Atlanta Public Schools

Teacher's Curriculum Supplement

Mathematics I: Unit 4
The Chances of Winning

This document has been made possible by funding from the GE Foundation Developing Futures grant, in partnership with Atlanta Public Schools. It is derived from the Georgia Department of Education Math I Frameworks and includes contributions from Georgia teachers. It is intended to serve as a companion to the GA DOE Math I Frameworks Teacher Edition. Permission to copy for educational purposes is granted and no portion may be reproduced for sale or profit.

Preface

We are pleased to provide this supplement to the Georgia Department of Education's Mathematics I Framework. It has been written in the hope that it will assist teachers in the planning and delivery of the new curriculum. This document should be used with the following considerations.

- The importance of working the tasks used in these lessons cannot be overstated. In planning for the teaching of the Georgia Performance Standards in Mathematics teachers should work the task, read the teacher notes provided in the Georgia Department of Education's Mathematics I Framework Teacher Edition, and *then* examine the lessons provided here.

- This guide provides day-by-day lesson plans. While a detailed scope and sequence and established lessons may help in the implementation of a new and more rigorous curriculum, it is hoped that teachers will assess their students informally on an on-going basis and use the results of these assessments to determine (or modify) what happens in the classroom from one day to the next. Planning based on student need is much more effective than following a pre-determined timeline.

- It is important to remember that the Georgia Performance Standards provide a balance of concepts, skills, and problem solving. Although this document is primarily based on the tasks of the Framework, we have attempted to help teachers achieve this all important balance by embedding necessary skills in the lessons and including skills in specific or suggested homework assignments. The teachers and writers who developed these lessons, however, are not in your classrooms. It is incumbent upon the classroom teacher to assess the skill level of students on every topic addressed in the standards and provide the opportunities needed to master those skills.

- In most of the lesson templates, the sections labeled *Differentiated support/enrichment* have been left blank. This is a result of several factors, the most significant of which was time. It is also hoped that as teachers use these lessons, they will contribute their own ideas, not only in the areas of differentiation and enrichment, but in other areas as well. Materials and resources abound that can be used to contribute to the teaching of the standards.

On the topic of differentiation, it is critical to reiterate that many of the strategies used in a standards-based mathematics classroom promote differentiation. These strategies include

- the use of rich tasks with multiple points of entry and more than one path to a solution
- flexible grouping of students,
- multiple representations of mathematical concepts,
- writing in mathematics,
- monitoring of progress through on-going informal and formative assessments, and
- analysis of student work.

We hope that teachers will incorporate these strategies in each and every lesson.

It is hoped that you find this document useful as we strive to raise the mathematics achievement of all students in our district and state. Comments, questions, and suggestions for inclusions related to the document may be emailed to Dr. Dottie Whitlow, Executive Director, Mathematics and Science Department, Atlanta Public Schools, dwhitlow@atlantapublicschools.us

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Explanation of the Terms and Categories Used in the Lesson Template

Task: This section gives the suggested number of days needed to teach the concepts addressed in a task, the task name, and the problem numbers of the task as listed in the Georgia Department of Education's Mathematics I Framework Teacher Edition (GaDOE TE).

In some cases new tasks or activities have been developed. These activities have been named by the writers.

Standard(s): Although each task addresses many Math I standards and uses mathematics learned in earlier grades, in this section, only the key standards addressed in the lesson are listed.

New Vocabulary: Vocabulary is only listed here the first time it is used. It is strongly recommended that teachers, particularly those teaching Math Support, to use interactive word walls. Vocabulary listed in this section should be included on the word walls.

Mathematical concepts/topics: Here are listed the major concepts addressed in the lesson whether they are Math I concepts or were addressed in earlier grades.

Prior knowledge: Prior knowledge includes only those topics studied in previous grades. It does not include Math I content taught in previous lessons.

Essential Question(s): Essential questions may be daily and/or unit questions.

Suggested materials: In an attempt to list all materials that will be needed for the lesson, including consumable items, such as graph paper, and tools, such as graphing calculators and compasses. This list did not include those items that should always be present in a standards-based mathematics classroom such as markers, chart paper and rulers.

Warm-up: A suggested warm-up is included with every lesson. Warm-ups should be brief and should focus student thinking on the concepts that are to be addressed in the lesson.

Opening: Openings should set the stage for the mathematics to be done during the work time. The amount of class time used for an opening will vary from day-to-day but this should not be the longest part of the lesson.

Worktime: The problem numbers have been listed and the work that students are to do during the work time have been described. A student version of the day's activity follows the lesson template in every case. In order to address all of the standards in Math I, some of the problems in some of the tasks have been omitted and, in a few instances, substituted less time consuming activities for tasks. In many instances, in the student versions of the tasks, the writing of the original tasks has been simplified. In order to preserve all vocabulary, content, and meaning it is important that teachers work the original tasks as well as the student versions included here.

Teachers are expected to both facilitate and provide some direct instruction, when necessary, during the work time. Some suggestions, related to student misconceptions, difficult concepts, and deeper meaning in this section have been included. However, the teacher notes in the GaDOE Math I Framework are exceptional. In most cases, there is no need to repeat the information provided there. Again, it is imperative that teachers work the tasks and read the teacher notes that are provided in GaDOE support materials.

Questioning is extremely important in every part of a standards-based lesson. We included suggestions for questions in some cases but did not focus on providing good questions as extensively as we would have liked. Developing good questions related to a specific lesson should be a focus of collaborative planning time.

Closing: The closing may be the most important part of the lesson. This is where the mathematics is formalized. Even when a lesson must be continued to the next day, teachers should stop, leaving enough time to "close", summarizing and formalizing what students have done to that point. As much as possible students should assist in presenting the mathematics learned in the lesson. The teacher notes are all important in determining what mathematics should be included in the closing.

Homework: In some cases, additional written homework suggestions are provided or used the homework provided in the GaDOE sample lessons. We hope that you will use your resources, including your textbook, to assign homework related to the lesson that addresses the needs of your students.

Homework should be focused on the skills and concepts presented in class, relatively short (30 to 45 minutes), and include a balance of skills and thought-provoking problems.

Differentiated support/enrichment: On the topic of differentiation, it is critical to reiterate that many of the strategies used in a standards-based mathematics classroom promote differentiation. These strategies include

- the use of rich tasks with multiple points of entry and more than one path to a solution
- flexible grouping of students
- multiple representations of mathematical concepts
- writing in mathematics
- monitoring of progress through on-going informal and formative assessments
- and analysis of student work.

Check for understanding: A check for understanding is a short, focused assessment-a ticket out the door, for example. The Coach Book may be a good resource for these items.

Resources/materials for Math Support: Again, in some cases, we have provided materials and/or suggestions for Math Support. This section should be personalized to your students, class, and/or school, based on your resources.

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Unit 4

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Task 1: SAD to MAD: Examining Mean and Variability at Different Levels
This task has been adapted from the work of Christine Franklin and Gary Kader. It uses the approach recommended by the Pre-K Statistics Education Framework outlined in the GAISE document in Levels A and B.

Students begin by examining the mean as a "fair share" and the variability from the mean by considering which of two or more distributions is "more fair". Students are then encouraged to think of the mean as a balancing point and to quantify variability in relation to this *balancing point*. Finally the concepts and formula for mean absolute deviation are developed.

Task 2: Sampling Rectangles
This task can be found on pages 13-19 and 94-97 of *Navigating through Data Analysis in Grades 9-12* published by NCTM. The publication can be ordered from the NCTM website.

Students choose subjective and random samples from a population of one hundred rectangles and compare sample means to determine which kind of sample is more representative of the population based on the means.

Task 3: Using Spinners with Congruent Sectors
This task appears in the GaDOE ACC Math I Framework as *Spinner Task I.* Minor modifications have been made to notation used in the original task. Teacher notes in the GaDOE TE are applicable.

Task 4: Using Spinners with Sectors of Different Sizes
This task appears in the GaDOE ACC Math I Framework as *Problems 7-12* of *Spinner Task 2.* Questions 1-6 of the original task appear in this document as the homework that follows Task 3, *Using Spinners with Congruent Sectors*. Teacher notes from the GaDOE TE are applicable.

Task 5: Shooting Free Throws
This task can be found on pages 53-55 and pages105-108 of *Navigating through Probability in Grades 9-12* published by NCTM. The publication can be ordered from the NCTM website.

Using the context of Basketball Foul Shots, students use simulation to investigate expected value and to distinguish between expected value and the most likely outcome.

Task 6: Oops! Bankrupt!
This task appears in the GaDOE ACC Math I Framework as *Spinner Task 4.* Problems 7 and 9 of the original task have been omitted in the interest of time. Teacher notes from the GaDOE TE are applicable.

GaDOE *Spinner Task 5* is included in this document as a possible assessment.

Task 7: Marbles, Cards, and Surveys
This task combines three GaDOE learning tasks in a manner intended to maintain rigor and reduce class time. Teacher notes may be found between on pages 26 – 29 of the GaDOE ACC Math I TE.

Task 8: The Supermarket
This task is not included in the GaDOE Framework. It addresses permutations (including permutations of repeated objects and circular permutations), combinations, and probability involving both permutations and combinations.

Task 9: What's the Probability of a Hit?
This task can be found on pages 40-42 and pages 94-97 of *Navigating through Probability in Grades 9-12* published by NCTM. The publication can be ordered from the NCTM website.

This task lays the foundation for teaching binomial probability through use of the multiplication principle and counting permutations of sets of objects with repeated elements. Students may then generalize the concepts they have learned in the task to the formula for binomial probability, if teachers feel the formula is necessary.

Applications of Probability
These problems can be found in the GaDOE ACC Math I Unit 4 TE on pages 55 and 56. They may be used in class or as outside assignments to give students added opportunities to apply the probability they have learned in this unit.

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Teacher's Curriculum Supplement

Mathematics I: Unit 4
The Chances of Winning

Task 1: SAD to MAD: Examining Mean and Variability at Different Levels

Mathematics I
Task 1: SAD to MAD: Examining Mean and Variability at Different Levels Day 1/1

Standard(s): MA1D4. Students will explore variability of data by determining the mean absolute deviation (the averages of the absolute values of the deviations).

New vocabulary: measures of spread, deviation from the mean, absolute values of the deviations from the mean, sum of the absolute values of the deviations from the mean (SAD), the average of the absolute values of the deviations from the mean or mean absolute deviation (MAD)

Mathematical concepts/topics: the mean as fair share, the mean as a balancing point, variability of a distribution, representations of distributions using models, dot plots, measures of spread, deviation from the mean, sum of the absolute values of the deviations from the mean (SAD), mean absolute deviation (MAD)

Prior knowledge: calculation of the mean, dotplots, range, interquartile range, absolute value, operations with positive and negative numbers

Essential question(s): How can I quantify the variability of a distribution?

Suggested materials: snap cubes, chart paper, 1" or 2" post-it notes, markers

Warm-up: Post the following:

Perform the following operations.

1. $^-2 + {^-8}$
2. $^-11 + 7$
3. $|{^-7}| - 2$
4. $^-|3| - 10$
5. $\dfrac{-12}{-2}$

Opening: Discuss the warm-up, reviewing absolute value and the rules for operating with positive and negative numbers.

Explain to students that their task today will involve examining the means of various sets of data and determining ways of quantifying how much the data in a distribution varies from the mean.

Worktime: Students should work in groups of 4 or 5 for this task. Provide students with snap cubes or other "building" materials for problems 1 and 2. Monitor group discussions for these two problems carefully to be sure that students begin the task with a reasonable understanding of the mean.

Provide chart paper and markers for problem 3. Once students have completed problem 3, have a whole group discussion of problems 1 – 3. (See teacher notes immediately following student task in this supplement). The definition of "fair share" provided in the teacher notes should be established during this discussion.

Allow students to work on problems 4 – 6. They will need a clean piece of chart paper for this activity. Carefully monitoring groups in order to address all concepts discussed in the teacher notes.

Problem 7 may be used as a closing activity in which the mean absolute deviation is ultimately introduced. Suggestions for addressing this problem are given.

Problem 8 may be used to assess student understanding of the mean absolute deviation and other concepts developed in this task.

Closing: See teacher notes for problem 7.

Homework: Math I semester grades for Avery and Jonas are shown below. Their teacher must use these grades to pick one of them as the final member of the math team. Who should she choose? Justify your decision using the ideas discussed in today's task. Your justification should include the concepts of mean, median, range, the SAD, the MAD, and any other information that you feel is important..

Avery: 70, 100, 98, 72, 90, 85, 80
Jonas: 86, 84, 85, 85, 84, 86, 85

Differentiated support/enrichment:

Check for Understanding: Problem 8 of the task.

Resources/materials for Math Support: Students may need to review absolute value and operations with signed numbers. They may also need to review calculation of the mean, median, and range.

The vocabulary and notation in this task is extensive. Make sure students have an opportunity to preview both the new vocabulary and the notation used to represent new statistical concepts, including summation notation.

