the concept that they are solving for the coefficients *a*, *b*, and *c* to find the equation of the parabola formed by the given points.

For *Item 6*, each group of three students should be given a bag containing three different kinds of miniature chocolate candy bars. There should be a total of 9 pieces of candy in each bag. On the front of the bag should be stapled a chart containing nutrition information for that particular bag of candy. A sample chart is shown below.

Bag number ___________1___

Types of Candy ___Reese's, KitKat, Whoppers___

Total pieces ________9___

Total number of calories _____522____

Total fat grams ________54____________

Total cholesterol ____10 mg________________

Sodium _____660 mg______

Total number of carbohydrates ____208 mg_____

Total protein ______16 g___

Each group of students should also receive a general nutrition table containing information for each type of candy in the bag. A sample table is shown below.

Candy	Hershey's	Whoppers	Reese's	KitKat
Fat grams	12 g	7 g	5 g	11 g
Cholesterol	10 mg	0 mg	0 mg	5 mg
Sodium	35 mg	115 mg	60 mg	30 mg
Carbohydrates	25 g	31 g	10 g	27 g
Protein	3 g	1 g	2 g	3 g
Calories	67	33	90	70

Student instructions are given in the student task. It is particularly effective if you specify that each of the three students in the group determine the number of each kind of candy in the bag using a different combination of equations. Students should realize that all three systems should have the same solution. Not only does this increase student accountability but it is also a good way to "catch" and help rectify mistakes.

Homework: Students should complete *Items 4a* and *4b* of the task if not completed in class. See attached.

Differentiated support/enrichment:

Check for understanding:

Resources/materials for Math Support: Students should preview:

- slope-intercept form of an equation of a line
- solving systems of two linear equations in two unknowns
- standard form of an equation of a parabola

CCGPS Mathematics III

Writing and Solving Matrix Equations

Student Task

A system of equations such as $\begin{cases} ax + by = c \\ dx + ey = f \end{cases}$ can be written as a matrix equation where $\begin{bmatrix} a & b \\ d & e \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \\ f \end{bmatrix}$ or $CV = A$ where C is the coefficient matrix, V is the variable matrix, and A is the answer matrix.

1. Explain why multiplying each side of the matrix equation by the inverse of C, as shown below, isolates the variable matrix.

$$CV = A$$
$$C^{-1}CV = C^{-1}A$$
$$V = C^{-1}A$$

2. Consider the system of equations $\begin{cases} 5x - y = 7 \\ 2x + 3y = -1 \end{cases}$

   a. Write the system as a matrix equation in the manner described above.

   b. Use what you have learned about matrices to this point to find and multiply both sides of the matrix equation by the inverse of the coefficient matrix.

   c. Did you obtain a solution for the system of two equations in two unknowns? Show how you know.

Solving systems of equations of higher order can be accomplished using the procedure described in *Problem 1* and a graphing calculator.

3. Write the following system of three equations in three unknowns as a single matrix equation and then use your calculator to help you solve the system.

$$\begin{cases} x - 2y + 3z = 3 \\ 2x + y + 5z = 8 \\ 3x - y - 3z = -22 \end{cases}$$

4. Solve each of the following systems by first writing the system as a matrix equation.

a. $\begin{aligned} 2x+3y&=2 \\ 4x-9y&=-1 \end{aligned}$

b. $\begin{aligned} 9x-7&=5 \\ 10x+3y&=-16 \end{aligned}$

c. $\begin{aligned} 5x-4y+3z&=15 \\ 6x+2y+9z&=13 \\ 7x+6y-6z&=6 \end{aligned}$

Systems of equations can be used to write an equation for a graph given coordinates of points on the graph. Suppose you are asked to find an equation of a line passing through the points (2, -5) and (-1, -4). Knowing the slope-intercept form of the equation of a line is $y = mx + b$, you can form a system of equations $\begin{cases} -5 = 2m + b \\ -4 = -1m + b \end{cases}$. Solving this system gives $m = \frac{-1}{3}$ and $b = -4\frac{1}{3}$ and an equation of $y = \frac{-1}{3}x - 4\frac{1}{3}$.