Mathematics I
SAD to MAD: Examining Mean and Variability at Different Levels
Student Task

In elementary school, young children are taught to understand the mean of a distribution as "fair share". For example, suppose the following group of five children is exploring the number of letters in their first names: Jonathan, Hosea, Michael, Ann, and Sharquinta.

The children are asked to build snap cube towers representing the lengths of their names. They are then asked questions such as:

- "Who has the longest name?"
- "Who has the shortest name?"
- "What is the typical or average name length of your group?"

1. How do you think the children might determine an answer to the last question? Use snap cubes or draw a model to illustrate your reasoning.

2. How does the method you used in problem 1 relate to the idea of "fair share"? How does the method relate to the formula for computing the mean of a distribution?

Suppose two groups of 8 students have the given name lengths:
Group 1: 4, 4, 4, 4, 5, 5, 6, 8
Group 2: 3, 3, 4, 5, 5, 5, 7, 8

3. "Build" or draw the distribution for each group.
   - What is the "fair share" or the mean for each group?
   - Which group do you think is closer to being "fair"? Explain your thinking and illustrate with a model.

In the first three problems you examined how the ideas of *mean* and *variability* are introduced to young children. In the next few problems, you will examine the two ideas using a more sophisticated approach.

4. Look again at the name length distribution for the students in Group 1 of problem 3. On chart paper, your group is to represent the distribution of name lengths for the 8 students using a dotplot.
   - Scale your plot assuming that no name will have less than 3 letters or more than 11 letters.
   - Represent each data value with post-it notes rather than drawing on the plot so that different plots may be represented on your chart paper as you work through the remainder of the task.

   Compare your plot with those of other groups. Do they all look the same?

5. Use your same chart paper to create a dotplot consisting of 8 students each of which has a name length of 6 letters.
   a. What is the mean of the name lengths in this group of students?
   b. Create a different plot consisting of 8 students with the same mean name length. Explain how you arrived at your plot.

6. Your group will now be asked to create a dotplot representing 8 students with a mean name length of 6 letters based on a specific restriction provided by your teacher. Be prepared to explain how you arrived at your plot.

7. Share the distribution your group created in problem 6 with the distributions of other groups. Does it make sense to compare our various distributions using the mean? Why or why not? What ways, other than using the mean, might we compare the distributions?

8. Your class discussion of the questions in problem 7 should have led you to discover statistical concepts often labeled the SAD and the MAD. Describe each concept using both words and symbols. Discuss the relationship between the two and explain why the MAD is a better measure of variability than the SAD.

Mathematics I
Task 1: Teacher Notes

SAD to MAD: Examining Mean and Variability at Different Levels

This task has been adapted from the work of Christine Franklin and Gary Kader. It uses the approach recommended by the Pre-K Statistics Education Framework outlined in the GAISE document in Levels A and B. Students begin by examining the mean as a "fair share" and the variability from the mean by considering which of two or more distributions is "more fair". Students are then encouraged to think of the mean as a balancing point and to quantify variability in relation to this *balancing point*. Finally the concept and formula for mean absolute deviation are developed.

This task addresses standard MMID4.

In elementary school, young children are taught to understand the mean of a distribution as "fair share". For example, suppose the following group of five children is exploring the number of letters in their first names: Jonathan, Hosea, Michael, Ann, and Sharquinta.

The children are asked to build snap cube towers representing the lengths of their names. They are then asked questions such as:

- "Who has the longest name?"
- "Who has the shortest name?"
- "What is the typical or average name length of your group?"

1. How do you think the children might determine an answer to the last question? Use snap cubes or draw a model to illustrate your reasoning.

Allow students to work in groups of four or five. Using snap cubes to build towers representing name lengths is particularly effective in helping even high school students understand the concept of mean. In determining the typical or average name length of a group of students, young children will usually take one of two approaches.

- They will pile all the cubes in the middle of the group and redistribute cubes one at a time to each member until each child has an equal number of cubes, leaving any "left over" cubes unassigned; or
- They will remove cubes from the taller "towers" and place them on the shorter "towers" until all "towers" are of an even height, leaving any "extra" cubes unassigned.

Have a discussion with your students about the mean name lengths of their groups. What might they do with any leftover cubes if the mean of their group is not a whole number? What if we were discussing candy bars? How could every student have a "fair share"?

2. How does the method you used in problem 1 relate to the idea of "fair share"? How does the method relate to the formula for computing the mean of a distribution?

Students should be able to verbalize the concept of summing all the data values (in this case the total number of letters in all the names) and then dividing this sum by the number of data values (the number of students in the group), thus relating the ideas presented in problem 1 to the formula for calculating the mean.

$$\text{Mean} = \frac{\text{sum of data values}}{\text{number of data values}}$$

3. Suppose two groups of 8 students have the given name lengths:
   Group 1: 4, 4, 4, 4, 5, 5, 6, 8
   Group 2: 3, 3, 4, 5, 5, 5, 7, 8

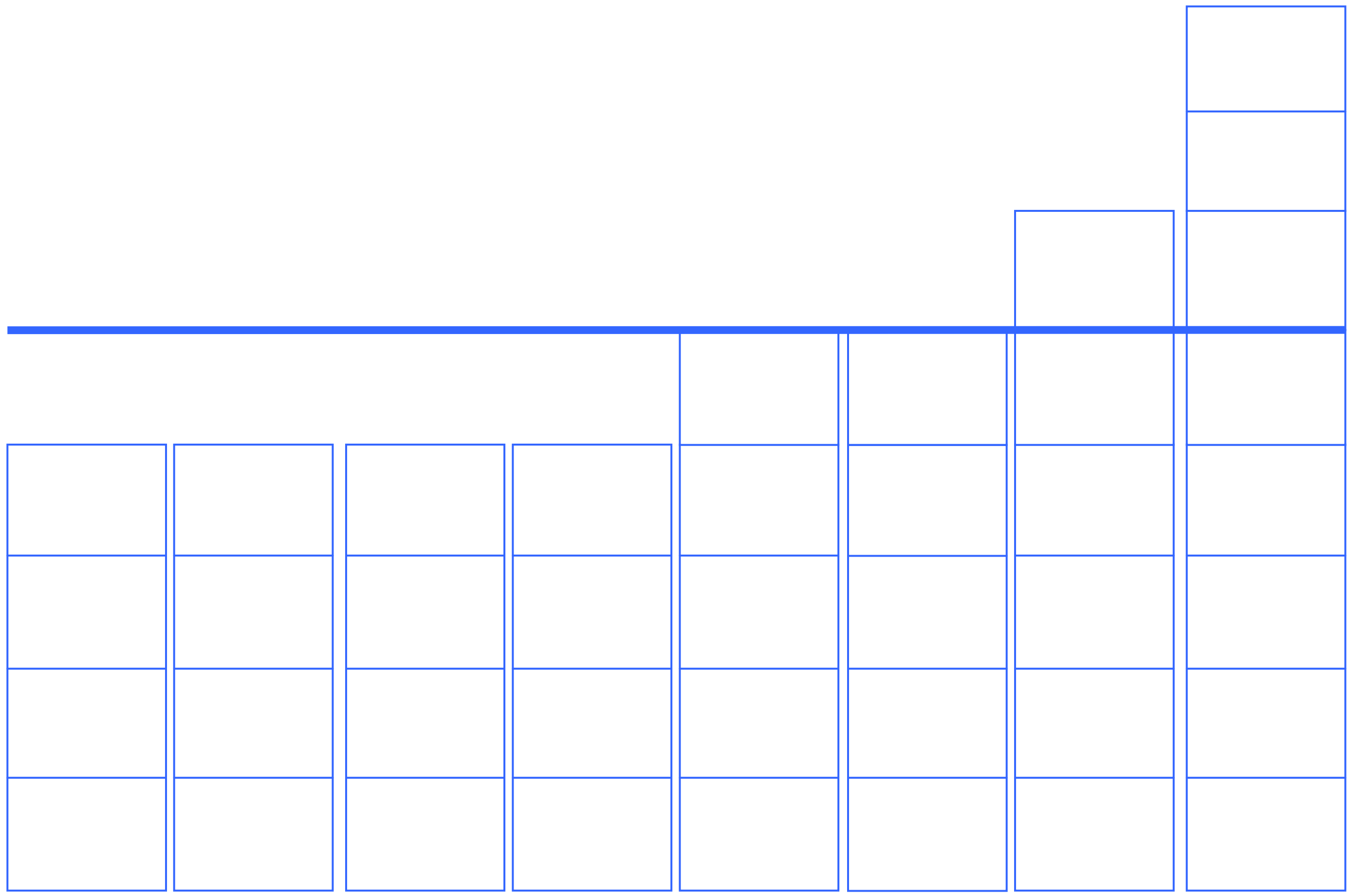
"Build" or draw the distribution for each group.

- What is the "fair share" or the mean for each group?
- Which group do you think is closer to being "fair"? Explain your thinking and illustrate with a model.

In this problem, students address the issue of variability by deciding whether the distribution in Group 1 or the distribution in Group 2 is "more fair". (You may want to relate the situation back to cookies or candy bars.)
The "fair share" or mean for each group is 5 letters.
We have not defined the term "fair" at this point. Students will have varying opinions about which group is more fair. Students are likely to share versions of the following ideas.

- Group 1 is "more fair" because (if still illustrating the name lengths with stacks of cubes) you only have to move four cubes to make all eight stacks even.
- Group 1 is "more fair" because, in Group 2, some people only have 3 letters and another has 8. That is a bigger difference than occurs in Group 1 where the least number of letters is 4 and the greatest number is 8.
- Group 2 is "more fair" because more people started out with the "fair share" value of 5 letters.

Allow students to present their models and share their thinking. All of the ideas above are valid. Various drawings or models using the cubes are also acceptable. Hopefully some students will present representations for both groups similar to the model below. Here a horizontal line is drawn at the mean or "fair share"



Obviously, there is a need to define what is meant by "more fair". Suppose we consider the move of one cube (or the redistribution of one letter in this case) as a *step*. Then we will agree that the fewer the number of steps needed to obtain the "fair share" (or the mean of a set of data), the more fair the distribution.

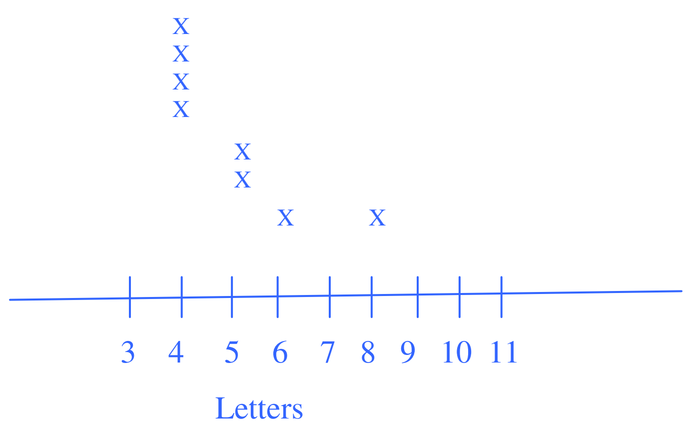
In this event, Group 1 is "more fair" because we only have to make four *moves (steps)* to obtain the fair share of 5 letters for each student. In Group 2, we have to make 5 *moves (steps)*. This idea is easy to visualize using the model above.

In the first three problems you examined how the ideas of *mean* and *variability in data* are introduced to young children. In the next few problems, you will examine the two ideas using a more sophisticated approach.

4. Look again at the name length distribution for the students in Group 1 of problem 3. On chart paper, your group is to represent the distribution of name lengths for the 8 students using a dotplot.
   - Scale your plot assuming that no name will have less than 3 letters or more than 11 letters.
   - Represent each data value with post-it notes rather than drawing on the plot so that different plots may be represented on your chart paper as you work through the remainder of the task.

   Compare your plot with those of other groups. Do they all look the same?

   The purpose of this problem is to be sure that all students know how to construct a dotplot. (These graphs are referred to as line plots in the GPS standards and are first addressed in the 6^{th} grade. Statisticians refer to them as dotplots.) Each group of students should receive a piece of chart paper and 1" by 1" post-it notes to use as their "dots" so that the plot may be varied in problems 5 and 6 without having to re-draw the graph. Using a scale of 3 to 11 is simply a logistical issue so that students have plenty of room to position the post-it notes on their plots as the graphs are varied. A plot of the distribution for Group 1 is shown below. Make sure students understand that the number of letters in a name is represented on the horizontal scale and the dots (or post-it notes in this case) represent students with the given name length.



5. Use your same chart paper to create a dotplot consisting of 8 students each of which has a name length of 6 letters.
    c. What is the mean name length for this group of students?
    d. Create a different plot consisting of 8 students with the same mean name length. Explain how you arrived at your plot.

The mean name length of this group of students is obviously 6. The purpose of this problem is for students to begin to think of the mean as a *balancing point*. They move from a distribution of 8 students all with a name length of 6, to a *different* distribution still of 8 students with a mean name length of 6 letters. Students will take different approaches to this problem. Strategies may include:

- The determination that there is a total of 48 letters to be shared by 8 students. Your students may assign different name lengths to the students in the problem, totaling letters as they go. Any combination of 8 students with a total of 48 letters in their names is acceptable.
- The mean may be considered as a "balancing point". For example, beginning with the existing plot (8 students all with a name length of 6), if a post-it note is moved one unit to the right (there is now one student with a name length of 7), a post-it note needs to be moved one unit to the left (one student with a name length of 5), in order to keep a mean (or a *balance*) of 6. If a post-it note is moved 2 units to the right (a student with a name length of 8), a post-it note must be moved two units to the left (a student with a name length of 4), etc.

Before moving on to problem 6, make sure that all students are comfortable with using some strategy to create a different plot. Do not "force" one particular strategy on students. Rather, look for different approaches that can be shared later.

6. Your group will now be asked to create a dotplot representing 8 students with a mean name length of 6 letters based on a specific restriction provided by your teacher. Be prepared to explain how you arrived at your plot.

Once you are sure that a group is comfortable with at least one strategy for determining a different distribution, still with 8 students and a mean name length of 6 letters, give the group one of the restrictions provided here.

Create a dotplot of 8 students with a mean name length of 6 letters in which:

- More names have more than 6 letters
- More names have less than 6 letters
- Exactly two names have 10 letters
- Exactly 3 names have 7 letters
- Exactly 4 names have 3 letters
- There are an odd number of names with 6 letters

(Similar restrictions may be added if more than 6 groups or pairs of students are working on the task.)

As groups construct their plots, ask them to *quantify* how they arrived at the plot. Lead (by asking guiding questions) some groups to think about representing their moves to the right and to the left, using the mean as a balancing point, as moves in the positive direction (represented with positive numbers) and moves in the negative direction (represented as negative numbers). Students should recognize that the sum of the moves in the positive direction and the moves in the negative direction should equal 0.

7. Share the distribution your group created in problem 6 with the distributions of other groups. Does it make sense to compare our various distributions using the mean? Why or why not? What ways, other than using the mean, might we compare the distributions?

It is not very helpful to compare these distributions using only the mean since the mean for all distributions is 6 letters.

As students share the distributions they have created in this part of the task, you will quantify the amount of variability that exists in each of the distributions. Quantities that measure the amount of variability in data are called *measures of spread*. In elementary school students are introduced to the range as a measure of spread. In middle school they investigate the interquartile range. Here they will be introduced to another measure of spread, the *mean absolute deviation*. The important concepts in this part of the task include:

- Deviations from the mean: *(value – mean)*= $x_i - \bar{x}$, where x_i is each individual data point, $\bar{x}$ is the sample mean
- Absolute values of the deviations from the mean: |*value – mean*|= $|x_i - \bar{x}|$
- The sum of the absolute values of the deviations from the mean (SAD): *total distance from the mean of all values* = $\sum_{i=1}^{N} |x_i - \bar{x}|$, where N is the sample size.
- The average (or mean) of the absolute values of the deviations from the mean or the **Mean Absolute Deviation** (MAD):

$$\frac{\text{total distance from the mean for all values}}{\text{number of data values}} = \frac{\sum_{i=1}^{n} |x_i - \bar{x}|}{N}$$

The following discussion provides ideas for the sharing discussed in problem 7:

Have students post their distributions, based on the given restrictions, around the room. Have one student from each group stand beside their work with a marker in hand. Ask a student from each group to share their restriction and explain how they arrived at their distribution. During the course of the presentations, focus on a group that has used positive and negative numbers to represent their deviations from the mean. Allow them to discuss their work being sure to bring out the following points:

- The sum of the deviations from the mean in the positive direction is the opposite of the sum of the deviations from the mean in the negative direction (balance).
- The sum of the deviations from the mean is 0.

Have this group find the total distance from the mean for all data values (the sum of the absolute values of the deviations from the mean). *There is no need to introduce formal notation yet.*

Go back to each group, no matter what strategy was used, and ask each student with the marker to determine the total distance from the mean of all values. Have them write and circle this number in the corner of each poster. When all groups have presented and determined the total distance from the mean for their distribution, ask the students the following questions:

- *Which group is "most fair" based on our definition of fair in problem 3?*
  The group that is most fair will be the group with the least deviation (smallest total distance) from the mean.
- *Which group is least fair?*
  The group that is least fair will be the group with the highest deviation (greatest total distance) from the mean.

Explain to students that distance from the mean is referred to as **deviation from the mean** and share the notation $x_i - \bar{x}$ **.** Also share the concepts and notations for **absolute values of the deviations from the mean** and **the sum of the absolute values of the deviations from the mean (SAD)**.

Help students equate the SAD to the number circled on their charts.

Ask students if they can think of any shortcomings in using the SAD to quantify the variability from the mean in different distributions? Using the SAD to quantify the variability of the data from the mean in the distributions created in this activity is reasonable because each distribution has 8 data values. However, it is *not* so useful in comparing distributions in general because not all distributions are the same size. More data values could naturally cause more variability. An *average* of the deviation from the mean per data value would be much more useful. Thus we find the mean of the absolute deviations or the **Mean Absolute Deviation (MAD)**.

Share the concept and notation for the MAD with students and have the students with the markers calculate the MAD for each distribution.

8. Your class discussion of the questions in problem 7 should have led you to discover statistical concepts often labeled the SAD and the MAD. Describe each concept using both words and symbols. Discuss the relationship between the two and explain why the MAD is a better measure of variability than the SAD.

   This question could be used as a ticket out the door or a homework assignment to summarize the material discussed in class and to assess student understanding.

   Discussions should include the following ideas:
   - Both the SAD and the MAD are measures of spread.
   - The sum of the positive deviations from the mean of a distribution is the opposite of the sum of the negative deviations from the mean.
   - The sum of the deviations from the mean of a distribution is 0.
   - The SAD is the sum of the absolute values of the deviations from the mean.
   - Although the SAD is a measure of spread, its usefulness is limited because different distributions are often of different sample sizes.
   - The MAD is the average of the absolute values of the deviations from the mean of a distribution.

   Notation and formulas should be included. Students should also be encouraged to provide examples.

Mathematics I
SAD to MAD
Day 1 Homework

Math I semester grades for Avery and Jonas are shown below. Their teacher must use these grades to pick one of them as the final member of the math team. Who should she choose? Justify your decision using the ideas discussed in today's task. Your justification should include the concepts of mean, median, range, the SAD, the MAD, and any other information that you feel is important..

Avery: 70, 100, 98, 72, 90, 85, 80
Jonas: 86, 84, 85, 85, 84, 86, 85

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Mathematics I: Unit 4
The Chances of Winning

Task 2: Sampling Rectangles

Mathematics I
Task 2: Sampling Rectangles **Day 1/2**
(This task can be found on pages 13-19 and 94-97 of *Navigating through Data Analysis in Grades 9-12* published by NCTM. The publication can be ordered from the NCTM website.)

Standard(s): MM1D3. Students will relate samples to a population.
a. Compare summary statistics (mean, median, quartiles, and interquartile range) from one sample data distribution to another sample data distribution in describing center and variability of the data distributions.
b. Compare the averages of summary statistics from a large number of samples to the corresponding population parameters.
c. Understand that a random sample is used to improve the chance of selecting a representative sample.

New vocabulary: convenience sample, voluntary sample, random sample, representative sample, sampling distribution, random number generator

Mathematical concepts/topics: simple random samples; subjective samples; bias in the sampling process; comparison of graphical representations including boxplots and dotplots

Prior knowledge: mean, median, mode, 5-number summary, boxplots, and dotplots

Essential question(s): How can I use samples to help make reasonable statements about a population? Do subjective or random samples give better estimates of population parameters?

Suggested materials: For each student:
- Copy of activity pages from *Navigating through Data Analysis in Grades 9-12* (These blackline masters can be downloaded and printed from the CD provided with the publication).
  - Random Rectangles (pg 94)
  - Sampling Rectangles (pg 95)
  - Distribution of Sample Mean Areas of Rectangles (pg 97)
- copy of the student task, immediately following this lesson plan
- graphing calculators or statistical software

Warm-up: Assess students' understanding of the concepts of random sampling by having them answer the following questions given at the beginning of the *Sampling Rectangles* activity.

- *Why do polls use data from people selected randomly?*
- *Do you think most people can do a good job of choosing a "typical" group of people for a poll?*
- *Why do scientists choose random samples when they are designing an experiment, and how do they make their selections?*

Opening: Discuss the warm-up and incorporate the discussion of *random* versus *accidental sampling* and *convenience* and *voluntary* samples contained in the first paragraph of page 15 in the Navigations publication.

Worktime: Students may only complete problems 1-3 of the student task, depending upon how much time they need to review the construction of boxplots. Both boxplots and dot plots will continue to be used throughout the mathematics curriculum so it is important that students master these skills.

Make sure every student records the class data for both sample distributions.

Closing: Review the 5 number summary and summarize what students have learned to this point.

Homework:

For the data distribution below:

1. Draw a dotplot of the data.

2. Generate a five number summary for the data.

3. Overlay a boxplot of the data on your dotplot.

4. Examine your graphs. How many data values fall at or below Q_1? On or between Q_1 and the median? On or between the median and Q_3? At or above Q_3? Explain.

5. Find the mean and the mean absolute deviation for the set of data.

6. Describe the position of the mean in relation to the median. Explain why this is the case.

7. What does the mean absolute deviation tell you?

10, 12, 9, 4, 6, 8, 7, 6

Differentiated support/enrichment:

Check for Understanding:

Resources/materials for Math Support: Students will probably need to review finding a 5 number summary and drawing both boxplots and dotplots. They should be provided multiple opportunities to compare data distributions using measures of center and spread.

Previewing new vocabulary will be important throughout this unit. Much of the statistical vocabulary is very new to students.

Mathematics I
Sampling Rectangles
Day 1 Student Task

1. Complete problem 1 of *Sampling Rectangles.* Record your information in the allotted space on the activity sheet.

2. Use the random number generator explained by your teacher to complete problem 2 of *Sampling Rectangles*. Record your information in the allotted space on the activity sheet.

3. Each student has reported to the class the mean area of their subjectively chosen rectangles and results have been compiled. Using the top scale on the page labeled *Distribution of Sample Mean Areas of Rectangles*, represent the class results for subjectively chosen rectangles with both a dotplot and a boxplot. Overlay the boxplot on the dotplot in order to facilitate the comparison of the two representations. Draw both plots without the use of technology.

Mathematics I
Sampling Rectangles
Day 1 Homework

For the data distribution below:

1. Draw a dotplot of the data.

2. Generate a five number summary for the data.

3. Overlay a boxplot of the data on your dotplot.

4. Examine your graphs. How many data values fall at or below Q_1? On or between Q_1 and the median? On or between the median and Q_3? At or above Q_3? Explain.

5. Find the mean and the mean absolute deviation for the set of data.

6. Describe the position of the mean in relation to the median. Explain why this is the case.

7. What does the mean absolute deviation tell you?

10, 12, 9, 4, 6, 8, 7, 6

Mathematics I
Task 2: Sampling Rectangles **Day 2/2**
(This task can be found on pages 13-19 and 94-97 of *Navigating through Data Analysis in Grades 9-12* published by NCTM. The publication can be ordered from the NCTM website.)

Standard(s): MM1D3. Students will relate samples to a population.
a. Compare summary statistics (mean, median, quartiles, and interquartile range) from one sample data distribution to another sample data distribution in describing center and variability of the data distributions.
b. Compare the averages of summary statistics from a large number of samples to the corresponding population parameters.
c. Understand that a random sample is used to improve the chance of selecting a representative sample.

New vocabulary: summary statistic, population parameters

Mathematical concepts/topics: simple random samples; subjective samples; bias in the sampling process; comparison of graphical representations including boxplots and dotplots; variability in sampling techniques; comparing summary statistics from one sample data distribution to another using center and spread; comparing averages of summary statistics from samples to the population parameters

Prior knowledge: mean, median, mode, 5-number summary, boxplots, and dotplots

Essential question(s): How can I use samples to help make reasonable statements about a population? Do subjective or random samples give better estimates of population parameters?

Suggested materials: For each student:

- Activity pages from *Navigating through Data Analysis in Grades 9-12* distributed in previous lesson:
  - Random Rectangles (pg 94)
  - Sampling Rectangles (pg 95)
  - Distribution of Sample Mean Areas of Rectangles (pg 97)
- copy of problems 4 -8 of the student task, immediately following this lesson plan
- graphing calculators or statistical software

Warm-up: Have students compare homework from the previous lesson with a partner.

Opening: Discuss the homework. Overlaying the boxplot on the dotplot, students should see that the data values can be grouped so that 2 of the values (25%) fall *at* or below the first quartile, two of the values fall *on* or between the first quartile and the median, etc., thus re-enforcing the characteristics of a boxplot.