5. Consider the following graph. A parabola passes through the points (-3, 5), (1, 1), and (2, 10). Write a system of equations for this graph. Solve the system and write an equation for the parabola. Justify your answer.

6. Knowing how to write and solve matrix equations is really going to pay off for you and your partners today, particularly if you are hungry. Each group of three students will get one bag of candy. Before the bag can be opened, you must be able to tell your teacher exactly how many pieces of each type of candy are in your bag. You must show your work and every student in the group must be able to justify that work. Each bag holds 3 different types of candy and a total of 9 pieces of candy.

CCGPS Mathematics III
Writing and Solving Matrix Equations
Homework

1. Write and solve a matrix equation to find the equation of the parabola that goes through the points (1, 2), (0, 3), (-1, 6). (Note: If you do not have a graphing calculator, just write the matrix equation. You will be given time to use a calculator to solve the equation during the next warm-up.)

2. You have been asked to solve a system of two linear equations in two unknowns by writing and solving a matrix equation. However, once you have written the matrix equation, you find that the coefficient matrix has no inverse. What does this tell you about the system of two equations in two unknowns? Give a graphical and an algebraic response.

3. Write your own system of two equations in two unknowns for which the coefficient matrix has no inverse.

ATLANTA PUBLIC SCHOOLS
Mathematics & Science Initiative
Making A Difference

Task 4: An Okefenokee Food Web

CCGPS Mathematics III
Task 4: ***An Okefenokee Food Web*** **Day 1/1**
(GaDOE TE Problems 1- 8)

CCSS Standard(s):
Vector Quantities and Matrices N-VM
Perform operations on matrices and use matrices in applications.
6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
8. (+) Add, subtract, and multiply matrices of appropriate dimensions.

GPS Standard(s):
MM3A4. Students will perform basic operations with matrices.
a. Add, subtract, multiply, and invert matrices, when possible, choosing appropriate methods, including technology.
MM3A7. Students will understand and apply matrix representations of vertex-edge graphs.
a. Use graphs to represent realistic situations.
b. Use matrices to represent graphs, and solve problems that can be represented by graphs.

New vocabulary: vertex-edge graph, digraph, adjacency matrix

Mathematical concepts/skills:
- use vertex-edge graphs to represent realistic situations
- use adjacency matrices to represent information given in vertex-edge graphs
- use vertex-edge graphs and associated matrices to answer questions related to realistic situations

Prior knowledge:
- basic arithmetic

Essential question(s): How can I use vertex-edge graphs to represent and solve problems?

Suggested materials:
- graphing calculators

Warm-up: Ask students to read the scenario for *The Okefenokee Food Web*.

Opening: This topic may be very interesting to students, particularly in light of the oil spill in the Gulf of Mexico. Discuss the scenario in detail. Make sure students understand what is meant by a vertex-edge graph, a digraph, and an adjacency matrix.

Worktime: Students should complete *Items 1 - 8* of the task.

After students have had time to complete *Items 1- 4*, have a whole class discussion to be sure that all students understand the questions asked in *Items 1 – 3*. Ask students to explain *why* F^2 indicates indirect food sources through one intermediary? Ask what F^3 and F^4 tell us.

In *Item 5*, students should use their calculators to compute powers of *F* at least through the 4th power.

Closing: Allow students to share their responses to *Items 5 – 7*. You might want to give 15 – 20 minutes to write the report described in *Item 8* and then share reports.

Homework: See attached.

Differentiated support/enrichment:

Check for understanding:

Resources/materials for Math Support: Give students opportunities to:

- read information and draw directed graphs representing that information,
- construct adjacency matrices based on directed graphs, and
- draw graphs given an adjacency matrix.