The median of this set of data is 7.5. The mean is 7.75. The mean is slightly larger than the median. Although this is a very small set of data, and the difference between the median and the mean is small, student should still be able to see that there is a little more variability in the values above the median than below the median. Values above the median are more spread out. This can also be visualized by examining the part of the boxplot above the median.

This discussion will help students answer the questions in the remainder of the task.

Worktime: Students should complete problems 4 - 8 of the student task. In problem 4 students are instructed to produce plots using technology. It may be necessary at this point to have a mini-lesson related to the technology being used in your classroom.

The discussion held in the opening of the lesson should help students begin to answer the questions asked in problems 5 – 7. Monitor student discussions carefully to be sure that important points are addressed in comparing and contrasting both the representations of the data (boxplots and dotplots) and the data distributions themselves.

Closing: Project the plots of the two sampling distributions so that all students can view them during the closing discussion.

Have students compare and contrast the two simulated sampling distributions by sharing their answers to problems 5 – 7. (See the teacher notes following this task and pages 11-19 of *Navigating through Data Analysis in Grades 9-12*).

Once the two simulated sampling distributions have been compared and contrasted, students will compare the results of their samples to the actual population parameters. Allow them to share their responses to the questions in problem 8. *(The mean of the population is given in the teacher notes.)*

The following concepts should have been carefully addressed throughout this task and summarized in the closing:

- *Subjectively chosen samples may be biased.*
- *True random samples are in fact carefully chosen to ensure that every set of a specified size consisting of individuals in a population under consideration is equally likely to be selected.*
- *Though an individual event is not predictable, there is an underlying pattern in long-term random behavior.*
- *Estimates obtained from samples rarely give the exact mean of a population but a random sample provides an unbiased estimate of the population mean.*
- *Different representations of a data distribution can provide different information. It is often useful to examine a distribution using more than one type of representation.*

Homework:

Differentiated support/enrichment:

Check for Understanding:

Resources/materials for Math Support: Students should continue to preview vocabulary and prior knowledge identified in this and the previous lesson.

Mathematics I
Sampling Rectangles
Day 1 Student Task

4. Using the bottom scale on the activity page labeled *Distribution of Sample Mean Areas of Rectangles*, represent the class results for randomly chosen rectangles with both a dotplot and a boxplot. Use your calculator (or other technology) to draw these plots and then copy them onto your activity sheet.

5. For each of the two *simulated sampling distributions* you have represented, what does the boxplot tell you that the dotplot does not tell you? What does the dotplot tell you that the boxplot does not tell you?

6. Find the mean and the mean absolute deviation for each distribution above.

7. Using the information you have obtained in problems 3 – 6, make at least five true statements that compare and contrast the two sampling distributions.

8. How well do you think the two distributions predict the actual mean area of the 100 rectangles shown on the page *Random Rectangles*? Do you think a subjectively chosen sample or a randomly chosen sample would give a better estimate of the actual population mean?

Your teacher KNOWS the truth! Ask your teacher for the true average area of the 100 rectangles shown on the page *Random Rectangles*. Which sampling technique, random sampling or subjective sampling, provided a better estimate of the true population mean in this case? Why do you think this is true?

Mathematics I
Sampling Rectangles
Teacher Notes

This task has been adapted from the activity *Sampling Rectangles*, in Chapter 1 of *Navigating through Data Analysis in Grades 9-12*, a publication of NCTM. It is imperative that teachers read the notes related to this activity on pages 11-19 of the publication before using this task in the classroom.

1. Complete problem 1 of *Sampling Rectangles.* Record your information in the allotted space on the activity sheet.

   Have students complete problem 1 of *Sampling Rectangles* as written, recording information in the allotted space on the activity sheet. (Page 95)

2. Use the random number generator explained by your teacher to complete problem 2 of *Sampling Rectangles*. Record your information in the allotted space on the activity sheet.

   Before beginning problem 2, discuss with students at least one method of generating random integers. Instructions for the TI-83/84 families are included here.

   MATH→ PRB→ 5: randInt (lower bound, upper bound, number of trials)

   Note 1: Generally the number of trials entered into the calculator is slightly larger than the required number of data values. This is because, as a rule, repeated values are not used in a sample. For example, we might enter *randInt (1, 100, 8)* and then use the first 5 distinct values generated.

   Note 2: Monitor student results from the TI random integer generator carefully. If this is the first time the random generator has been used on the calculators, they will all generate the *same set of integers*! Have students generate sets of numbers several times so that sets of numbers are different, otherwise your data will be skewed.

3. Each student has reported to the class the mean area of their subjectively chosen rectangles and results have been compiled. Using the top scale on the page labeled *Distribution of Sample Mean Areas of Rectangles*, represent the class results for subjectively chosen rectangles with both a dotplot and a boxplot. Overlay the boxplot on the dotplot in order to facilitate the comparison of the two representations. Draw both plots without the use of technology.

   Compile the first **simulated sampling distribution** by having each member of the class report the mean area of their subjectively chosen rectangles. *Every student should record the data.* Compile the second **simulated sampling distribution** by having each member of the class report the mean area of their randomly chosen rectangles. *Every student should record this set of data as well.*

Once both data distributions have been compiled, students should represent each distribution with both a dotplot and a boxplot. Students have been asked to draw the first plots by hand in order to review material studied in the middle grades. It may be necessary to have a mini-lesson on computing the "5- number summary" (maximum value, minimum value, first quartile, median, and third quartile) needed for drawing a boxplot.

The homework provided with this lesson re-enforces drawing boxplots, comparing the mean and the median, and calculating the mean absolute deviation.

4. Using the bottom scale on the activity page labeled *Distribution of Sample Mean Areas of Rectangles*, represent the class results for randomly chosen rectangles with both a dotplot and a boxplot. Use your calculator (or other technology) to draw these plots and then copy them onto your activity sheet.

The importance of using technology to plot data and perform statistical calculations cannot be overstated. It is expected here that students already know, or will learn, how to use their calculators or other technology, to plot the data for randomly chosen rectangles. A mini-lesson on calculator operations or the use of another technology may be necessary.

5. For each of the two *simulated sampling distributions* you have represented, what does the boxplot tell you that the dotplot does not tell you? What does the dotplot tell you that the boxplot does not tell you?

Answers will vary. However, the following statements are usually true and should be included in the discussion in some form.

The boxplot:

- provides a picture of one measure of center (the median);
- provides a compact view of how the data are distributed over the range (quartiles and interquartile range);
- does not allow one to see individual data values.

The dotplot:

- displays individual data values;
- makes clusters and/or gaps apparent;
- does not display an obvious measure of center.

6. Find the mean and the mean absolute deviation for each distribution above.

Answers will vary. These values should be found with the aid of technology.

7. Using the information you have obtained in problems 3 – 6, make at least five true statements that compare and contrast the two sampling distributions.

Answers will vary. Comparisons should include the use of measures of center, interquartile ranges, ranges, mean absolute deviations, and any other important information. (See teacher notes in the publication.)

8. How well do you think the two distributions predict the actual mean area of the 100 rectangles shown on the page *Random Rectangles*? Do you think a subjectively chosen sample or a randomly chosen sample would give a better estimate of the actual population mean?

Your teacher KNOWS the truth! Ask your teacher for the true average area of the 100 rectangles shown on the page *Random Rectangles*. Which sampling technique, random sampling or subjective sampling, provided a better estimate of the true population mean in this case? Why do you think this is true?

The mean area of the 100 rectangles (shown on page 14 and again on page 94 of the publication) is 7.42 square units. Generally, the mean and median of the subjectively chosen sample mean areas tend to be much higher than the mean and median of the randomly chosen sample mean areas. This is due to the fact that the human eye is drawn to larger objects.

This task highlights five very important concepts:

1. Subjectively chosen samples may be biased.
2. True random samples are in fact carefully chosen to ensure that every set of a specified size consisting of individuals in a population under consideration is equally likely to be selected.
3. Though an individual event is not predictable, there is an underlying pattern in long-term random behavior.
4. Estimates obtained from samples rarely give the exact mean of a population but a random sample provides an unbiased estimate of the population mean.
5. Different representations of a data distribution can provide different information. It is often useful to examine a distribution using more than one type of representation.

It is very important that these ideas are addressed in the closing.

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Mathematics I: Unit 4
The Chances of Winning

Task 3: Using Spinners with Congruent Sectors

Mathematics I
Task 3: Using Spinners with Congruent Sectors **Day 1/1**
(GaDOE Acc Math I TE Spinner Task 1)

Standard(s): MM1D1a. Students will determine the number of outcomes related to a given event.
a. Apply the addition and multiplication principles of counting.
MMID2a. Students will use the basic laws of probability.
a. Find the probabilities of mutually exclusive events.
c. Calculate conditional probabilities.

New vocabulary: conditional probability, multiplication principle of counting, mutually exclusive events

Mathematical concepts/topics: probability, conditional probability, tree diagrams, independent events, dependent events, multiplication principle of counting, probability of mutually exclusive events

Prior knowledge: simple probability, tree diagrams, independent events, dependent events, multiplication principle

Essential question(s): How can I tell if a spinner is really random? How do I know if a spinner is "fair"?

Suggested materials: For each student:
- protractors
- spinners or spinner master with paper clips or safety pins

Warm-up: Ask students to read the introduction to complete problems 1, 2 and 3 of the task.

Opening: Discuss the fact that students will be calculating various probabilities in this task. Ask them to talk with their partners concerning the meaning of phrases "at least", and "at most". Have some partners share their discussion with the teacher questioning for understanding if needed. This discussion will help students answer the questions in the remainder of the task.

Worktime: Students should complete problems 4 - 12 of the student task. In problem 4, the concept of mutually exclusive events may be introduced. The discussion held in the opening of the lesson should help students answer the questions asked in problems 4 – 12.

Problems 11 and 12 deal with conditional probability. Allow students time to determine their own methods of solving these problems before introducing formal terminology. It is certainly NOT necessary to introduce the formula for conditional probability at this point. Hopefully students will notice that in these problems they are simply looking at a smaller sample space for finding the indicated probabilities.

Closing: Teachers may want to randomly choose three groups to make the formal presentation of questions 5 – 6, 7 – 9, and 10 – 12. Other groups should be allowed to share any additional insights or make corrections.

Homework: Students should complete problems 1 – 6 from the GaDOE Spinner Task 2. Note modifications have been made to these questions to utilize proper probability notation.

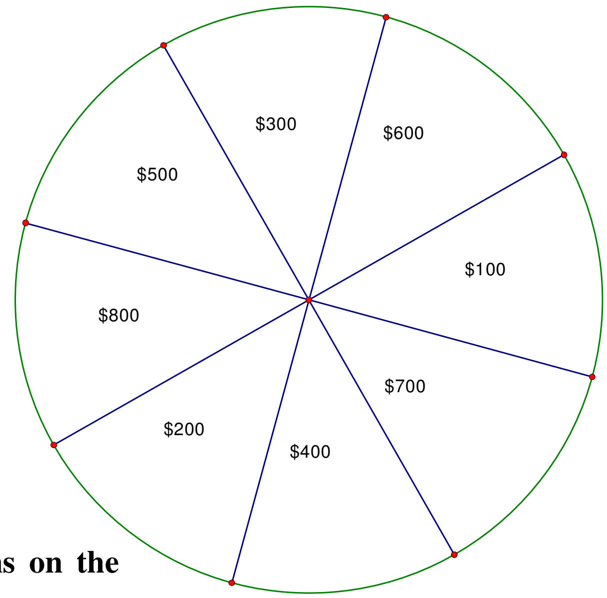
Differentiated support/enrichment:

Check for Understanding:

Resources/materials for Math Support: Students should continue to preview vocabulary and prior knowledge identified in this and the previous lesson.

Mathematics I
Spinner Task 1
Day 1 Student Task

Calculate the following probabilities given one spin on the spinner below (assuming the spinner is fair).



1) P($800) =

2) P($400) =

3) Does P($100) = P($800)? __________ Why or why not?

4) P(at least $500) =

5) P(less than $200) =

6) P(at most $500) =

Calculate the following probabilities given two spins on the above spinner.

7) P(sum of $200) =

8) P(sum of at most $400) =

9) P(sum of at least $1500) =

10) P(sum of at least $300) =

11) P(sum of $200 | first spin lands on $100) =

12) P(sum of at least $1000 | first spin lands on $800) =

Mathematics I
Spinner Task I
Day 1 Homework

Look at the spinner below. Determine the following probabilities if we assume the $100, $500, $700 and $600 regions are half the size of the $800, $200, $300 and $400 regions.

1) P($800) =

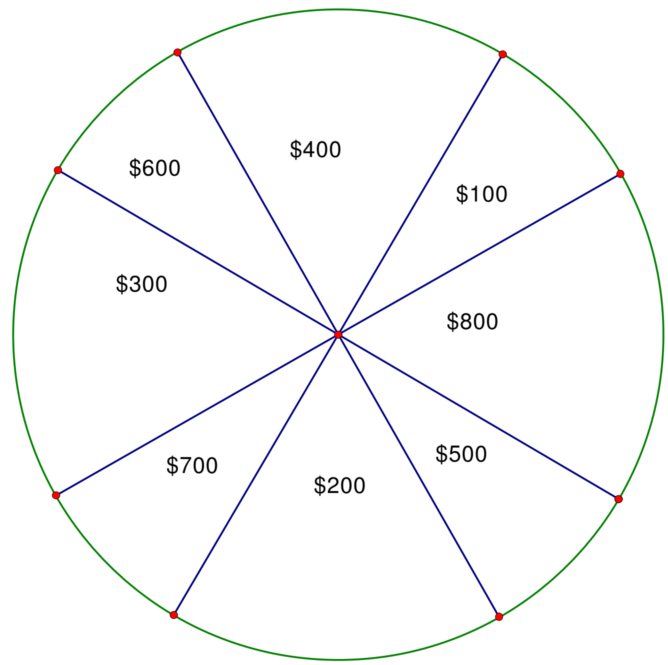
2) P($400) =

3) Does P($100) = P($800)?
Why or why not?
How does P($100) compare to P($800)?

4) P(at least $500) =

5) P(less than $200) =

6) P(at most $500) =



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Mathematics I: Unit 4
The Chances of Winning

Task 4: Using Spinners with Sectors of Different Sizes

Mathematics I
Task 4: Using Spinners with Sectors of Different Sizes **Day 1/1**
(GaDOE Acc Math I TE Spinner Task 2)

Standard(s): MM1D1a. Students will determine the number of outcomes related to a given event.
a. Apply the addition and multiplication principles of counting.
MMID2a. Students will use the basic laws of probability.
a. Find the probabilities of mutually exclusive events.
c. Calculate conditional probabilities.

New vocabulary:

Mathematical concepts/topics: probability, tree diagrams, independent events, dependent events, multiplication principle of counting, set notation, conditional probability

Prior knowledge: simple probability, tree diagrams, independent events, dependent events, addition and multiplication principle of counting, set notation

Essential question(s): How does dealing with outcomes of different weights affect probabilities? How do I find probabilities of events involving conditional probability?

Suggested materials:

Warm-up: Ask students to discuss the results of their homework (Questions 1-6 from Spinner Task 2).

Opening: Discuss the fact that students will be extending what they learned in the previous class to reflect the spinner with unequal sectors from the homework assignment. Be certain all misconceptions have been address from the homework assignment during the warm-up.

This discussion is critical to the success of the remaining portions of this task.

Worktime: Students should complete problems 1 – 6 the task. In problems 4 & 5, the concept of conditional probability is again addressed. Using carefully crafted teacher questioning, students should be able to derive the formula. This is a great opportunity for the set notations learned in Grade 8 to be revisited.

Closing: Allow students to share their work. (See GaDOE teacher notes.)

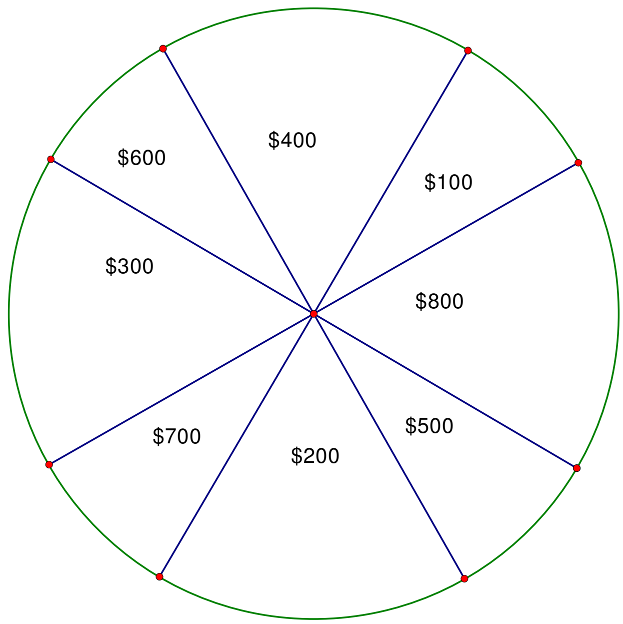
Homework:

Differentiated support/enrichment: See the GaDOE comments at the end of this task (Spinner Task 3 in the TE Framework) for another investigative task.

Check for Understanding:

Resources/materials for Math Support: Students should continue to preview vocabulary and prior knowledge identified in this and the previous lesson.

Mathematics I
Spinner Task 2
Student Task



Calculate the following probabilities given two spins on the above spinner.

1) P(sum of $200) =

2) P(sum of at most $400) =

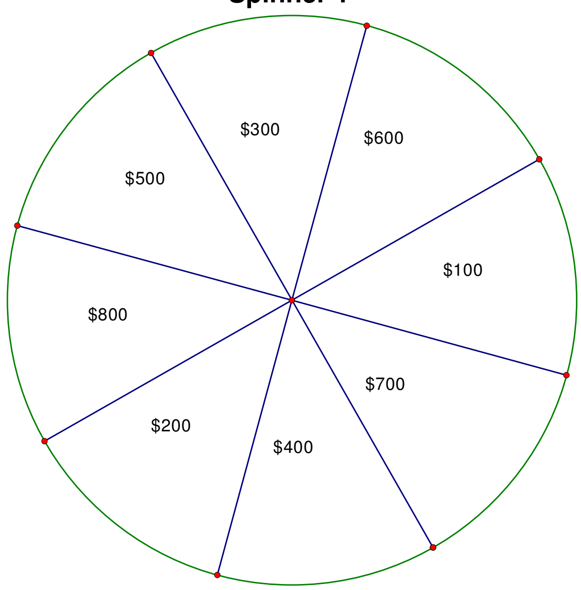
3) P(sum of at least $1500) =

4) P(sum of $200 | first spin lands on $100) =

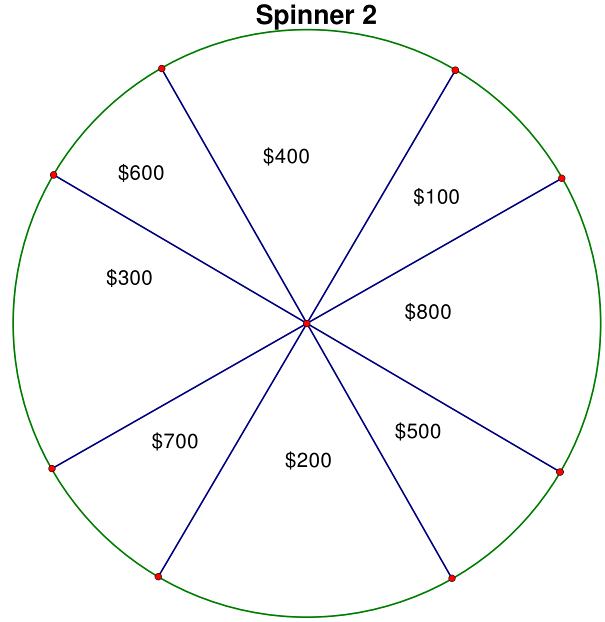
5) P(sum of $1500 | first spin lands on $800) =

6) If you were trying to win the most money, which of the spinners below would you choose and why?

Spinner 1



Spinner 2



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Mathematics I: Unit 4
The Chances of Winning

Task 5: Shooting Free Throws

Mathematics I

Task 5: Shooting Free Throws **Day 1/1**

(This task can be found on pages 53-55 and pages105-108 of *Navigating through Probability in Grades 9-12* published by NCTM. The publication can be ordered from the NCTM website.)

Standard(s): MM1D1a. Students will determine the number of outcomes related to a given event.

a. Apply the addition and multiplication principles of counting.

MMID2a. Students will use the basic laws of probability.

a. Find the probabilities of mutually exclusive events.
b. Find the probabilities of dependent events.
c. Calculate conditional probabilities.
d. Use expected value to predict outcomes.

New vocabulary: simulation, expected value

Mathematical concepts/topics: designing simulations, probability of independent events, theoretical probability, experimental probability, mean, expected value

Prior knowledge: percent, theoretical probability, experimental probability, mean, mode

Essential question(s): How can I determine the expected value (the mean number of points per opportunity) for foul shots if I know the success rate for the player shooting the foul shots?

Suggested materials: cards, random number tables, calculators with random number function, Set Up Simulation (from the CD accompanying NCTM's *Navigating through Probability in Grades 9-12*), Random Number Generator (from the CD accompanying NCTM's *Navigating through Probability in Grades 9-12*), and Binomial Distribution Simulator applet (from the CD accompanying NCTM's *Navigating through Probability in Grades 9-12*)

Warm-up: Post the following:

Your teacher uses the following weights to average your semester grade.

Tests: 80%
Quizzes: 15%
Homework: 5%

Calculate your final grade if you have the following averages.

Tests: 83%
Quizzes: 92%
Homework: 87%

Opening: Discuss the warm-up. Explain to students that a way that a weighted average can also be described as expected value.

Ask students to share what they know about basketball foul shots and free throws. During the discussion, be certain students understand what is meant by one-and-one opportunities and two-shot chances; how they are alike and how they are different. If appropriate for the class, it could be motivating to have a student model a few of these shots using a paper-wad and trashcan and determine his/her successful percent of free throws. If this would not be appropriate for the class, short (1- 3 minute) videos of basketball foul shots are easy to find on the Internet and may be substituted for the trashcan activity.

Ask students to carefully read the paragraph at the beginning of the task and to answer *Question 1*. Hold a class discussion about their predictions and why they picked the number of foul shots most likely to be made. Record the class choices for each option for comparison purposes at the end of the task.)

Worktime: Give students time to reflect and work on their response to question 2. After about 3-5 minutes, allow them to discuss their simulation design with a partner. After about 5 more minutes, bring the class together for students to share their ideas. Through carefully crafted questioning, the teacher should lead them (if needed) to a design that could be successful. (See the Set Up Simulation and Random Number Generator from the CD accompanying NCTM's *Navigating through Probability in Grades 9-12* for assistance.

Once the class understands how to conduct the simulation in a correct fashion (there is more than one way to do this correctly); students should continue working on the rest of the questions.

When the majority of students have completed #4, bring the large group together for sharing of results and ideas. Be certain students understand expected value correctly and all misconceptions are corrected before sending them back to the task. Do not tell them what their results should be. Instead, question to encourage sharing and student dialogue until there is a correct class consensus.

Less time will be needed for questions 5 & 6 than was needed for questions 1 & 2. Give students time to reflect and work on their response to question 5 and allow them to discuss their simulation design with a partner. Afterwards, bring the class together for students to share their ideas.

Allow time for students to complete the task.

Closing: Allow students to share their work again listening for possible misconceptions to be cleared through teacher questioning and student dialogue. Students should understand that expected value does not have to be a possible value of the random variable. Be sure students are able to explain how and why this makes sense to help them think more clearly about the concept of expected value as a long-term average outcome.

Homework: Students should complete problems 1 – 4 of GaDOE Accelerated Mathematics I Spinner Task 3..

Differentiated support/enrichment:

Check for Understanding:

Resources/materials for Math Support: Students should continue to preview vocabulary and prior knowledge identified in this and the previous lesson.

Mathematics I
Shooting Free Throws
Day 1 Student Task

1. Carefully read the paragraph at the beginning of the task and complete question 1. Be sure you are able to articulate your reasoning clearly concerning why you chose your response to question 1.

2. Complete 2 – 4 of *Shooting Free Throws.* Record your information in the allotted space on the activity sheet.

3. Compare and discuss your results with your partner.

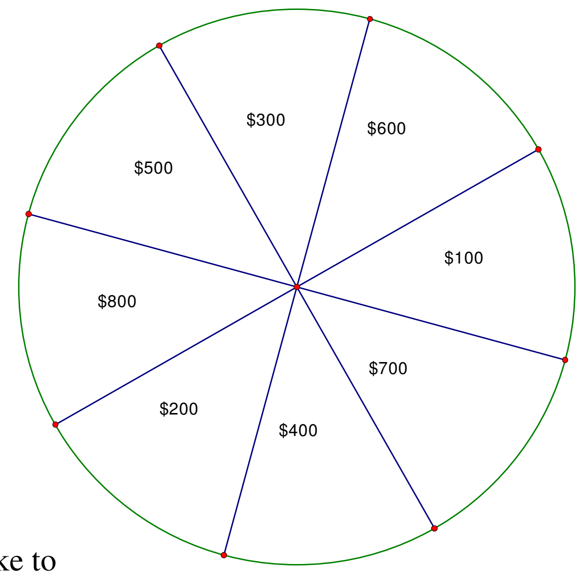
4. Complete problem 5 and be sure you are able to articulate your reasoning clearly concerning why you chose your response.

.

4. Complete 6 – 8 of *Shooting Free Throws.* Record your information in the allotted space on the activity sheet.

6. Compare your results with your partner's results. Be prepared for both you and your partner to possibly be asked to present your findings to the class and explain your reasoning and findings.

Mathematics I
Shooting Free Throws
Homework



The spinner shown at the right is divided into eight congruent sectors. Use the spinner to answer the following questions.