CCGPS Mathematics III
An Okefenokee Food Web
Student Task

Recent weather conditions have caused a dramatic increase in the insect population of the Okefenokee Swamp area. The insects are annoying to people and animals and health officials are concerned there will be an increase in disease. Local authorities want to use an insecticide that would literally wipe out the entire insect population of the area. You, as an employee of the Environmental Protection Agency, must determine how detrimental this would be to the environment. Specifically, you are concerned with the effects on the food web of six animals known to populate the swamp.

Consider the following digraph of a food web for the six animals and the insects that are causing the problem.

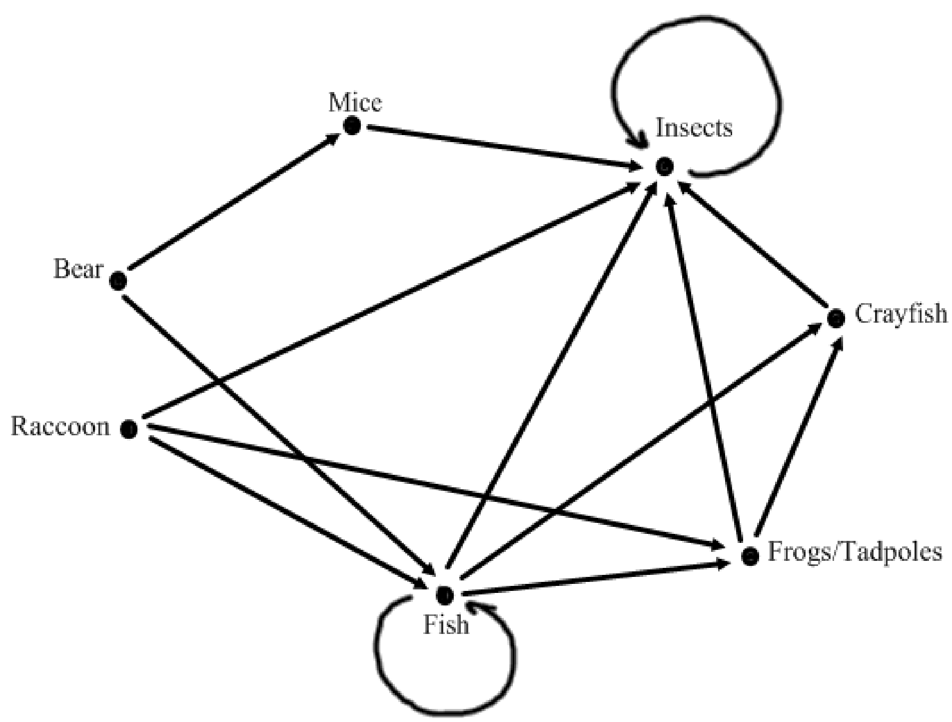
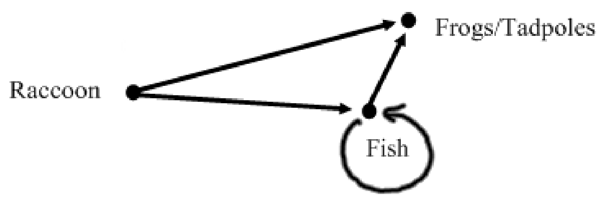


figure 1

A **vertex-edge graph** is a diagram consisting of a set of points (called vertices) along with segments or arcs (called edges) joining some or all of the points. A **digraph** is a directed vertex edge graph. Here each vertex represents an animal or insects. The direction of an edge indicates whether an animal preys on the linked animal. For example, raccoons eat fish. (Note: The food web shown is simplified. Initial producers of nutrients, plants, have not been included.)

An **adjacency matrix** is a matrix used to represent the information in a vertex-edge graph. Suppose we consider the graph below dealing with only raccoons, fish, and frogs.