1) If you spin the spinner once, how much money, on average, would you expect to receive?

2) If you spin the spinner twice, how much money, on average, would you expect to receive?

3) If you spin the spinner 10 times, how much money, on average, would you expect to receive?

4) On average what is the fewest number of spins it will take to accumulate $1000 or more?

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Mathematics I: Unit 4
The Chances of Winning

Task 6: Oops! Bankrupt!

Mathematics I
Task 6: Oops! Bandrupt! **Day 1/1**
(GaDOE Acc Math I TE Spinner Learning Task 4 #'s 1-6, and 8)

Standard(s): MM1D1a. Students will determine the number of outcomes related to a given event.
a. Apply the addition and multiplication principles of counting.
MMID2a. Students will use the basic laws of probability.

a. Find the probabilities of mutually exclusive events.
b. Find the probabilities of dependent events.
c. Calculate conditional probabilities.
d. Use expected value to predict outcome

New vocabulary:

Mathematical concepts/topics: probability, independent events, dependent events, multiplication principle of counting, set notation, conditional probability, center (mean or median), shape (symmetric, somewhat symmetric, or skewed), spread (range, mean deviation, or interquartile range), expected value

Prior knowledge: simple probability, independent events, dependent events, addition and multiplication principle of counting, set notation, symmetry, mean, median, range, interquartile range, measuring with a protractor, proportions

Essential question(s): How can I find the probability of spinning a certain amount when the sectors are all different and appear to be unrelated to each other? How can I find the expected value of an event? How can simulations help me answer questions related to probability?

Suggested materials: post-it notes, protractors

Warm-up: Ask students to discuss the results of their homework (Questions 1-4 from GaDOE Acc Math I Spinner Task 3).

Opening: Have students use simulation to verify the results of their homework. (See GaDOE Acc Math I TE notes page 19.) Use the suggestion from GaDOE notes to form a histogram with student post-it notes. Use the results of the histogram for a discussion of center, shape, and spread.

Use the correct response to #1 to make sure students understand the basic concept of expected value

Worktime: Students should complete problems 1 – 7 of the task. In problem 7, students should explore simulations.

Closing: Allow students to share their work. (See GaDOE Acc Math I teacher notes.)

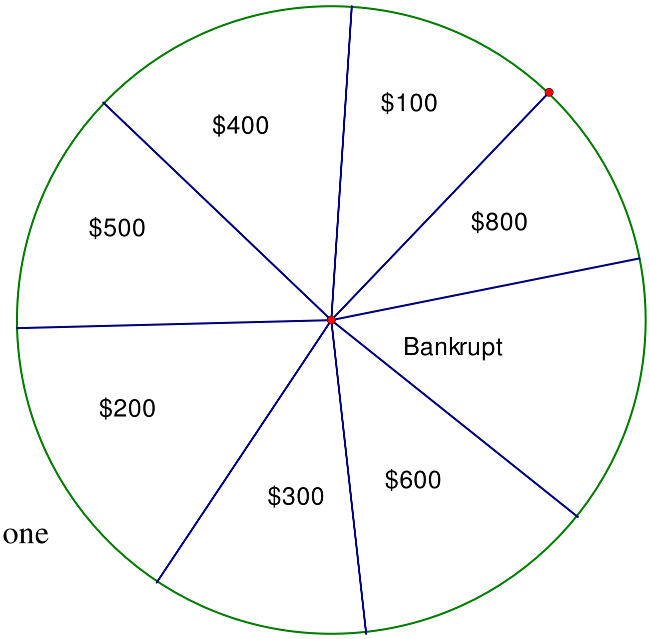
Homework:

Differentiated support/enrichment:

Check for Understanding: The GaDOE Spinner Task 5 from the Acc Math I Framework is included for use as a possible assessment. See the GaDOE teacher notes for assessment evaluation ideas.

Resources/materials for Math Support: Students should continue to preview vocabulary and prior knowledge identified in this and the previous lesson

Mathematics I
Task 6: Spinner Task 4
Student Task



This spinner is different than the spinners from previous tasks because the sectors have different areas and one sector is labeled 'Bankrupt'.

1) How can you determine the probability of landing on the individual amounts? Determine a method you could use and describe it.

2) Calculate the following probabilities given just one spin:

   a. P($100) =
   b. P($200) =
   c. P($300) =
   d. P($400) =
   e. P($500) =
   f. P($600) =
   g. P($800) =
   h. P(bankrupt) =

3) Are you just as likely to land on $100 as on $800?

4) Calculate the following based on one spin:

   a. P(at least $500) =
   b. P(less than $200) =
   c. P(at most $500) =

5) Calculate the following using two spins. But remember, if you land on Bankrupt you lose your second spin.

   a. P(a sum of $200) =

   b. P(a sum of at most $200) =

   c. P(a sum of at least $1500) =

   d. P(a sum of at most $200 | you land on $100 on the first spin) =

   e. P(a sum of at least $1500 | you land on $800 on the first spin) =

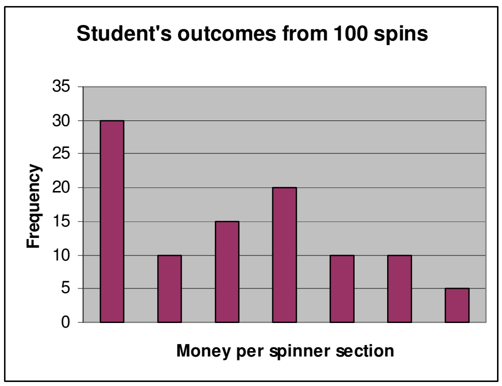
6) If you spin the spinner once, how much money, on average, would you expect to receive?

7) If you spin the spinner three times, how much money, on average, would you expect to receive?

Mathematics I
Spinner Assessment

A student created a spinner and records the outcomes of 100 spins in the table and bar graph below.

Amount of Money on each section of the spinner	\$0	\$100	\$200	\$300	\$400	\$500	\$600
Number of times the student lands on that section	30	10	15	20	10	10	5



1. Based on the table and graph above, calculate the experimental probabilities of landing on each section of the spinner. Use these probabilities to draw what the spinner would look like.

2. Calculate the average amount of money a person would expect to receive on each spin of the spinner.

3. Calculate P(at least \$400) on your first spin.

4. If you land on \$300 on the first spin, calculate P(sum of your first two spins is at least \$600).

5. Calculate P(sum of two spins is \$400 or less).

6. You propose a game using the spinner formed in #1:
   A person pays $350 to play. If the person lands on $0, $200, $400, or $600, then they get that amount of money and the game is over. If the person lands on $100, $300, $500, then they get to spin again, and they will receive the amount of money for the sum of the two spins.

   In the long run, would the player expect to win or lose money at this game? If the player played this game 100 times, how much would he/she be expected to win or lose?

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Mathematics I: Unit 4
The Chances of Winning

Task 7: Marbles, Cards and Surveys

Mathematics I
Task 7: Marbles, Cards and Surveys Task **Day 1/1**
(GaDOE Acc Math I TE Tasks between pages 26 and 28 of the Framework.)

Standard(s): MMID2 Students will use the basic laws of probability
b. Find the probabilities of dependent events.

New vocabulary: dependent events

Mathematical concepts/topics: probability, independent events, dependent events, conditional probability, set notation

Prior knowledge: probability, independent events, set notation

Essential question(s): How does not replacing objects from a set affect probability related to that set?

Suggested materials: marbles (various colors of cm cubes or other items may be substituted for marbles), decks of cards

Warm-up: Place three different color markers in a cup or bag. Tell the students what is in the cup or bag including the three different colors of the markers. Ask them what the probability would be for you to draw one of the colors. Actually draw a marker out and show it to the students. Without replacement, ask them to write on their paper what the probability would be of your drawing each of the original colors of markers. Once they have written these probabilities down, have them compare their response with a partner.

Opening: Allow time for whole group discussion concerning the results of the warm-up. If needed, ask guided questions to assure all students understand what is happening when there is no replacement. Include in the discussion and questioning reasons why this is called dependent events and compare them with the independent events already included in previous tasks within the unit.

Worktime: Ask students to complete the Marbles part of the task. Have small groups share out their responses prior to moving on to the Cards part of the task.

Closing: Allow students to share their work. (See GaDOE Acc Math I teacher notes.)

Homework:

Differentiated support/enrichment:

Check for Understanding:

Resources/materials for Math Support: Students should continue to preview vocabulary and prior knowledge identified in this and the previous lesson

Mathematics I
Task 7: Marbles, Cards, Surveys, and Dice Task
Day 1 Student Task

So far, we have looked at mostly independent events. For example, each spin of the spinner can be considered as independent because the outcome of the 2nd spin does not rely on the outcome of the first spin. The same can be said about rolling dice. Each roll is independent of each other.

Now we will look at calculating dependent probabilities. When you draw a sample and do not replace it the probabilities change. For example, suppose you have a deck of 52 cards. The probability of drawing a queen is 4/52. Suppose you drew a queen but did not replace the card. What is the probability of drawing a queen now? It is 3/51. Note that this probability depends on the outcome of the first draw. Therefore, the two events are dependent.

Dependent events can occur in many situations. The key is distinguishing between dependent and independent events. Keep this in mind as you work though the next set of tasks.

Drawing Marbles

1. There are 21 marbles in a bag. Seven are blue, seven are red, and seven are green. If a blue marble is drawn from the bag and not replaced, calculate the probability of drawing the following marbles on the second draw.
   a) P(blue)
   b) P(blue or green)
   c) P(not blue)

2. There are 21 marbles in a bag. Seven are blue, seven are red, and seven are green. If the marbles are not replaced once they are drawn, what is the probability of drawing the following sets of marbles in the given order?

   a) P(red, blue)

   b) P(red, blue, green)

   c) P(red or blue, green)

   d) P(red| first draw was a red marble)

Drawing Cards

Using a standard deck of 52 cards, 3 cards are dealt without replacement.

3. What is the probability that all three cards are queens?

4. If the first card is an Ace, what is the probability that the second card will not be an Ace?

5. If the first two cards are queens, what is the probability that you will be dealt three queens?

6. If two of the three cards are 8's, what is the probability that the other card is not an 8?

7. How do the probabilities calculated above change if each card is replaced in the deck (and the deck is well shuffled) after being dealt?

8. What is the probability of being dealt 3 of a kind? (all 8's or all 2's, etc) Is this an independent or dependent event?

9. What is the probability of getting 3 cards from the same suit? Is this an independent or dependent event?

10. What is the probability of getting a run of 3, 4, 5? Is this an independent or dependent event?

Student Survey

Probabilities can also be calculated from survey data. A student conducted a survey with a randomly selected group of students. She asked freshmen, sophomores, juniors, and seniors to tell her whether or not they liked the school cafeteria food.
The results were as follows:

	Freshmen	Sophomores	Juniors	Seniors
Liked food	85	50	77	82
Did not like food	44	92	56	78

11. Use the table above to answer the following questions.

a) What is the probability that a randomly selected student is a freshman?

b) What is the probability that a randomly selected student likes the food?

c) What is the probability that randomly selected student is a freshman and likes the food?

d) If the randomly selected student likes the food, what is the probability that he/she is a freshman?

e) Are the events "freshman" and "likes food" independent or dependent?

f) What conclusions might you draw from this survey data?

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Mathematics I: Unit 4
The Chances of Winning

Task 8: The Supermarket

Mathematics I
Task 8: The Supermarket **Day 1/2**

Standard(s): MA1D1 Students will determine the number of outcomes related to a given event.

a. Apply the addition and multiplication principles of counting
b. Calculate and use simple permutations and combinations

MA1D2. Students will use the basic laws of probabilities.

a. Find the probabilities of mutually exclusive events

New vocabulary: permutation, factorial

Mathematical concepts/topics: counting principle of multiplication, factorial notation, number of permutations on n objects, number of permutations of n objects taken r at the time, notation for permutations, number of permutations of repeated objects, circular permutations, probability involving permutations

Prior knowledge: counting principle of multiplication

Essential question(s): How can I determine the number of possible arrangements of *n* objects taken *r* at the time?

Suggested materials: colored tiles, graphing calculator

Warm-up: Have students read and answer *Problem 1* of the task.

Opening: Allow students to share their responses to the warm-up. Focus on sketches or illustrations representing the possible arrangements.

Read *Problem 2* aloud. Give students time to work on this problem and then discuss their answers again focusing on sketches or models that allow students to "see" all of the possible arrangements.

Worktime: Students should complete problems 3 – 11 of the task in class. Problems 12 – 14 may be assigned as homework.

In working through this task, students should be given time to develop the understandings needed to derive, on their own, the formula for the number of permutations of *n* objects taken *r* at the time.

Closing: Allow students to share their work. Discuss *Problem 6* thoroughly.

Homework: Students should complete *Problems 12 – 14* of the task.

Differentiated support/enrichment:

Check for Understanding:

Resources/materials for Math Support: Students should continue to preview vocabulary and prior knowledge identified in this and the previous lesson. Use of colored tiles or other manipulatives is important in helping students visualize numbers of possible arrangements.