The adjacency matrix would be $\begin{matrix} & R & FT & F \\ R & 0 & 1 & 1 \\ FT & 0 & 0 & 0 \\ F & 0 & 1 & 1 \end{matrix}$.

1. Construct a matrix F to represent the entire Okefenokee food web shown in figure 1. What does a row containing a single one indicate? What does a column of zeros indicate?

2. Which animals have the most direct sources of food? How can this be determined from the matrix? Find the number of direct food sources for each animal.

3. The insect column has the most ones. What does this suggest about the food web?

4. The matrix F^2 denotes indirect (through one intermediary) sources of food. For example, the fish relies on insects for food, and the bear relies on the fish for food, so the insect is an indirect source of food for the bear. Find F^2. Notice that the insect column contains all nonzero numbers. What does this indicate?

5. Compute additional powers of the food web matrix to represent the number of direct and indirect sources of food for each animal. Which animal has the most food sources?

If an insecticide is introduced into the food web, killing the entire insect population, several animals will lose a source of food.

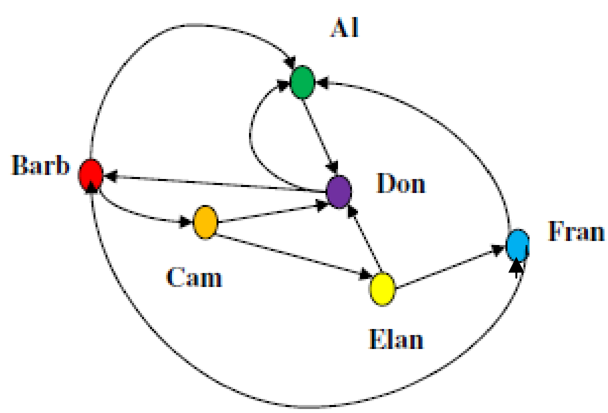
6. Construct a new matrix *G* to represent the food web with no insects. What effect does this have on the overall animal population? What has happened to the row sums? Compare these with those of the original matrix. What does a row sum of zero indicate?

7. Will all the animals be affected by the insecticide? Which animal(s) will be least affected?

8. Organize and summarize your findings in a brief report to the health officials. Take and support a position on whether using an insecticide to destroy the insect population is harmful to the environment.

CCGPS Mathematics III
An Okefenokee Food Web
Homework

1. One of the main problems with cell phones is that there is no phone directory. You cannot call 411 to get a friend's cell number. A directed edge in the following digraph indicates that one person knows another person's phone number. For example, Barb knows Al's number.



a. Construct the adjacency matrix associated with this graph. (Assume that no person will call themselves. In other words, we should have all zeros on the main diagonal of the matrix.) What does a row of the matrix tell you? What does a column tell you?

b. Powers of adjacency matrices can tell you many things. Use your calculator and the matrix you have created to answer the following questions:

i. Can you get a message from Fran to Cam? How many calls will it take?

ii. In how many different ways can you get a message from Barb to Elan in three or fewer calls?

2. The following is an airline route matrix. A "1" in a row indicates that there is a direct flight from the city represented by the row to the city represented by the column.

	Atlanta	*Chicago*	*Omaha*	*New York*
Atlanta				
Chicago				
Omaha	0	1	0	0
New York	1	1	1	0

   a. Draw a digraph for the matrix.

   b. How many ways are there to get from Atlanta to Omaha with exactly one-stop? Show how you know.

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Making A Difference

Task 5: Building Skateboards

CCGPS Mathematics III
Task 5: *Building Skateboards* **Day 1/1**
(GaDOE TE Problems 1- 9)

CCSS Standard(s):
Creating Equations A-CED
Create equations that describe numbers or relationships.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
Represent and solve equations and inequalities graphically
12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

GPS Standard(s):
MM3A6. Students will solve linear programming problems in two variables.
a. Solve systems of inequalities in two variables, showing the solutions graphically.
b. Represent and solve realistic problems using linear programming.

New vocabulary: linear programming model, decision variable, objective function, constraints, feasible region, line of constant profit

Mathematical concepts/skills:
- graph solution sets of systems of linear inequalities in two variables
- use a linear programming model to represent and solve problems

Prior knowledge:
- graphing linear equations in two variables
- solving systems of two linear equations in two unknowns
- graphing solution sets of systems of linear inequalities in two variables

Essential question(s): How can I use a linear programming model to represent and solve problems?

Suggested materials:
- graph paper
- graphing calculators

Warm-up: Ask students to read the scenario for *Building Skateboards* silently, making notes of all pertinent information as they read.

Opening: Discuss the scenario. As you question students about their reading, chart the most important information on an Anchor Chart so that students can refer back to it as they work.

Questions that might be asked include:

- What is the name of this company?
- What do they manufacture?
- What are the components of a skateboard?
- How many different kinds of boards does the company make?
- What is Mr. Hurley's biggest concern?
- How much profit does each board generate?
- What is a meant by a *constraint*?
- What resources constrain the number of boards that SK8MAN can produce?
- How long does it take a worker to shape a Sporty board?
- How long does it take to shape a Fancy board?
- How much time do the workers work each week?

Worktime: Students should complete *Items 1 - 9* of the task.

After the opening discussion, have students complete *Items 1 – 5* alone or with a partner then allow them to share responses, particularly their answers to questions *4* and *5*.

Once students have finished discussing *Items 1 – 5*, conduct a short mini-lesson addressing the information provided in the task between *Items 5* and *6*. Allow students to graph the system of linear inequalities in *Item 6*. Make sure that all students have graph paper.
(Note: Students have spent extensive time in previous grades graphing both systems of linear equations and systems of linear inequalities. Many students may need to be reminded of these skills but those who struggle with the concepts should be provided additional support outside of the regular class time.)

You will probably need to stop students after they have completed *Item 6* and again after they have completed *Item 7* to discuss the concepts addressed in the task. These discussions should help students clarify the following ideas:

- The coordinates of every point on a particular line of constant profit represent a mix of products that generates that amount of profit.
- If we were to continue to graph lines with increasingly large values for profit, we would eventually happen upon a line that intersects the feasible region at just **one** point. If the value of z were made any larger, none of the points on the new line would intersect the feasible region.
- *The point that maximizes (or minimizes) the objective function of a linear programming problem is always a vertex of the feasible region.*

Closing: Allow students to share their responses to *Items 8 – 9*.

Homework: See attached.

Differentiated support/enrichment:

Check for understanding:

Resources/materials for Math Support: Give students opportunities to:

- graphing linear equations in two variables
- solving systems of two linear equations in two unknowns
- graphing solution sets of systems of linear inequalities in two variables

CCGPS Mathematics III
Building Skateboards
Student Task

SK8MAN, Inc. manufactures and sells skateboards. A skateboard is made of a deck, two trucks that hold the wheels (see Figure 1.2.1), four wheels, and a piece of grip tape. SK8MAN manufactures the decks of skateboards in its own factory and purchases the rest of the components. Currently, SK8MAN manufactures two types of skateboards: Sporty (Figure 1.2.2, top) and Fancy (Figure 1.2.2, bottom).

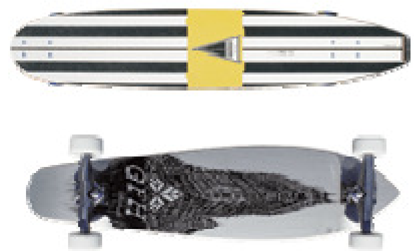

Figure 1.2.2: Sporty and Fancy

Figure 1.2.1: A skateboard truck

G. F. Hurley, the production manager at SK8MAN, needs to decide the production rate for each type of skateboard in order to make the most profit. Each Sporty board earns $15 profit, and each Fancy board earns $35 profit. However, Mr. Hurley might not be able to produce as many boards of either style as he would like, because some of the necessary resources are limited. We say that the production rates are *constrained* by the availability of the resources.