Mathematics I
Task 8: The Supermarket
Day 1 Student Task

Jack and Jill have part-time jobs at a supermarket. Today, Jack is working on a display in the bakery. He is arranging containers of cookies for a sale. The sale includes chocolate chip, oatmeal, sugar, and peanut butter cookies.

1. Jack is arranging the cookies on a shelf. How many arrangements can he make? Illustrate your answer with a sketch.

2. Jill comes by to see what Jack has done. She thinks that his display looks too crowded and suggests that he use only three of the four types. How many arrangements can Jack make using three types of the four types of cookies?

3. Jack's supervisor decides to add macaroons and brownies to the sale. How many arrangements of 3 types of cookies can Jack make if he select from 5 types? When he can select from 6 types?

4. Arrangements, like these, are called **permutations.** Numbers of possible arrangements are represented by nPr where n= the number of items in the set and r = the number of items being considered. Notation for the number of arrangements in *Question 1* is ${}_4P_4$ and notation for the number of arrangements in Q*uestion 2* is ${}_4P_3$. What is the notation for the number of arrangements in *Question 3*?

5. Based on your experience above, explain what ${}_5P_5$ means and evaluate it. Explain and evaluate ${}_5P_4$. Evaluate ${}_5P_3$, ${}_5P_2$, and ${}_5P_1$.

6. Make and test a conjecture for calculating the number of arrangements when there are n items in a set and r items are being considered.

7. Mathematicians have a shortcut for indicating that consecutive numbers are to be multiplied. The shortcut is called **factorial** and its notation is *!*. For example, 5! means *5x4x3x2x1*.
   a. What does *8!* mean?

   b. Notice that $nPn = n!$ How can you use factorial notation to represent nPr? [Hint: ${}_6P_2$ = 6x5 = 6!/4!]

8. Jack has 6 types of cookies and he is planning to display 3 on one shelf.
   a. What is the probability that Jack picks oatmeal, macaroon, and brownie in that order?
   b. What is the probability that he picks chocolate chip as one of the three types?

9. Jack is preparing to replenish the sale display. The baker just gave him 1 container of chocolate chip, 1 container of oatmeal and 1 container of sugar cookies. Jack has to place the containers on the shelf one at a time. How many different arrangements could he make using the 3 containers of cookies?

10. Looking closer, Jack realizes that that he has 2 identical containers of chocolate chip cookies, and 1 container of sugar cookies. List the number of different arrangements Jack could use to place these cookies. How does your list compare with your answer in *Question 1*? Explain.

11. Suppose the baker gave Jack 2 containers of chocolate chip, 1 container of oatmeal, and 1 container of sugar cookies. How many different arrangements could he use to place these containers?
12. Complete the table below.

Total number of containers	Number of arrangements if all containers are different	Number of identical containers	Number of arrangements with identical containers	Factorial notation for the number of arrangements with identical containers
3		2 chocolate chip		
4	4!	2 chocolate chip	12	
4		2 chocolate chip 2 sugar	6	$\frac{4!}{2!\,2!}$
4		3 chocolate chip		
5		4 chocolate chip		
5		3 chocolate chip	20	
5		3 chocolate chip 2 sugar		
n	n!	q		
n		q, p		

13. From the table, we see that the number of distinguishable permutations of n objects where one object is repeated q times and another object is repeated p times is $\frac{n!}{?}$.

14. The floral department wants to display 3 identical houseplants, 2 identical flower bouquets, and 2 identical dish gardens. How many distinguishable displays are possible?

Mathematics I
Task 8: The Supermarket **Day 2/2**

Standard(s): MA1D1 Students will determine the number of outcomes related to a given event.

c. Apply the addition and multiplication principles of counting
d. Calculate and use simple permutations and combinations

MA1D2. Students will use the basic laws of probabilities.

b. Find the probabilities of mutually exclusive events

New vocabulary: combination

Mathematical concepts/topics: circular permutations, number of combinations of n objects, number of combinations of n objects taken r at the time, notation for combinations, probability involving combinations

Prior knowledge: counting principle of multiplication

Essential question(s): How can I determine the number of possible combinations of n objects taken r at the time?

Suggested materials: colored tiles, graphing calculator

Warm-up: Have students work with a partner to compare Problems 15 – 17 assigned as homework following the previous lesson.

Opening: Discuss the homework.

Worktime: Students should complete problems 18 - 33 of the task in class. Problems 34 and 35 may be assigned as homework.

In working through this task, students should be given time to develop the understandings needed to derive, on their own, the formula for the number of combinations of n objects taken r at the time. Monitor student work carefully asking guiding questions that will help students answer the questions posed.

Have a whole class discussion after *Problems 24* to insure that students understand the idea of circular permutations before moving on to combinations.

Closing: Allow students to share their work. Discuss *Problem 33* thoroughly.

Homework: Students should complete *Problems 34* and *35* of the task.

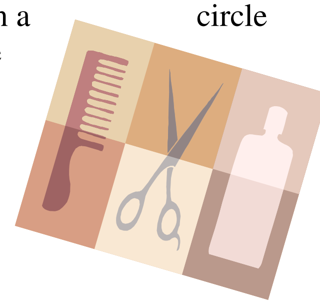
Differentiated support/enrichment:

Check for Understanding:

Resources/materials for Math Support: Students should continue to preview vocabulary and prior knowledge identified in this and the previous lesson. Use of colored tiles or other manipulatives is important in helping students visualize numbers of possible arrangements and combinations.

Mathematics I
Task 8: The Supermarket
Day 2 Student Task

The supermarket has round display tables in the pharmacy. Jill has been asked to make a display of Oil of Okay beauty products: shampoo, rinse, mousse, and lotion.



15. Use the letters *S*, *R*, *M*, and *L* to make as many different arrangements in a circle as you can. Compare with members of the class to make sure you have all possible arrangements and no repeats. How many arrangements can Jill make of the four items in a circle?

16. How many arrangements could Jill make if she just placed the four products in a row? How does this number compare with your results from *Question 1*?

17. Jack suggests that Jill also include Oil of Okay shower gel (*G*) in the display. How many arrangements could Jill make of the five items in a row? How many can she make if she places them in a circle?

18. Oil of Okay also makes sunscreen (*U*). Jill's supervisor thinks that six items could be arranged in a circle in *5!* or *120* ways. Is she correct? Without drawing or making 120 arrangements, support your answer.

19. Based on the examples in *Questions 1 - 4*, make a conjecture about the number of circular arrangements you can make with *n* items.

20. How would you change your formula in *Question 5* if the items are like keys on a ring or beads on a necklace, that is, the arrangement does not change if you turn the ring over?

21. If Jill is arranging five Oil of Okay products (*S*, *R*, *M*, *L*, and *G*) in a circle, what is the probability that the shampoo is between the conditioner and the mousse?

Before they leave the supermarket, Jack and Jill decide to buy a few items for dinner. They grab a basket and put in fried chicken, Cole slaw, baked beans, Hawaiian rolls, pears, cake, and iced tea.

22. Does the order that the food is placed in the basket matter? In other words, do Jack and Jill have the same items in their basket despite the order in which the items were selected?

23. The lines at the cashiers are quite long. Jill suggests that they put back some things so that they can use the express lane. The express lane allows 5 or fewer items. How many items must they choose to return? Does the order in which they choose the return items matter?

24. Subsets of a given set, like the possible sets of items chosen for return in *Question 2*, are called **combinations**. The number of subsets of size *r* that can be formed from *n* total items is represented nCr or $\binom{n}{r}$. What would be the notation for the number of combinations of return items in *Question 2*?

25. Explain ${}_4C_4$, ${}_4C_3$, ${}_4C_2$, and ${}_4C_1$, ${}_4C_0$.

26. Jack and Jill decided to leave the pears and the Cole slaw and make fruit salad when they arrived home. They have apples, bananas, cherries, and mangos. Below is a list of the fruit salads they can make if they "stack" or arrange the fruits.

ABCM	*BCMA*	*CMAB*	*MABC*
ABMC	*BCAM*	*CMBA*	*MACB*
AMBC	*BACM*	*CBMA*	*MCAB*
ACBM	*BMCA*	*CAMB*	*MBAC*
AMCB	*BAMC*	*CBAM*	*MCBA*
ACMB	*BMAC*	*CABM*	*MBCA*

How many salads can they make if they just mix all 4 ingredients together?

27. Below is a list of the fruit salads they can make if they "stack" or arrange 3 of the 4 fruits.

ABC	*BCM*	*CMA*	*MAB*
ABM	*BCA*	*CMB*	*MAC*
AMB	*BAC*	*CBM*	*MCA*
AMC	*BAM*	*CBA*	*MCB*
ACB	*BMC*	*CAM*	*MBA*
ACM	*BMA*	*CAB*	*MBC*

Draw rings around all the salads that have apples, bananas, and cherries. Draw a box around all the salads that have apples, bananas, and mangoes. Draw an X through all the salads that have bananas cherries, and mangoes. Draw a check next to all the salads that have apples, cherries and mangoes.

How many <u>different</u> combinations of ingredients do we have when order does not matter?

28. Below is a list of the fruit salads they can make if they "stack" or arrange 2 of the 4 fruits.

AB	*BC*	*CM*	*MA*
AM	*BA*	*CB*	*MC*
AC	*BM*	*CA*	*MB*

Draw rings around all the salads that have apples and bananas. Draw a box around all the salads that have apples and mangoes. Draw an X through all the salads that have cherries and mangoes. Draw a check next to all the salads that have apples and cherries. Draw a + next to all the salads that have bananas and cherries. Draw a @ next to all the salads that have bananas and mangoes. How many <u>different</u> combinations of ingredients do we have when <u>order does not matter</u>?

29. Use *Questions 5 – 7* to help you do the following:
    a. Explain what ${}_4C_4$ means. Evaluate ${}_4C_4$.
    b. Explain what ${}_4C_3$ means?
    c. Evaluate ${}_4C_2$.
    d. Evaluate ${}_4C_1$. Explain why your answer makes sense. Illustrate your explanation with a sketch.

30. To figure out how many **combinations** of fruit salad we have, we started with **permutations** and then eliminated the arrangements that had the same ingredients. Mathematically, we can represent these steps ${}_4C_3 = \frac{{}_4P_3}{6} = \frac{4!/(4-3)!}{3!}$

$${}_4C_2 = \frac{{}_4P_2}{2} = \frac{4!/(4-2)!}{2!} \qquad {}_4C_1 = \frac{{}_4P_1}{1} = \frac{4!/(4-1)!}{1!}$$

Using these examples, write a general formula for ${}_nC_r$..
(Note: To eliminate the salads with the same ingredients we divided by *r*! because *r* objects can be arranged in *r*! ways.)

31. Suppose that Jack and Jill want to invite 4 of their 5 friends to join them for dinner. In how many different ways can 4 friends be invited?

32. Their friends decide to surprise Jack and Jill by bringing dinner to them. How many ways could they order 7 different dishes from a menu of 20 entrees?

33. To figure out how many **combinations** of fruit salad we have, we started with **permutations** and then eliminated the arrangements that had the same ingredients. Mathematically, we can represent these steps ${}_4C_3 = \frac{{}_4P_3}{6} = \frac{4!/(4-3)!}{3!}$

$${}_4C_2 = \frac{{}_4P_2}{2} = \frac{4!/(4-2)!}{2!} \qquad {}_4C_1 = \frac{{}_4P_1}{1} = \frac{4!/(4-1)!}{1!}$$

Using these examples, write a general formula for ${}_nC_r$..
(Note: To eliminate the salads with the same ingredients we divided by *r*! because *r* objects can be arranged in *r*! ways.)

34. Suppose that Jack and Jill want to invite 4 of their 5 friends to join them for dinner. In how many different ways can 4 friends be invited?

35. Their friends decide to surprise Jack and Jill by bringing dinner to them. How many ways could they order 7 different dishes from a menu of 20 entrees?

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Mathematics I: Unit 4
The Chances of Winning

Task 9: What's the Probability of a Hit?

Mathematics I
Task 9: What's the Probability of a Hit? Day 1/1
(This task can be found on pages 40-42 and pages 94-97 of *Navigating through Probability in Grades 9-12* published by NCTM. The publication can be ordered from the NCTM website.)

Standard(s): MM1D1. Students will determine the number of outcomes related to a given event.
b. Apply the addition and multiplication principles of counting.
c. Calculate and use simple permutations and combinations
MMID2. Students will use the basic laws of probability.
a. Find the probabilities of mutually exclusive events.

New vocabulary: binomial probability, binomial experiment, binomial distribution, relative frequencies

Mathematical concepts/topics: simulation of a binomial distribution, examining binomial probability through simulation and though an analysis of outcomes (sample spaces), comparing experimental and theoretical probability, counting using permutations of repeated elements, addition and multiplication principles

Prior knowledge: relative frequencies, addition and multiplication principles, sample space

Essential question(s): How can I determine the probability of a baseball player getting a hit or not getting a hit if I know his current batting average?