To produce a skateboard deck, the wood must be glued and pressed, then shaped. After a deck has been produced, the trucks and wheels are added to the deck to complete a skateboard. Skateboard decks are made of either North American maple or Chinese maple. A large piece of maple wood is peeled into very thin layers called veneers. A total of seven veneers are glued at a gluing machine and then placed in a hydraulic press for a period of time (See Figure 1.2.3). After the glued veneers are removed from the press, eight holes are drilled for the trunk mounts. Then the new deck goes into a series of shaping, sanding, and painting processes. Figure 1.2.4 shows a deck during the shaping process.

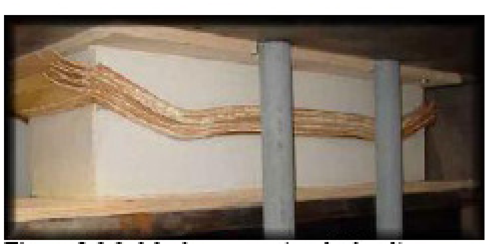

Figure 1.2.3: Maple veneers in a hydraulic press

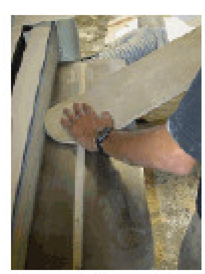

Figure 1.2.4: Shaping a deck

The Sporty board is a less expensive product, because its quality is not as good as the Fancy board. Chinese maple is used in the manufacture of Sporty decks. North American maple is used for Fancy decks. Because Chinese maple is soft, it is easier to shape. On average, it takes a worker 5 minutes to shape a Sporty board. However, a Fancy board requires 15 minutes to shape. SK8MAN, Inc., is open for 8 hours a day, 5 days a week.

To make decisions in order to maximize the company's weekly profit, operations researchers use a technique known as *linear programming*. Answering the following questions will help you understand this technique.

One way to approach the problem of maximizing profit is to make some guesses and test the profit generated by each guess. Suppose SK8MAN decides the company should make 200 of each model per week.

1. How much profit would be generated?

2. Is there enough available time to shape that number of each model?

3. Answer the same two questions if SK8MAN decides to make:

   a. 50 Sporty and 350 Fancy
   b. 350 Sporty and 50 Fancy

4. Can you find a production mix for which there is enough shaping time?

5. How much profit do the production rates you found generate for the company?

The first step in the formulation of a linear programming problem is to define the ***decision variables*** in the problem. The decision variables are then used to define the ***objective function***. This function captures the goal in the problem. In the SK8MAN problem, the goal is to maximize the company's profits per week. Therefore, the objective function should represent the weekly profit. That profit comes from the sale of the two different styles of skateboards, so we may begin the problem formulation with:

Let: x = the weekly production rate of Sporty boards,
y = the weekly production rate of Fancy boards, and
z = the amount of profit SK8MAN, Inc earns per week.

Now, since we know the profit for each style of skateboard, we can write the objective function by expressing z in terms of x and y. The objective or profit function is ______________________.

The last step in the formulation of the problem is to represent any constraints in terms of the decision variables. In this case, Mr. Hurley cannot just decide to make as many boards as he wants, because the number made is *constrained* by the available shaping time. Eight hours per day, five days per week is a 40-hour workweek. However, the information about shaping time is expressed in minutes, so we convert 40 hours to 2,400 minutes. Now, if SK8MAN makes x Sporty Skateboards and y Fancy Skateboards per week, that uses ____________ minutes of shaping time. Thus, the shaping time ***constraint*** is ______________.

There are also two not-so-obvious, but completely logical, constraints. Since the production rate cannot be a negative number for either type of skateboard, $x \geq 0$ and $y \geq 0$. We call these last two inequalities the ***non-negativity constraints***.

Finally, the problem formulation looks like this:

Let: x = the weekly production rate of Sporty boards,
y = the weekly production rate of Fancy boards, and
z = the amount of profit SK8MAN, Inc earns per week.

Maximize the objective function: $z = 15x + 35y$, subject to the constraints:
$5x + 15y \leq 2400$, $x \geq 0$, and $y \geq 0$.

We will solve the problem graphically by first graphing the system of linear inequalities created by the constraints.

6. Graph the system of inequalities:

$5x + 15y \geq 2400$
$x \geq 0$
$y \geq 0$

The solution set of this system of inequalities is a shaded region containing infinitely many points. This region is called the ***feasible region*** for the linear programming problem. Each ordered pair in the feasible region represents a combination of Fancys and Sportys that SK8MAN, Inc., could produce without violating any of the constraints. The solution to our problem requires us to pick the one point from our feasible region that will generate the most profit, or in other words, the one mix of Sportys and Fancys that will maximize our objective function.

Let's begin finding a solution by using a trial-and-error approach.

7. On the same set of axes you used to graph your feasible region, graph the three different lines that result from the objective function when the profit for SK8MAN is $1750, $3500, and $7000. What do you notice about these lines?

The coordinates of every point on a particular line of constant profit represent a mix of products that generates that amount of profit. When the weekly profit is assumed to be $3,500, for example, you can see there is an infinite number of points on that line that lie within the feasible region. If we were to continue to graph lines with increasingly large values for profit, we would eventually happen upon a line that intersects the feasible region at just **one** point. If the value of z were made any larger, none of the points on the new line would intersect the feasible region. This means that *the point that maximizes (or minimizes) the objective function of a linear programming problem is always a vertex of the feasible region.*

8. List the vertices of your feasible region.

9. Now that you know that the ordered pair that maximizes an objective function must be a vertex of the feasible region, how many Sporty and how many Fancy skateboards do you think SK8MAN, Inc. should build to maximize their profit? Show how you know. What is the maximum profit?

CCGPS Mathematics III
Building Skateboards
Homework

Let's review what you have learned. A linear programming problem consists of an objective function to be optimized (maximized or minimized) and a set of linear inequalities that represent constraints on the situation being studied.

To solve a linear programming problem:

1. Graph the system of linear inequalities created by the constraints to determine a feasible region.
2. Find the vertices of the feasible region.
3. Evaluate the objective function at each vertex of the feasible region to determine which vertex yields the maximum (or minimum) value of this function.

It is important to remember that if a linear programming problem has a solution, it will occur at a vertex of the feasible region. If the problem has more than one solution, at least one of them will occur at a vertex of the feasible region.

1. A linear program yields an objective function $z = 5x + 3y$ subject to the constraints:

   $x + y \leq 30$
   $2x + y \leq 40$
   $x \geq 0$
   $y \geq 0$

   a. Graph the system of linear inequalities.

   b. Determine the vertices of the feasible region. (There are four.)

   c. Determine which vertex maximizes the objective function. What is the maximum value of the objective function?

2. A linear program yields an objective function $z = 4x + 7y$ subject to the constraints:

   $x + y \geq 30$
   $3x + 2y \geq 90$
   $x \geq 0$
   $y \geq 0$

   a. Graph the system of linear inequalities.

   b. Determine the vertices of the feasible region. (There are three.)

   c. Determine which vertex minimizes the objective function. What is the minimum value of the objective function?

3. Paul has a small tailor shop where he makes shirts and ties. A shirt requires 2 hours on the sewing machine and a tie requires 1 hour on the machine. A shirt requires 3 ounces of dye and a tie requires 2 ounces. Paul can spend 30 hours per week on his sewing machine and can order 50 ounces of dye per week. He can sell all the shirts and ties he makes. If a shirt sells for $36 and a tie sells for $20, how many of each item should he make per week to maximize his profit?