Suggested materials: graphing calculator, activity sheets

Warm-up: Post the following:

*In how many ways can the letters in the word **probability** be arranged?*

Opening: Discuss the warm-up making sure that students understand how repeated elements affect the number of permutations of a given set of objects. This will be important in helping students determine the number of elements in the sample spaces used to find binomial probabilities.

Ask students to read the first two sentences of the task carefully and then to complete the first question (parts a-c) with a partner.

When most of the students have completed *Problem 1 a - c*, have a whole class discussion of this problem. Before moving on, all students should understand why the assignment of digits described in Problem 1 represents the player's batting average, how to generate an appropriate set of random digits using the graphing calculator, and how to translate the sequence of digits from the calculator into a sequence of "hits" (H) or "no hits" (N).

Worktime: (See content notes following the closing).
Students should continue to work on the task. Monitor student work carefully to be sure students understand the questions being asked and are correctly applying previously learned concepts.

Have a class discussion of student work after problem 4 and again after problem 6.

Closing: Allow students to share their work for parts 7 and 8.

It is important for students to be able to generalize the ideas they have learned to this point. Ask them how they might find the probability of exactly 3 hits out of four times at bat, 2 hits out of 5 times at bat, etc. See content notes below.

Content notes on binomial probability:
Every binomial probability situation (often referred to as Bernoulli Trials) has two common characteristics:

- There are a certain number *n* of repeated, independent trials.
- For any single trial, only two outcomes are possible-"success" or "not success"-in our case, "hit" or "no hit". If the probability of success is *p*, the probability of "not success" is *p-1*.

The formula for binomial probabilities is daunting. Rather than giving students this formula, we want them to develop the understandings illustrated in the following discussion about our batter.

The batter's constant batting average is given to be 0.4. Suppose we want to know the probability of his getting exactly 3 hits in his next five at bats. There are many possibilities for his next 5 at bats that include exactly 3 hits. With *H* standing for a "hit", and *N* standing for "no hit", his 5 at bats could include *HHHNN*, *HNHNH*, *NHNHH*… As a matter of fact, based on permutations of a set of objects with repeated elements, there are $\frac{5!}{3!2!}$ ways our batter could get exactly three hits. The probability of each of these possibilities is (0.6)(0.6)(0.6)(0.4)(0.4) or $(0.6^3)(0.4^2)$, based on the multiplication principle. So the theoretical probability of our player getting exactly three hits is $\frac{5!}{3!2!}(0.6)^3(0.4)^2$.

Students are much more likely to understand binomial probability using these ideas than by being given the formula $nCr\ p^r(p-1)^{(n-r)}$. The formula is certainly appropriate after students have developed the concepts.)

Homework: See the homework following the student task.

Differentiated support/enrichment: The CD included with *Navigating through Probability in Grades 9-12* includes several binomial activities developed by Shaughnessy and ARcidiacoo (1993).

Check for Understanding:

Resources/materials for Math Support: Students should continue to preview vocabulary and prior knowledge identified in this lesson. *The Supermarket Revisited* included in this document as an alternate task for *What's the Probability of a Hit* may be used in Math Support to help students develop concepts related to binomial probability.

Mathematics I
Task 9: What's the Probability of a Hit?
Student Task

1. Read the opening paragraph of the task carefully and complete part 1a-c of the task. Be prepared to share your thinking.

2. Continue to work on the task. Your teacher will let you know when it is time to discuss part 3c, part 6, and part 8 of the task with your classmates.

In problem 3 below will help you generalize what you have learned in this task to other situations.

3. Suppose you roll a fair die.
   a. What is the probability you will land on a 6?
   b. What is the probability you do not land on a 6?
   c. Suppose you roll the die 4 times. What is the probability that you land on a 6 exactly 3 of the four rolls?

Mathematics I
What's the Probability of a Hit
Homework

1. Chipper Jones currently has 160 hits for 440 at bats.
    a. What is his current batting average?
    b. What is the probability that he will get exactly 3 hits in his next 5 at bats?
    c. What is the probability that he will get at least 4 hits in his next five at bats?

2. Your teacher gives you a multiple choice pre-test with 6 questions. There are four choices for each question. You have never seen the material and you have to guess at every question.
    a. What is the probability that you will get any given question right?
    b. What is the probability that you will get exactly two questions right?
    c. What is the probability that you will make 100%?

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Mathematics I: Unit 4
The Chances of Winning

Applications of Probability

Applications of Probability

1. Cancer Diagnosis

   A patient is tested for cancer. This type of cancer occurs in 5% of the population. The patient has undergone testing that is 90% accurate and the results came back positive. What is the probability that the patient actually has cancer?

   (Hint: The question asks for P(cancer given the test is positive).)

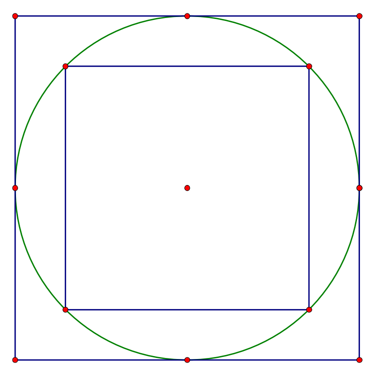
   Work in small groups to solve this problem.

2. Parachuting Safely

   A parachutist jumps from an airplane and lands in a square field that is 0.5 miles on each side. In each corner of the field there is a large tree. The parachutist's rope will get tangled in the tree if she lands within 1/10 of a mile of its trunk. What is the probability she will land in the field without getting tangled in the tree?

Dart Games

3. A dart board is created by inscribing a circle in a square with side lengths of 2 units. A smaller square is inscribed in the circle.



   1. If you throw a dart at the board, and it lands in the large square, what is the probability that it lands in the circle?
   2. If you throw a dart at the board, and it lands in the large square, what is the probability that it lands in the small square.
   3. If you throw a dart at the board and it lands in the circle, what is the probability that it does not land in the small square?

4. What if a simple square dartboard was placed on a coordinate grid with boundaries $-1 \le x \le 1$ and $0 \le y \le 3$. Let x be the x-coordinate and y be the y-coordinate of the location of a dart that lands on the board. Find the following:

   a) P(y >1 and x >0)

   b) P(y >2 or x >0)

   c) P($y > x$)

   d) P(y >2 given $y > x$)

5. The bull's eye of a standard dartboard has a radius of about 1 inch. The inner circle has a radius of 5 inches, and the outer circle has a radius of 9 inches.

   a) Assuming a thrown dart hits the board, what is the probability it lands on the bull's eye?

   b) Assuming a thrown dart hits the board , what is the it lands between the inner and outer rings?

6. Imagine you are playing darts and you hit the bull's eye 6 out of 20 times.

   a) What is the probability that you hit the bull's eye with your first shot?

   b) What is the probability that you hit the bull's eye with your 21st dart?

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Mathematics I: Unit 4
The Chances of Winning

Resources

TI-83/TI-84 Calculator Commands
from the website: **http://w3.salemstate.edu/~jbelock/MAT%20247/TIcommands.htm**

To enter data into a list:

Press STAT > EDIT, then type all the values in your data set into a single column.

To clear a list:

Go to the spreadsheet by pressing STAT > EDIT. Use the up arrow to place the cursor on the name of the list you want to erase. Press CLEAR, then arrow back down into the list.

To find descriptive statistics:

First, enter the data into a list. Quit to the home screen. Then press STAT > CALC 1:1-var stats and hit ENTER. The command is pasted onto the home screen, and enter the name of the list where your data is.

To make a graphical display:

All the statistical graphs are under 2nd STATPLOT. You must turn on the graph you want, select the appropriate type of graph and enter the correct list. Then press either GRAPH or ZOOM 9: Zoom stat to see your graph.

What if you get an error? Check that the graph is turned on, make sure no other graphs are on (including on the Y = screen), make sure you've entered the correct list, in the case of a scatter plot make sure both the X and Y lists have the same number of values, try hitting ZOOM 9 because sometimes it just needs to be recentered.

To find values from the Normal Distribution:

1. If you know an interval and are looking for area (percentage), use 2nd DISTR 2:normalcdf and hit ENTER. This pastes it onto the home screen. You must then enter as follows:

Normalcdf(*lowerbound*,*upperbound*,μ,σ) then hit ENTER

Remember that μ is the mean of the normal distribution and σ is its standard deviation.

2. 2. If you know the area and are looking for values along the horizontal (i.e. percentiles) use 2nd DISTR 3:invNorm and hit ENTER. This pastes it to the home screen. You must then enter as follows:

InvNorm(*Area to the left of the value you have,*μ,σ) then hit ENTER.

To find the correlation coefficient:

First, your calculator must be set up to display the correlation. (You only have to set it up once, so if you've done it in class, skip this part. Sometimes if you change batteries you have to do it again.) Hit 2^{nd} CATALOG (this is over the 0 button). Go down to DiagnosticOn, hit ENTER then ENTER again. It is now set up to display correlation with the regression line.

Enter the X values in one list and the Y values in another. Go to STAT>CALC 8:LinReg (a+bx) and hit ENTER. It is now pasted to the home screen. You must input the names of the list containing the X values followed by a comma then the list containing the Y values. For example, if my X values are in L1 and Y values are in L2, I would enter

LinReg(a+bx) L1,L2

TI-83/TI-84 Calculator Commands

from the website: http://pasles.org/Ti83.html

Calculating mean and standard deviation on the TI-83:

Let's say we have a table of data describing four birds of different species (units of height and weight have been left out here):

height	weight
2	5
3	6
1	5
4	5

You can think of the two columns as representing variables *x* and *y*.

First we have to enter the data. Hit the STAT button and you will see the options EDIT, CALC and TESTS atop the screen. Use the left and right arrows (if necessary) to move the cursor to EDIT, then select 1:Edit...

Now you will see a table with the headings L_1 and L_2. Enter the x values under L_1, the y values under L_2. (If you want to clear pre-existing data first, move the cursor to the top of the column, hit CLEAR and then ENTER.)

(If there were only a single variable, we could enter the data as x-values and leave the 2^{nd} column blank.)

Once all the data is entered, go back to the STAT menu, but this time move the cursor to CALC instead of EDIT. If you can't find your way there, remember: Every TI graphing calculator is equipped with CLEAR, QUIT and/or EXIT commands for getting back out of tough situations.

Once you're in the CALC menu, select *2-Var Stats*. (If we had only entered a single column of data, *1-Var Stats* would be the appropriate choice instead.) Then hit ENTER.

The calculator will display the x-mean (= 2.5), some other stuff, and then the standard deviation (s_x=1.29). Note that s_x is what we called s in class; the calculator refers to it as s_x so we know that this is the standard deviation of the variable x and not that of y (which will be denoted by s_y). This is followed by something called σιγμα $_x$ (which is what you would get as standard deviation if you had used n instead of $n-1$), and finally the sample size (there are $n = 4$ observations). Use the down arrow to get the corresponding information about y. (Its mean is 5.25, and its standard deviation is s_y =.5).

Now go back to the STAT menu and select CALC, 1-Var stats. See what happens: we get the mean and standard deviation of x, and also its five-number summary! If you want the five-number summary for y, try STAT, CALC, 1-Var stats and then type L_2 (which is the 2^{nd} function on the 2 key) before hitting ENTER.

Valuable Resources for the Statistics Teacher

Books

- Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A Pre-K-12 Curriculum Framework; Franklin, Christine, Gary Kader, etc; American Statistical Association. 2007.

  This report may also be assessed online: http://www.amstat.org/education/gaise

- Navigating through Data Analysis in Grades 9-12; Burrill, Gail, Christine Franklin, etc; NCTM Navigation Series, 2003.

- Navigating through Probability in Grades 9-12; Peck, Roxy, etc; NCTM Navigation Series, 2003

- Navigating through Data Analysis in Grades 6-8

- Navigating through Probability in Grades 6-8

- Thinking and Reasoning with Data and Change: 68th NCTM Yearbook; 2006.

- Quantitative Literacy Ser4ies; Pearson Learning

- Data Driven Mathematics; Pearson Learning

Websites

- Causeweb: Consortium for the Advancement of Statistics Education (great resource!) http://www.causeweb.org/

- Ten Websites Every Statistics Instructor Should Bookmark (by Robin Lock) http://it.stlawu.edu/%7Erlock/10sites.html

- ASA Section on Statistical Educatio – the website has links to excellent resources http://www.amstat.org/sections/educ/

- College GAISE report: This report is the next level beyond the Pre-K-12 GAISE report: http://www.amstat.org/education/gaise/

Teachers in Georgia

- The new Georgia Performance Standards: http://www.georgiastandards.org/math.asp

Journals

- *Mathematics Teacher* published by NCTM.
- *Mathematics Teaching in the Middle Schools* published by NCTM
- *Journal of Statistics Education* published by ASA. This is a free online journal. http://www.amstat.org/publications/jse/
- *Teaching Statistics*. An international journal. http://www.rsscse.org.uk/ts/